Week 9

Variations of these exercises will appear on the final. Most of the exercises are in the main textbook.

Complex eigenvalues

- Find the general solution of the system. Describe the asymptotic behaviour. Are the trajectories forming a spiral source, a spiral sink or concentric circles?

  1. $$\mathbf{x}' = \begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix} \mathbf{x}.$$
  2. $$\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 1 & 1 \end{pmatrix} \mathbf{x}.$$
  3. $$\mathbf{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \mathbf{x}.$$

- (*) Find the general solution of the system. Find the bifurcation value or values of α where the qualitative nature of the phase portrait for the system changes. Draw a phase portrait for a value of α slightly below, and for another value slightly above, each bifurcation value.

$$\mathbf{x}' = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} \mathbf{x}.$$

- (*)Consider the circuit

$$\frac{\mathrm{d}}{\mathrm{dt}} \begin{pmatrix} I \\ V \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{8} \\ 2 & \frac{-1}{2} \end{pmatrix} \begin{pmatrix} I \\ V \end{pmatrix}.$$

  Solve and determine long term behaviour. Is it asymptotically stable?

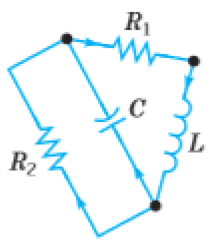

Figure 0.1: The circuit with complex eigenvalues.

Repeated eigenvalues

- Find the general solution of the system. Describe the asymptotic behaviour. Are the trajectories forming a source or sink behaviour wrt the origin?

  1. $$\mathbf{x}' = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x}.$$
  2. $$\mathbf{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \mathbf{x}.$$
  3. Find the particular solution and determine the asymptotic behaviour as above:

$$\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}, \mathbf{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$