Weeks 13

This is the last set of exercises. Variations of these exercises will appear on the final. Most of the exercises are in the main textbook.

Lyapunov Linearize around origin and also identify stability using Lyapunov:

- Consider the system:
$$\frac{\mathrm{d}x}{\mathrm{dt}} = y - 2x, \frac{\mathrm{d}y}{\mathrm{dt}} = 2x - y - x^3$$
with Lyapunov $V = (x+y)^2 + \frac{x^4}{2}$.
- Consider the linear harmonic oscilator:
$$\frac{\mathrm{d}x}{\mathrm{dt}} = y, \frac{\mathrm{d}y}{\mathrm{dt}} = -x - y^3(1+x^2).$$
with Lyapunov function $V = \frac{1}{2}x^2 + \frac{1}{2}y^2$.
- Consider the system:
$$\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix} \mathbf{x},$$
for Lyapunov function $V = \frac{1}{2}x^2 + \frac{1}{3}xy + \frac{1}{3}y^2$.
- Consider the system:
$$\frac{\mathrm{d}x}{\mathrm{dt}} = -x + 2y + y^4, \frac{\mathrm{d}y}{\mathrm{dt}} = 2y - 2x + x^4$$
for Lyapunov function $V = \frac{5}{2}x^2 - 3xy + \frac{7}{4}y^2 = \frac{5}{2}(x^2 - \frac{6}{5}xy + \frac{7}{10}y^2)$.

(Textbook problems 9.6 1-3) Construct a suitable Liapunov function of the form $ax^2 + cy^2$, where a and c are to be determined. Then show that the critical point at the origin is of the indicated type.

- show origin is asymptotically stable:
$$\frac{\mathrm{d}x}{\mathrm{dt}} = -x^3 + xy^2, \frac{\mathrm{d}y}{\mathrm{dt}} = -2x^2y - y^3.$$
- show origin is stable:
$$\frac{\mathrm{d}x}{\mathrm{dt}} = -x^3 + 2y^3, \frac{\mathrm{d}y}{\mathrm{dt}} = -2xy^2.$$
- show origin is unstable:
$$\frac{\mathrm{d}x}{\mathrm{dt}} = x^3 - y^3, \frac{\mathrm{d}y}{\mathrm{dt}} = 2xy^2 + 4x^2y + 2y^3.$$