Weeks 10

Variations of these exercises will appear on the final. Most of the exercises are in the main textbook.

Nonhomogeneous: diagonalization (*)Find the general solution for the following systems and limiting behaviour (which one is the dominating term or what type of spiral is it and centered around what?):

1. $$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} e^t \\ t \end{pmatrix}.$$

2. $$\mathbf{x}' = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t.$$

3. $$\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{x} + \begin{pmatrix} t \\ 0 \end{pmatrix}.$$

4. $$\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 8sin(t) \\ 0 \end{pmatrix}.$$

Autonomous systems and stability

- Stability classification
  1. Find the critical points.
  2. From the plots below determine whether each critical point is asymptotically stable, stable, or unstable. Is it also spiral sink, source or concentric circles?
     The blue dots denote the initial position of the solution.

  ☐ Consider the system

$$\frac{\mathrm{d}x}{\mathrm{dt}} = (2-x)(y-x), \frac{\mathrm{d}y}{\mathrm{dt}} = y(2-x-x^2).$$

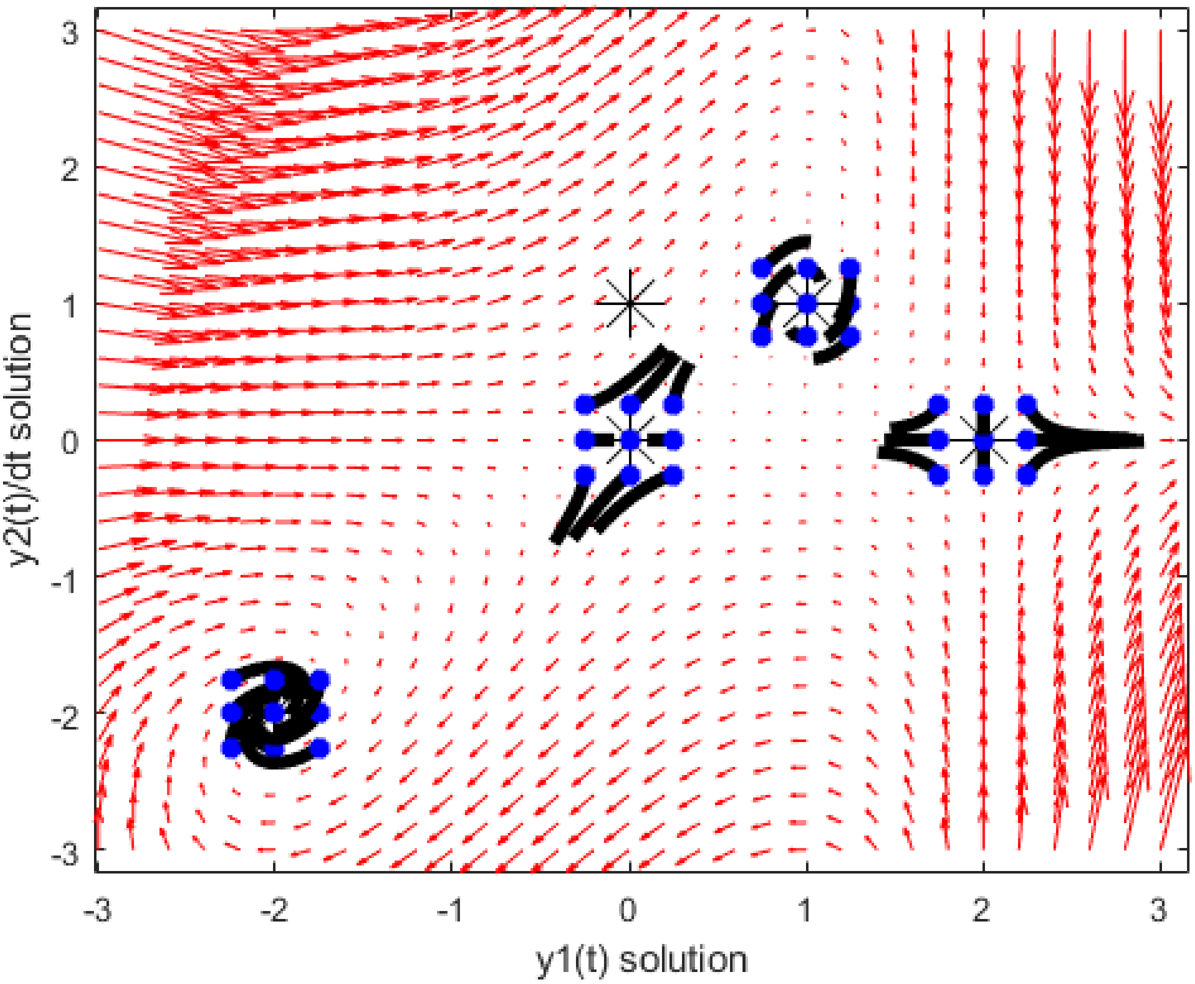

Figure 0.1: Global behaviour

- (*)Find an equation of the form $H(x, y) = c$ satisfied by the equations. Do a nullcline analysis around the critical point (i.e. looking at all possible cases on the sign of the pair $(\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t})))$. Draw your conclusions around the critical point.

  - ☐ Consider the system
  $$\frac{\mathrm{d}x}{\mathrm{d}t} = 2y, \frac{\mathrm{d}y}{\mathrm{d}t} = -8x.$$
  - ☐ Consider the system
  $$\frac{\mathrm{d}x}{\mathrm{d}t} = x, \frac{\mathrm{d}y}{\mathrm{d}t} = -5y + x^3.$$
  - ☐ Consider the system
  $$\frac{\mathrm{d}x}{\mathrm{d}t} = -x + xy, \frac{\mathrm{d}y}{\mathrm{d}t} = -y + y^2/2.$$
  - ☐ Consider the system
  $$\frac{\mathrm{d}x}{\mathrm{d}t} = y, \frac{\mathrm{d}y}{\mathrm{d}t} = -sin(x)$$
  and draw only around the origin.