Week 3

Exercises marked (*) are harder and will not appear on quizzes. But variations of them may appear on the midterm. Exercises marked (**) are above the required level and will not appear on midterms or final exam. Most of the exercises are in the main textbook.

- **Autonomous equations**

  Identify which solutions are asymptotically stable/unstable.

  1. $\frac{dy}{dt} = ay + by^2, a > 0, b > 0, -\infty < y_0 < \infty$,
  2. (*)$\frac{dy}{dt} = y(y-1)(y-2), y_0 \geq 0,$
  3. (*) $\frac{dy}{dt} = e^y - 1, -\infty < y_0 < \infty$.

- **Real distinct roots**

  1. Find the solution and describe its asymptotic behaviour
     (a) $y'' + y' - 2y = 0, y(0) = 1, y'(0) = 1,$
     (b) $y'' + 3y' = 0, y(0) = -2, y'(0) = 3.$
  2. (*) Solve

     $$y'' - y' - 2y = 0, y(0) = a, y'(0) = 2$$

     and determine for which a, the solution goes to zero as $t \to +\infty$.