Week 6

Exercises marked (*) are harder and will not appear on quizzes. But variations of them will appear on the midterm. Most of the exercises are in the main textbook.

- Find the general solution of the system. Describe the asymptotic behaviour (what is the dominating term and the limit). Draw the two eigenvectors spans and draw arrows towards the dominating term. Is it a saddle or a sink to the origin?

  1. $$\mathbf{x}' = \left(\begin{smallmatrix} 3 & -2 \\ 2 & -2 \end{smallmatrix}\right)\mathbf{x}.$$
  2. $$\mathbf{x}' = \left(\begin{smallmatrix} 1 & -2 \\ 3 & -4 \end{smallmatrix}\right)\mathbf{x}.$$

- (*) Find the particular solution of the system. Describe the asymptotic behaviour (what is the dominating term and the limit). Draw the two eigenvectors spans and draw arrows towards the dominating term. Is it a saddle or a sink to the origin?

  1. $$\mathbf{x}' = \left(\begin{smallmatrix} 5 & -1 \\ 3 & 1 \end{smallmatrix}\right)\mathbf{x}, \mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix},$$
  2. $$\mathbf{x}' = \left(\begin{smallmatrix} -2 & 1 \\ -5 & 4 \end{smallmatrix}\right)\mathbf{x}, \mathbf{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

- (*) For some $a \in [\frac{1}{2}, 2]$ consider the system

  $$\mathbf{x}' = \left(\begin{smallmatrix} -1 & -1 \\ -a & -1 \end{smallmatrix}\right)\mathbf{x}.$$

  Find the general solution in terms of a. Determine the asymptotic behaviour for $a = \frac{1}{2}$ and for 2, and find the $a_* \in [\frac{1}{2}, 2]$,called the bifurcation value, where the asymptotic behaviour changes.

- (*)The amounts of salt $x_1(t), x_2(t)$ in the two tanks satisfy the equations

  $$\frac{\mathrm{d}x_1}{\mathrm{dt}} = -k_1 x_1, \frac{\mathrm{d}x_2}{\mathrm{dt}} = k_1 x_1 - k_2 x_2 \text{ with } x_1(0) = 15, x_2(0) = 0,$$

  where $k_1 = \frac{r}{V_1} = \frac{1}{5}, k_2 = \frac{r}{V_2} = \frac{2}{5}$. Find the particular solution and determine the asymptotic behaviour. What does it tell you about the tank's salt concentration?

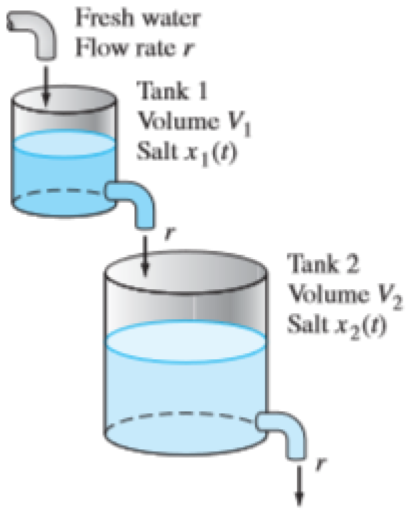

Figure 0.1: The two brine tanks.