Week 12

Variations of these exercises will appear on the final. Most of the exercises are in the main textbook.

Nonhomogeneous: Variation of parameters

- Obtain the general solution for the following systems and identify the dominating term:

  1. $$\mathbf{x}' = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix}\mathbf{x} + \begin{pmatrix} t \\ 0 \end{pmatrix}.$$

  2. $$\mathbf{x}' = \begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} ln(t) \\ t \end{pmatrix}.$$

  3. $$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 1 & -3 \end{pmatrix}\mathbf{x} + \begin{pmatrix} e^{-2t} \\ e^{-2t} \end{pmatrix}.$$

Laplace transform

- Use the Laplace transform to solve the following 2nd-order equations (chapter 6.2):

  1. $y'' - 2y' + 2y = 0, y(0) = 0, y'(0) = 1.$
  2. $y'' + 4y = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}, y(0) = 0, y'(0) = 0.$
  3. $y'' + y = \begin{cases} t, & 0 \le t < 1 \\ 2 - t, & 1 \le t < 2 \\ 1, & t \ge 1 \end{cases}, y(0) = 0, y'(0) = 0.$

- Use the Laplace transform to solve the following systems (assume $\mathbf{x}(0) = 0$):

  1. system with repeated eigenvalue:

     $$\mathbf{x}' = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix}\mathbf{x} + \begin{pmatrix} t \\ 0 \end{pmatrix}.$$

     Compare your answer with the variation of parameters above.
  2. system with piecewise forcing:

     $$\frac{\mathrm{d}\mathbf{x}}{\mathrm{dt}} = \begin{pmatrix} -2 & 1 \\ 0 & -1 \end{pmatrix}\mathbf{x} + \begin{pmatrix} f_{piece}(t) \\ 0 \end{pmatrix},$$

     where $f_{piece} := \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$
  3. for $y'' + 4y = f_{step}$ we use the transformation $x_1 = y, x_2 = y'$ to obtain the 2d system:

     $$\frac{\mathrm{d}\mathbf{x}}{\mathrm{dt}} = \begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ f_{step}(t) \end{pmatrix}.$$

     Compare your answer with $y'' + 4y = f_{step}$.