$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

1) Find the eigvl

$\cancel{(I-A)\lambda}$ $\quad Av = \lambda v \Rightarrow (A - I\cdot\lambda)v = 0$

$$\Rightarrow \left| \begin{pmatrix} -1-\lambda & 2 \\ 1 & -3-\lambda \end{pmatrix} \right| = 0$$

$$(-1-\lambda)(-3-\lambda) - 2\cdot 1 = 0 \Rightarrow \lambda = -2 \pm \sqrt{3}.$$

2) Find eigvs

$$(A - \lambda I)v = 0 \Rightarrow \left(A - (-2-\sqrt{3})I\right)\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -1-(-2-\sqrt{3}) & 2 \\ 1 & -3-(-2-\sqrt{3}) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} (1-\sqrt{3})v_1 + 2v_2 = 0 \\ v_1 + (-1-\sqrt{3})v_2 = 0 \end{cases} \Rightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1-\sqrt{3} \\ 1 \end{pmatrix}$$

2b) $\left(A - (-2+\sqrt{3})I\right)u = 0$

$$\begin{pmatrix} -1-(-2+\sqrt{3}) & 2 \\ 1 & -3-(-2+\sqrt{3}) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 1+\sqrt{3} \\ 1 \end{pmatrix}$$

3) $\lambda_1 = -2+\sqrt{3}$, $V_1 = \begin{pmatrix} 1+\sqrt{3} \\ 1 \end{pmatrix}$, $\lambda_2 = -2-\sqrt{3}$

$$u = \begin{pmatrix} 1-\sqrt{3} \\ 1 \end{pmatrix}$$

$$X = C_1 e^{\lambda_1 t} V + C_2 e^{\lambda_2 t} u$$

$$= C_1 e^{(-2+\sqrt{3})t} \begin{pmatrix} 1+\sqrt{3} \\ 1 \end{pmatrix}$$

$$+$$

$$C_2 e^{(-2-\sqrt{3})t} \begin{pmatrix} 1-\sqrt{3} \\ 1 \end{pmatrix}$$

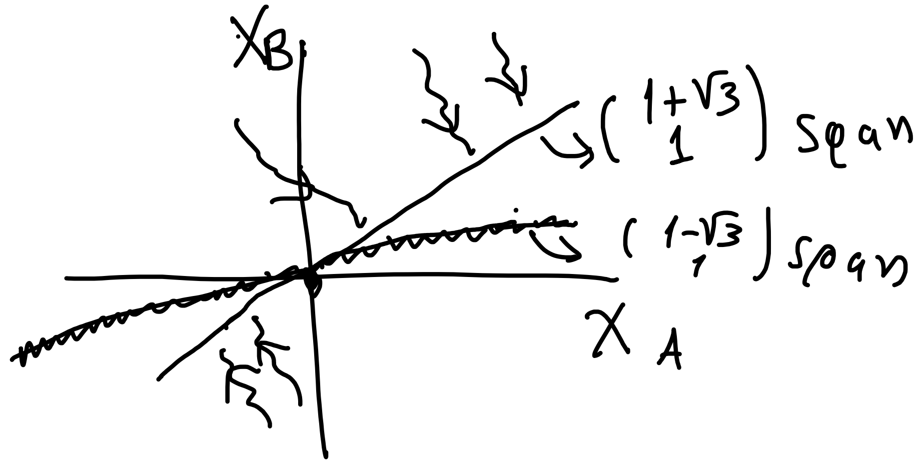
general s|n for

$$X' = \begin{pmatrix} -1 & 2 \\ 1 & -3 \end{pmatrix} X.$$

4) $e^{(-2+\sqrt{3})t}$ vs $e^{(-2-\sqrt{3})t}$

b/c $-2+\sqrt{3} > -2-\sqrt{3}$, the first term will dominate as $t \to +\infty$.

$$x \approx e^{(-2+\sqrt{3})t}\begin{pmatrix}1+\sqrt{3}\\ 1\end{pmatrix} = \varphi_1(t)$$



The $\varphi_1(t)$ solution is asymptotically Stable.

$$\frac{d}{dt}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-1 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$$

1) Find eigenvalues, eigenvectors and write the general solution

2) Which term dominates?

1) $\det\begin{pmatrix}-\lambda-1 & 0\\ 0 & -2-\lambda\end{pmatrix} = 0 \Rightarrow$

$$(-\lambda-1)\cdot(-2-\lambda) = 0$$
$$\Rightarrow \lambda_1 = -1,\ \lambda_2 = -2$$

Matrix is triangular $\Longrightarrow$
so the diagonal entries are the eigenvalues.

2) $\underline{\lambda_1 = -1}$

$$\begin{pmatrix} -1-(-1) & 0 \\ 0 & -2+1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow 0 \cdot v_1^{(0 \cdot 2)} + 0 \cdot v_2 = 0$$

$$\underset{0 \cdot 2}{0 \cdot v_1} - v_2 = 0$$

$$\Rightarrow v_2 = 0$$

v_1 = anything

$$v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

For $\lambda_2 = -2$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$X = C_1 e^{-t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + C_2 e^{-2t} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\approx e^{-t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{b/c} \quad -1 > -2$$

$\longrightarrow 0$ as t goes to infinity.