Week 4

- **Wronskian**

  1. Consider the equation $y'' - y' - 2y = 0$
     (a) Show that $y_1(t) := e^{-t}, y_2(t) := e^{2t}$ form a fundamental solution.
     (b) Show that each of $y_3(t) := -2e^{2t}, y_4(t) := y_1(t)+2y_2(t)$ and $y_5(t) := 2y_1(t)-2y_3(t)$ are solutions to the above ode.
     (c) Which of the following pairs give rise to a fundamental solution:
     $$\{y_1, y_3\}, \{y_2, y_3\}, \{y_1, y_4\}, \{y_4, y_5\}$$

- **Complex roots**

  1. Imagine a spring satisfying the following equations. Find the solution, do a rough sketch and describe its asymptotic behaviour (steady/growing/decaying oscilation). Compare this with the stability criterion.
     (a) $y'' + 4y = 0, y(0) = 0, y'(0) = 1$,
     (b) $y'' + 2y' + 2y = 0, y(\pi/4) = 2, y'(\pi/4) = -2$.

- **Repeated roots**

  1. Find the solution, do a rough sketch and describe its asymptotic behaviour. Compare this with the stability criterion.
     (a) $9y'' - 12y' + 4y = 0, y(0) = 2, y'(0) = -1$,
     (b) $y'' + 4y' + 4y = 0, y(-1) = 2, y'(-1) = 1$.
  2. Consider the problem
     $$y'' - y' + \frac{y}{4} = 0, y(0) = 2, y'(0) = b$$
     Find the solution and determine for which b, the solution remains positive for all $t > 0$. Compare this with the stability criterion.

- Let the demand and supply be
  $$D(P) = 9 - P + P' + 3P'' \text{ and } S(P) = -1 + 4P + 2P' + 5P''$$
  with $P(0) = 4, P'(0) = 4$

  1. Derive the price ode (see notes), and find the price solution.
  2. Does it have a globally stable solution as $t \to +\infty$? What does the stability criterion tell you?