ECO 2901
EMPIRICAL INDUSTRIAL ORGANIZATION

Lecture 6:
Market entry and spatial competition

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Lecture 6: Models of Firms' Spatial Location: Outline

1. Introduction

2. Models with single-store (product) firms

3. Models with multiple-store (product) firms

1. Firms' Spatial Location Introduction

Firms' Spatial Location: Introduction

- Consider the decision of a retail firm (e.g., coffee shop, restaurant, supermarket, apparel) of where to open a new store within a city.
- Different factors can play an important role:
  - Demand: what is the consumer traffic at different locations;
  - Rental prices
  - Location of competitors
- Geographic distance can be an important source of product differentiation. Ceteris paribus, a firm's profit increases with its distance to competitors.

Space: Beyond geographic location of stores

- Models for the geographic location of stores can be applied to study firms' decisions on product design.
- We only need to replace geographic space with the space of product characteristics.
- The following factors play an important role in firms' product location decisions:

  - Consumer demand at different locations;

  - Costs of entry and producing different bundles of characteristics;

  - Location of competitors in the product space.

Empirical questions

- How do profits decline when stores (products) get closer?
- **Cannibalization**: to what extend a multi-product firm is concerned with competition between its own products?
- **Economies of scope**: Do the costs of a new store/product decline with the number of other stores/products the firm has?
- **Economies of density:** Do the costs of a new store/product decline with the spatial proximity to other stores/products the firm has?
- Effect on competition of **a change in the geography of the city**, e.g., new neighborhoods. Similarly, effect of an expansion in the space of **technologically feasible product characteristics**.

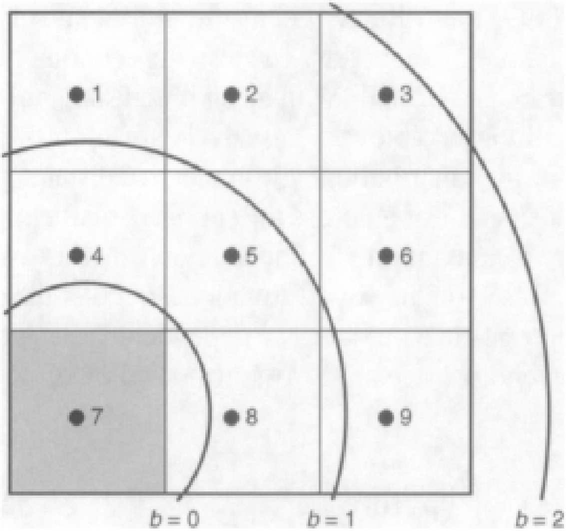
2. Models of Firms' Spatial Location: Single product (store) firms

Space of feasible store locations (the city)

- From a geographical point of view, a market (city) is a set, for instance **a rectangle**, in the space $\mathbb{R}^2$.
- Suppose that we divide this city/rectangle into L small squares, each one with its center.
- Each of these squares is a submarket (or neighborhood, or location).
- A market/city can have hundreds or thousands of these submarkets/locations.
- We index these locations by $\ell = \in \{1, 2, ..., L\}$

FIGURE 1

IMPACT ON PROFITS OF COMPETITORS' LOCATIONS: ILLUSTRATION



Product Space instead of Geographic Space

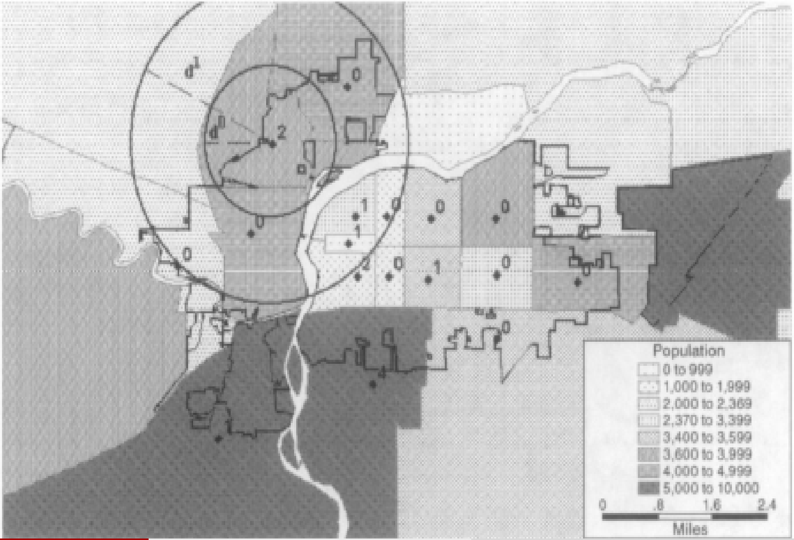
- Suppose that the spatial location refers to the location of products in the space of characteristics.
- Let $\mathbf{z} \in \mathbb{R}^K$ be a vector that describes the K observable (for the researcher) characteristics of a product.
  - For instance, for automobiles, horse power, max speed, physical dimensions, consumption, etc.
- The space of feasible characteristics is compact set in $\mathbb{R}^K$.
- We divide this space into L small hyper-squares in $\mathbb{R}^K$. Each of these hyper-squares is a submarket or product location.
- We index these product locations by $\ell = \in \{1, 2, ..., L\}$

Space of feasible store (product) locations

- Each location has some exogenous characteristics that can affect demand and costs of a firm in that location:
  - Population; demographic characteristics of the population; rental prices.
- We represent the exogenous characteristics of location ℓ using the vector $\mathbf{x}_\ell$.
- Therefore, we can see a city as a landscape of the characteristics $\mathbf{x}_\ell$ over the L locations.

FIGURE 2

SAMPLE MARKET: GREAT FALLS, MONTANA



Model: Firms

- There are N potential entrants in this industry (e.g., supermarkets) and city (Toronto).
- In the simpler version of the model, each potential entrant has only one possible store: no multi-store firms (no chains).
- For the moment, we consider this simpler version.
- Let a_i represent the entry / location decision of firm i.

$$a_i \in \{0, 1, ..., L_m\}$$

- $a_{im} = 0$ represents "no entry";
- $a_{im} = \ell > 0$ represents entry in location ℓ.

Model: Profit function

- What is the profit of firm i if it opens a store in location ℓ?
- In principle, we could consider a model of consumer choice of where to purchase (e.g., logit), a model of price competition between active firms; obtain the Bertrand equilibrium of that game, and the corresponding equilibrium profits.
- This approach requires having data on prices and quantities at every location.
- Instead, Seim (2006) considers a convenient shortcut.
- Her model does not specify (explicitly) consumer choices and price competition, but it incorporates the idea that geographic distance with competitors (spatial differentiation) can increase a firm's profit.

Model: Profit function [2]

- Suppose that we draw a circle of radius d around the center point of location ℓ, e.g., a radius of 1km.
- From the point of view of a store located at ℓ, we can divide its competitors in two groups:
  - Close competitors: within the circle of radius d.
  - Far away competitors: outside the circle of radius d.
- Let $N_\ell(close)$ and $N_\ell(far)$ be the number of close and far away competitors relative to location ℓ.
- We can consider a profit function that depends on $\gamma_{close}\ N_\ell(close) + \gamma_{far}\ N_\ell(far)$, where γ_{close} and γ_{far} are parameters to estimate.
- We expect $\gamma_{close} < \gamma_{close} < 0$. The difference between γ_{close} and γ_{far} tell us how important is geographic distance as a form of differentiation to increase profits.

Model: Profit function [3]

- We can generalized this idea to allow for multiple circles, with different radius, around a location the center point of a location ℓ.

- Let $d_1 < d_2 < ... < d_B$ be B different radius of increasing magnitude, e.g., $d_1 = 0.2$ km, $d_2 = 0.5$ km, ..., $d_{10} = 20$ km.

- We can construct the number of firms within the bands defined by these radii:

  $N_\ell(1) =$ Number of firms within the circle of radius d_1;

  $N_\ell(2) =$ Number of firms within the band defined by the circles with radii d_1 and d_2;

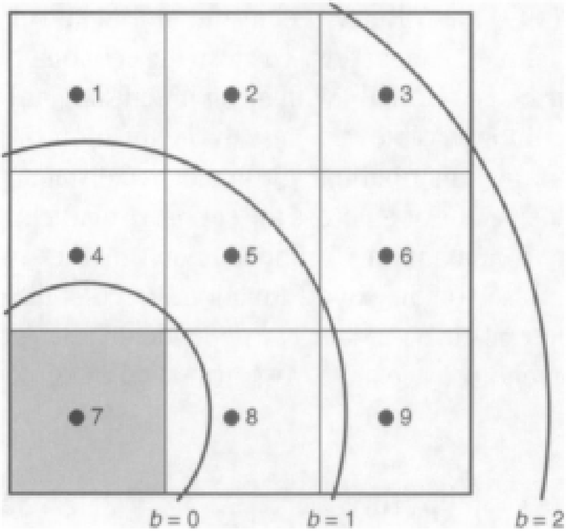
  ...

  $N_\ell(B) =$ Number of firms within the band defined by the circles with radii d_{B-1} and d_B;

  $N_\ell(B+1) =$ Number of firms outside the circle with radius d_B.

FIGURE 1

IMPACT ON PROFITS OF COMPETITORS' LOCATIONS: ILLUSTRATION



Model: Profit function [4]

- Profit of an active firm at location ℓ is:

$$\Pi_{i\ell} = x_{\ell}\ \beta + \xi_{\ell} + \sum_{b=1}^{B} \gamma_b\ N_{\ell}(b) + \varepsilon_{i\ell}$$

- We expect:

$$\gamma_1 < \gamma_2 < \ldots < \gamma_B < 0$$

- ξ_{ℓ} represents attributes of location ℓ which are known to firms bur unobserved to the researcher.

- ε_{i1}, ε_{i2}, ..., ε_{iL} are assumed iid over firms and locations with extreme value distribution.

Profit function - Space of product characteristics

- We can apply this approach to the model of spatial location in product space.
- Let $\mathbf{z}_\ell$ be the vector of observable characteristics that define a product in location ℓ.
- Given the B radii $d_1 < d_2 < ... < d_B$, we can define:

$$N_\ell(b) \equiv \text{Number of existing products with with characteristics } \mathbf{z} \text{ such that } d_{b-1} < \|\mathbf{z}_\ell - \mathbf{z}\| \leq d_b$$

- Profit of an active firm at location ℓ is:

$$\Pi_{i\ell} = x_\ell\ \beta + \xi_\ell + \sum_{b=1}^{B} \gamma_b\ N_\ell(b) + \varepsilon_{i\ell}$$

Model: Equilibrium (1)

- The game is of incomplete information. Firms do not know the actual numbers $N_\ell(1)$, ..., $N_\ell(B)$. Instead, they now the expected value:

$$\mathbb{E}\left[N_\ell(b)\right] = N_\ell^e(b; \mathbf{P}) \equiv N \sum_{\ell'=1}^{L} 1\left\{d_{b-1} < \left\|\mathbf{z}_{\ell'} - \mathbf{z}_\ell\right\| \leq d_b\right\} \; P_{\ell'}$$

  where P_ℓ is the equilibrium probability of opening a store at location ℓ; and $\mathbf{P} = \{P_\ell : \ell = 1, 2, ..., L\}$.

- Then, given that a firm has beliefs $\mathbf{P}$ about entry probabilities at every location, the firm's best expected profit of entry in a location ℓ is:

$$\Pi_{i\ell} = x_\ell \; \beta + \xi_\ell + \textstyle\sum_{b=1}^{B} \gamma_b \; N_\ell^e(b; \mathbf{P}) + \varepsilon_{i\ell}$$

  including the possibility of no entry, $a_i = 0$ with $\Pi_i(0) = 0$.

Model: Equilibrium (2)

- Given the logit assumption on $\varepsilon's$ the **best response** probability of entry at location ℓ is:

$$P_\ell = \frac{\exp\left\{x_\ell\ \beta + \xi_\ell + \sum_{b=1}^{B} \gamma_b\ N_\ell^e(b; \mathbf{P})\right\}}{1 + \sum_{j=1}^{L} \exp\left\{x_j\ \beta + \xi_j + \sum_{b=1}^{B} \gamma_b\ N_j^e(b; \mathbf{P})\right\}}$$

- The equilibrium of the model is described by L simultaneous equations, one for the share/probability of each location ℓ, P_ℓ. In compact form:

$$\mathbf{P} = \Lambda\left(\mathbf{P};\ \mathbf{x}, \boldsymbol{\xi}, \beta, \gamma\right)$$

- This is a continuous mapping and compact space. By Brower's, the model has at least one equilibrium.

Model: Equilibrium (3)

- In equilibrium, a change in x_ℓ in a single location affects the entry probabilities $P_{\ell'}$ at every location ℓ' in the city.
- Example: Policy that encourages entry in location 1.
  - Direct substitution effect: Keeping all $N_\ell^e(b; \mathbf{P})$ constants, the increase in $x_1\beta$ generates a substitution from other locations into location 1.
  - Indirect equilibrium effect: the increase in P_1 implies an increase in $N_\ell^e(b; \mathbf{P})$ that include location 1; implies a reduction in entry probabilities in locations ℓ nearby location 1.

Data and Estimation

- Suppose that we have data from an industry (e.g., supermarkets) in a city (or one network). We observe:

$$\text{Data} = \{x_\ell, \; n_\ell : \ell = 1, 2, ..., L\}$$

Given these data, we can construct shares: $s_\ell : \ell = 1, 2, ..., L$:

$$s_\ell = \frac{n_\ell}{N} \quad \text{and} \quad s_0 = \frac{N - n_1 - ... - n_L}{N}$$

- We distinguish three cases for the estimation of the model:

Case 1: 1 network. $L \rightarrow \infty$ & Large $\dfrac{N}{L}$ such that $s_\ell > 0$ at every ℓ.

Case 2: 1 network. $L \rightarrow \infty$ & Small $\dfrac{N}{L}$ such that $s_\ell = 0$ for some ℓ.

Case 3: M networks. $M \rightarrow \infty$. Small N, L.

Estimation: One Network; Large L; Large N/L

- With large N/L, such that $n_\ell > 0$ and $s_\ell > 0$ at every location ℓ, the model implies that:

$$\ln\left(\frac{s_\ell}{s_0}\right) = x_\ell\ \beta + \sum_{b=1}^{B} \gamma_b\ N_\ell^e(b;\mathbf{s}) + \xi_\ell$$

- This is a linear regression model with regressors x_ℓ, $N_\ell(1)$, ..., $N_\ell(B)$, and error term ξ_ℓ.

- Remember that:

$$N_\ell^e(b;\mathbf{s}) \equiv N \sum_{\ell'=1}^{L} 1\left\{d_{b-1} < \|\mathbf{z}_{\ell'} - \mathbf{z}_\ell\| \leq d_b\right\}\ s_{\ell'}$$

- Therefore, this is system of L simultaneous equations, and variables $N_\ell^e(b;\mathbf{s})$ are endogenous regressors.

One Network; Large L; Large N/L: OLS

$$\ln\left(\frac{s_\ell}{s_0}\right) = x_\ell\ \beta + \sum_{b=1}^{B} \gamma_b\ N_\ell^e(b; \mathbf{s}) + \xi_\ell$$

- Regressors $N_\ell^e(b; \mathbf{s})$ are endogenous: they are correlated with ξ_ℓ. OLS estimator is inconsistent.
- We expect: $cov(N_\ell^e(1; \mathbf{s}), \xi_\ell) > 0$ and $cov(N_\ell^e(1; \mathbf{s}), \xi_\ell) > cov(N_\ell^e(2; \mathbf{s}), \xi_\ell) > ... > cov(N_\ell^e(B; \mathbf{s}), \xi_\ell)$
- This implies that OLS estimator of γ_1 is upward biased, and $bias(\gamma_1) > bias(\gamma_2) > ... > bias(\gamma_B)$
- We might wrongly conclude that distance does not affect competition.

One Network; Large L; Large N/L: IV

$$\ln\left(\frac{s_\ell}{s_0}\right) = x_\ell \ \beta + \sum_{b=1}^{B} \gamma_b \ N_\ell^e(b; \mathbf{s}) + \xi_\ell$$

- Model implies instruments for the endogenous regressors.
- Market characteristics x_j in locations other than ℓ do not enter in the equation for location ℓ but affect the equilibrium values $N_\ell^e(b; \mathbf{s})$.
- Let $\overline{x}_\ell(b)$ be the mean value of x_j in those locations that belong to the band b around location ℓ:

$$\overline{x}_\ell(b) = \frac{\sum_{j=1}^{L} 1\{\text{location } j \text{ belongs to band } b \text{ around } \ell\} \ x_j}{\sum_{j=1}^{L} 1\{\text{location } j \text{ belongs to band } b \text{ around } \ell\}}$$

- We can use $\overline{x}_\ell(b)$ as an instrument for $N_\ell(b)$.

One Network; Large L; Large N/L: Consistency

- Consistency and root-L asymptotic normality of the IV estimator as $L \rightarrow \infty$ requires some conditions.
- **Haiqing Xu (IER, 2018)** establishes conditions for consistency.
- The key condition is the *Network Decaying Dependence (NDD) condition*.
- A sufficient condition for NDD is:

$$\left| \sum_{b=1}^{B} \gamma_b \right| < \frac{1}{N}$$

- Under this condition, spatial effects decay with distance and the spatial stochastic process implied by the equilibrium is ergodic.
- Interestingly, this is also a sufficient condition for equilibrium uniqueness.

One Network; Large L; Large N/L

- Note that this regression-like approach has two very important advantages.
- **Dealing with endogeneity.** We can deal with endogeneity using a standard IV method.
- **Computational simplicity.** For the estimation of the structural parameters, we don't need to solve for an equilibrium of the model even once.

Estimation: One Network; Large L; Small N

- We cannot use a regression-like approach.
- Now, $s_\ell = \dfrac{n_\ell}{N}$ is zero for many locations ℓ. Furthermore, $s_\ell = \dfrac{n_\ell}{N}$ is no longer a consistent estimator of P_ℓ.
- We can use a MLE but a key issue is how to deal with the endogeneity problem associated with the unobservables ξ_ℓ.
- We first describe the MLE under the assumption of $\xi_\ell = 0$ (**no unobserved location heterogeneity**) and then we relax this assumption.

One Network; Large L; Small N: epsi = 0

- Suppose that $\left|\sum_{b=1}^{B}\gamma_b\right| < \dfrac{1}{N}$ such that the model has a unique equilibrium. Suppose that we impose this restriction throughout the estimation of the model.

- Let θ be the vector of structural parameters, and let $\mathbf{P}(\theta)$ be the vector of equilibrium probabilities associated with θ. That is, $\mathbf{P}(\theta)$ solves

$$\mathbf{P}(\theta) = \Lambda\left(\mathbf{P}(\theta);\ \theta\right)$$

- Or equivalently, $\mathbf{P}(\theta)$ solves the system:

$$P_\ell(\theta) = \frac{\exp\left\{x_\ell\ \beta + \sum_{b=1}^{B}\gamma_b\ N_\ell^e(b; \mathbf{P}(\theta))\right\}}{1 + \sum_{j=1}^{L}\exp\left\{x_j\ \beta + \sum_{b=1}^{B}\gamma_b\ N_j^e(b; \mathbf{P}(\theta))\right\}}$$

One Net; Large L; Small N: epsi = 0 [2]

- According to the model,

$$[n_1, n_2, ..., n_L] \sim Multinomial\left(N;\ P_1(\theta), P_2(\theta), ..., P_L(\theta)\right)$$

- Therefore, the likelihood function is:

$$\mathcal{L}(\theta) = \prod_{\ell=0}^{L} P_\ell(\theta)^{n_\ell}$$

- or

$$\ln \mathcal{L}(\theta) = \sum_{\ell=0}^{L} n_\ell \ \ln P_\ell(\theta)$$

One Net; Large L; Small N: epsi = 0 [3]

- We can estimate θ by MLE using the **Nested Fixed Point algorithm**.
- We maximize $\ln \mathcal{L}(\theta)$ using a Newton's or **BHHH** iterative method:

$$\widehat{\theta}_{K+1} = \widehat{\theta}_K - \left[\sum_{\ell=0}^{L} \frac{\partial \ln P_\ell(\widehat{\theta}_K)}{\partial \theta} \frac{\partial \ln P_\ell(\widehat{\theta}_K)}{\partial \theta'} \right]^{-1} \left[\sum_{\ell=0}^{L} n_\ell \frac{\partial \ln P_\ell(\widehat{\theta}_K)}{\partial \theta} \right]$$

- At each iteration K, given $\widehat{\theta}_K$ we compute the equilibrium $\mathbf{P}(\widehat{\theta}_K)$.
- When L is large, the computation of an equilibrium can be computational demanding.
- To deal with this computational cost Haiqing Xu (IER, 2018) proposes approximating the equilibrium by using L local equilibria, on for each location. The local equilibrium at location ℓ is obtained using only location this location and its nearest neighbors.

One Net; Large L; Small N: epsi $/=$ 0

- Now, the equilibrium vector depends on the vector of unobservables $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_L)$. We have $\mathbf{P}(\theta; \boldsymbol{\xi})$.

- The log-likelihood function is integrated over the distribution of $\boldsymbol{\xi}$:

$$\ln \mathcal{L}(\theta) = \sum_{\ell=0}^{L} n_\ell \ \left[\int \ln P_\ell(\theta; \boldsymbol{\xi}) \ f(\boldsymbol{\xi}) \ d\boldsymbol{\xi} \right]$$

- Since L is large, the dimension of $\boldsymbol{\xi}$ and the integral is large. Very demanding computational problem.

- Two approaches to compute $I = \int \ln P_\ell(\theta; \boldsymbol{\xi}) \ f(\boldsymbol{\xi}) \ d\boldsymbol{\xi}$:

  - Monte Carlo simulation: $I \simeq \frac{1}{R} \sum_{r=1}^{R} \ln P_\ell(\theta; \boldsymbol{\xi}^{(r)})$.

  - Discrete ξ's. Binary, $K = 2^L$ points: $I = \sum_{k=1}^{K} \ln P_\ell(\theta; \boldsymbol{\xi}^{(k)}) \ \pi_k$

M Networks; Large M; Small N, L

- Now, we have M cities or networks and for each city m we observe $\{x_{m\ell}, n_{m\ell} : \ell = 1, 2, ..., L_m\}$.

- The log-likelihood function is: **without** $\xi's$:

$$\ln \mathcal{L}(\theta) = \sum_{m=0}^{M} \sum_{\ell=0}^{L_m} n_{m\ell} \ \ln P_{m\ell}(\theta)$$

- The estimation is the same as before, but for each trial value of θ we need to compute M equilibria, one for each city.

- **With** $\xi's$, the estimation is similar as described above for one single network, but again with as many equilibria as cities and values of $\boldsymbol{\xi}$ per city.

Seim (2006) application: Main Results

- Seim (2006) finds very significant results of spatial differentiation (γ parameters decline very significantly with distance)
- Market structure and spatial structure of stores under two different scenarios of city growth.
  - Growth in population but keeping city boundaries.
  - Growth in population and in city boundaries
- The model can be used to study how changes in the exogenous characteristics x_ℓ of a single location (e.g., new amenities, schools, new local regulations, transportation, developments) can affect the landscape of firms in the whole city.

3. Models of Firms' Spatial Location: Multi-product (store) firms

Model with Multi-Store Firms

- Consider the same spatial configuration as before, but now the N potential entrants can open as many stores as possible locations L.
- Now, the number of players N is very small (a few retail chains). For instance, two firms indexed by $i \in \{1, 2\}$.
- The decision variable for firm i:

$$\mathbf{a}_i = (a_{i1},\ a_{i2},\ ...,\ a_{iL})$$

where $a_{i\ell} = 1\{\text{Firm } i \text{ opens a store in location } \ell\} \in \{0, 1\}$.

Multi-Store Firms: Profit

- Now, the profit function should incorporate not only the competition effects from the stores of other firms but also the competition or/and spillover effects from the own stores.

- For instance (we can extend it to allow for B bands):

$$\Pi_i = \sum_{\ell=1}^{L} a_{i\ell} \left[x_\ell \beta_i + \xi_\ell + \gamma_i \; a_{j\ell} + \theta_i^{ED} \sum_{\ell'=1}^{L} \frac{a_{i\ell'}}{d_{\ell\ell'}} + \varepsilon_{i\ell} \right]$$

  where $d_{\ell\ell'} =$ distance between ℓ and ℓ'.

- θ_i^{ED} captures cannibalization effects (if $\theta_i^{ED} < 0$) or economies of scope/density (if $\theta_i^{ED} > 0$).

Best responses

- The space of the vector $\mathbf{a}_i = (a_{i1}, a_{i2}, ..., a_{iL})$ has 2^L possible points.
- For instance, Jia (2008) studies competition between in entry/location between Walmart and Kmart in $L = 2,065$ locations (US counties). This implies $2^L = 2^{2065} \simeq 10^{621}$.
- The computation of an equilibrium in this model is computationally very costly.
- Researchers have consider different approaches to deal with this issue.
  (a) Moment inequalities based on restrictions on the unobservables: Ellickson, Houghton, and Timmins (RAND, 2013)
  (b) Lattice theory approach: Jia (Econometrica, 2008); Nishida (Marketing Science, 2014)

Ellickson, Houghton, and Timmins (2013)

- Consider a game between N multi-store firms but ignore for the moment cannibalization and economies of scope/density such that:

$$\Pi_i = \sum_{\ell=1}^{L} a_{i\ell} \left[x_\ell \beta_i + \xi_\ell + \sum_{j \neq i} \gamma_{ij} \; a_{j\ell} + \varepsilon_{i\ell} \right]$$

- They assume that: $\varepsilon_{i\ell} = \alpha_i + \xi_\ell$. They also assume complete information.

- By revealed preference, the profit of the observed action of firm i, $\mathbf{a}_i$, should be larger than the profit of any alternative action, $\mathbf{a}_i'$:

$$\Pi_i \left(\mathbf{a}_i \right) - \Pi_i \left(\mathbf{a}_i' \right) \geq 0 \quad \text{for any } \mathbf{a}_i' \neq \mathbf{a}_i$$

- EHT (2013) consider hypothetical choices $\mathbf{a}_i'$ that difference out the error term such that we do not need to integrate over a space of 2^L unobservables.

Ellickson, Houghton, and Timmins [2]

- Suppose that the observe choice of firm i, $\mathbf{a}_i$, is such that $a_{i\ell} = 1$ and $a_{i\ell'} = 0$.
- Consider the hypothetical choice $\mathbf{a}_i'$ that consists in the relocation of a store from ℓ into ℓ', such that $a_{i\ell} = 0$ and $a_{i\ell'} = 1$. Then:

$$\Pi_i(\mathbf{a}_i) - \Pi_i(\mathbf{a}_i') =$$

$$[x_\ell - x_{\ell'}]\,\beta_i + \sum_{j \neq i} \gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + [\xi_\ell - \xi_{\ell'}] \geq 0$$

Ellickson, Houghton, and Timmins [3]

- Now, suppose that for a different firm, firm k, the observe choice, $\mathbf{a}_k$, is such that $a_{k\ell} = 0$ and $a_{k\ell'} = 1$.
- Consider the hypothetical choice $\mathbf{a}'_k$ that consists in the relocation of a store from ℓ' into ℓ, such that $a_{k\ell} = 0$ and $a_{k\ell'} = 1$.
- Then, for firm k we have:

$$\Pi_k\left(\mathbf{a}_k\right) - \Pi_k\left(\mathbf{a}'_k\right) =$$

$$\left[x_\ell - x_{\ell'}\right]\beta_k + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] + \left[\xi_{\ell'} - \xi_\ell\right] \geq 0$$

Ellickson, Houghton, and Timmins [4]

- Adding the inequalities:

$$[x_\ell - x_{\ell'}]\,\beta_i + \sum_{j \neq i} \gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + [\xi_\ell - \xi_{\ell'}] \geq 0$$

$$[x_\ell - x_{\ell'}]\,\beta_k + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] + [\xi_{\ell'} - \xi_\ell] \geq 0$$

- We have:

$$[x_\ell - x_{\ell'}]\ \ [\beta_i - \beta_k] + \sum_{j \neq i} \gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] \geq 0$$

Ellickson, Houghton, and Timmins [4]

- Using different pairs of locations and/or firms, we can construct many different inequalities like

$$[x_\ell - x_{\ell'}] \;\; [\beta_i - \beta_k] + \sum_{j \neq i} \gamma_{ij} \left[a_{j\ell} - a_{j\ell'}\right] + \sum_{j \neq k} \gamma_{kj} \left[a_{j\ell'} - a_{j\ell}\right] \geq 0$$

- Using these inequalities, we can estimate the parameters β and γ using the smooth Maximum Score estimator (Manski, 1975; Horowitz, 1992; Fox, 2010).

- EHT (RAND, 2013) apply this approach to study competition in entry/location between department store chains in US.