ECO 2901
EMPIRICAL INDUSTRIAL ORGANIZATION

Lecture 4:
Quantity and Price Competition with Incomplete Information

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Today's Lecture (Topic 4)

1. Competition with Incomplete Information: Introduction

2. Cournot competition with private information: Theory
Vives (RAND, 2002)

3. Econometric model

4. Cournot competition with private information: An application to airlines
Armantier & Richard (RAND, 2003)

1. Competition with incomplete information: Introduction

Competition with incomplete information: Introduction

- We have considered different factors that can affect price and quantity competition and market power in an industry:
  e.g., economies of scale and scope; firms' heterogeneity in marginal costs; product differentiation; multi-product firms; or conduct/form of competition.

- So far, all the models we have considered, **assume that firms have perfect knowledge** about demand, their own costs, and the costs of their competitors: **games of complete information**.

- This assumption can quite unrealistic in some industries. Firms have uncertainty about current and future realizations of demand, costs, market regulations, or the behavior of competitors.

- This uncertainty can have substantial implications for their decisions and profits, and for the efficiency of the market.

Competition with incomplete information [2]

- The assumption of firms' **complete information** has been the **status quo in empirical models of Cournot or Bertrand competition**.
- ... In contrast, games of incomplete information are the "default" when studying competition in auctions ...
- Today, we will study models and empirical applications of price and quantity competition that allow for firms' incomplete and private information.

2. Cournot competition with private information: Theory

Cournot competition with private information: Theory

- Based on Vives (RAND, 2002) who studies theoretically the importance of firms' private information as a determinant of prices, market power, and consumer welfare.
- Model of Cournot competition with free market entry where each firm has private information about random shocks affecting its marginal cost.
- **Main result.** There is a critical value for market size, M^* (that depends on the values of structural parameters), such that the effect of private information, on prices and consumer welfare, dominates the effect of market power if and only if market size is greater than this threshold value.
- In not too small markets, ignoring asymmetric information can imply more important biases than ignoring oligopoly competition and market power.

Vives (2002): Model

- Homogeneous product market with free entry.
- There are M consumers in the market. The inverse demand function is: $p = P(Q) = \alpha - \dfrac{\beta}{M}\ Q$.
- The cost function for firm i is:

$$C(q_i, \varepsilon_i) = \varepsilon_i\ q_i + \frac{\gamma}{2} q_i^2$$

- ε_i is a random shock that is private information of firm i.
- $\varepsilon_i \sim i.i.d.$ with mean μ_ε and variance σ_ε^2 which are common knowledge for all the firms.

Vives (2002): Model [2]

- **Two-stage game**.
- **First stage.** Firms decide whether to enter the market or not.
  - At this stage, potential entrants do not know yet the realization of their idiosyncratic ε's.
  - If a firm decides to enter, it pays a fixed cost $F > 0$.
- **Second stage**,
  - Each firm that has decided to enter observes its own ε_i (but not the ε's of the other active firms).
  - Firms compete according to a Bayesian Nash-Cournot equilibrium.

Vives (2002): Bayesian Nash Equilibrium - stage two

- Suppose that there are n firms active in the market.
- The expected profit of firm i is:

$$\begin{aligned}\pi_i^e &= \mathbb{E}\left[P(Q) \mid \varepsilon_i\right] \; q_i - \varepsilon_i \; q_i + \frac{\gamma}{2} q_i^2 \\ &= \left(\alpha - \frac{\beta}{M} \left(q_i + \mathbb{E}\left[\sum_{j \neq i} q_j\right]\right)\right) \; q_i - \varepsilon_i \; q_i + \frac{\gamma}{2} q_i^2\end{aligned}$$

- A **Bayesian Nash Equilibrium (BNE)** is an n-tuple of strategy functions, $[\sigma_1(\varepsilon_1)$, $\sigma_2(\varepsilon_1)$, ...,$\sigma_n(\varepsilon_n)]$, such that for every firm i:

$$\sigma_i(\varepsilon_i) = \arg\max_{q_i} \; \mathbb{E}\left[P(Q) \mid \varepsilon_i, \, \sigma_j \text{ for } j \neq i\right] \; q_i - \varepsilon_i \; q_i + \frac{\gamma}{2} q_i^2$$

- Since firms are identical up to ε_i, it seems reasonable to focus on symmetric BNE: $\sigma_i(\varepsilon_i) = \sigma(\varepsilon_i)$ for every i.

Bayesian Nash Equilibrium [2]

- F.O.C. for the best response of firm i implies:

$$\sigma(\varepsilon_i) = [\gamma + 2\beta_M\ ]^{-1} \left[\alpha - \varepsilon_i - \beta_M\ \sum_{j \neq i} \mathbb{E}\left(\sigma(\varepsilon_j)\right) \right]$$

- Taking expectations over ε_i,

$$\sigma^e \equiv \mathbb{E}\left(\sigma(\varepsilon_i)\right) = [\gamma + \beta_M\ (n+1)]^{-1} [\alpha - \mu_\varepsilon]$$

- Solving this expression into the F.O.C, we obtain the closed-form expression for the equilibrium strategy function:

$$q_i = \sigma(\varepsilon_i) = \frac{\alpha - \mu_\varepsilon}{\gamma + \beta_M\ (n+1)} - \frac{\varepsilon_i - \mu_\varepsilon}{\gamma + 2\beta_M}$$

First stage: Market entry

- The expected profit of firm i is:

$$\mathbb{E}\left[\pi(\varepsilon_i)\right] = \frac{\left[\alpha - \mu_\varepsilon^2\right]}{\left[\gamma + \beta_M \ (n+1)\right]^2} + \frac{\sigma_\varepsilon^2}{\left[\gamma + 2\beta_M\right]^2}$$

- Given a market of size M, the free-entry number of firms $n^*(M)$ is approximated by the solution to $\mathbb{E}\left[\pi(\varepsilon_i)\right] - F = 0$.

- Given the expression for the equilibrium profit, it is simple to verify that $n^*(M)$ is of the same order as market size M. That is, the ratio $n^*(M)/M$ of the firms per consumer is bounded away from zero and infinity.

$$0 < \frac{n^*(M)}{M} < \infty$$

Vives (2002): Market power & Incomplete info

- The optimal allocation in this industry can be achieved if firms share all their information and behave as price takers.

- Let us label this equilibrium as $CI - PT$ (*complete information & price taking*).

- To measure effects of incomplete information and Cournot behavior, it is convenient to define two models.
  - Model CI: Cournot competition but with complete information.
  - Model PT: Incomplete information but firms are price takers.

Market power & Incomplete info [2]

- Let p and W be the price and the total welfare under the "true" model with both Cournot behavior and incomplete info.

- Consider the decomposition:

$$\begin{aligned} p - p_{CI-PT} &= [p - p_{PT}] + [p_{PT} - p_{CI-PT}] \\ W_{CI-PT} - W &= [W_{CI-PT} - W_{PT}] + [W_{PT} - W] \end{aligned}$$

- $[p - p_{PT}]$ and $[W_{PT} - W]$ measure the effect of Cournot behavior (market power) on prices and Welfare.

- $[p_{PT} - p_{CI-PT}]$ and $[W_{CI-PT} - W_{PT}]$ measure the effect of incomplete information.

Market power & Incomplete info [3]

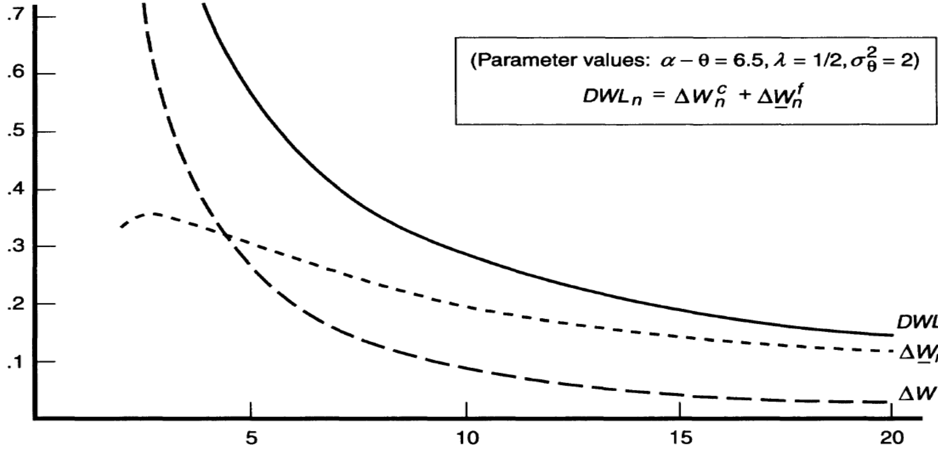
- In this model, as market size M (and therefore n) goes to infinity, these differences go to zero:

$$\left[p - p_{CI-PT}\right] \longrightarrow 0 \text{ and } \frac{W_{CI-PT} - W}{M} \longrightarrow 0$$

- As market size increases, market price and welfare per capita converge to the optimal allocation. That is, private information and Cournot behavior have an effect only when the market is not too large.

- **Main result.** There is a critical value for market size, M^* (that depends on the values of structural parameters), such that the effect of private information, on prices and consumer welfare, dominates the effect of market power if and only if market size is greater than this threshold value.

Market power & Incomplete info [4]



Policy implications

- Antitrust authorities look with suspicion the information exchanges between firms because they can help collusive agreements.
- The collusion concern is most important in the presence of a few players because collusion is easier to be sustained in this case.(repeated game).
- This paper shows that with few firms market power (Cournot) has the most important contribution to the DWL, so it seems reasonable to control these information exchanges.
- When market size and the number of firms increase, information asymmetry becomes a more important factor in the DWL. In this case, it seems optimal to allow for some information sharing between firms.

3. Econometric model

Econometric model

- Homogeneous product industry. Researcher observes prices, quantities, and exogenous characteristics over T periods or geographic markets:

$$\text{Data } = \{p_t,\ q_{it},\ x_t^D,\ w_{it} : i = 1, ..., N_t;\ t = 1, ..., T\}$$

- Here we extend the previous theory model to incorporate: exogenous observables; nonlinear demand; nonlinear marginal costs.

- **Demand:** $p_t = P(x_t^D, Q_t)$.

- **Cost function:** $C(\varepsilon_{it}, w_{it}, q_{it}) = \exp\{\varepsilon_{it} + w_{it}'\gamma_1\} \dfrac{1}{\gamma_2 + 1} q_{it}^{\gamma_2+1}$,

  where ε_{it} is private information and unobservable to the researcher; and w_{it} is assumed to be common knowledge.

- ε_{it} is independently distributed over markets and firms and independent of the common knowledge variables x_t^D and w_{it}.

Econometric model: BNE

- Information of firm i at period t is $(\mathbf{x}_t, \varepsilon_{it})$ with $\mathbf{x}_t = (x_t^D, w_{1t}, ..., w_{Nt})$.

- A firm's strategy function is $\sigma_i(\mathbf{x}_t, \varepsilon_{it})$ from $\mathcal{X} \times \mathbb{R} \rightarrow \mathbb{R}_+$.

- A BNE consists of strategy functions, $(\sigma_1, ..., \sigma_N)$, such that for any i:

$$\sigma_i(\mathbf{x}_t, \varepsilon_{it}) = \arg\max_{q_{it} \geq 0} \; P_{\sigma_{-i}}(\mathbf{x}_t, q_{it}) \; q_{it} - C(\varepsilon_{it}, w_{it}, q_{it})$$

- where $P_{\sigma_{-i}}(\mathbf{x}_t, q_{it})$ is the expected (inverse) demand function given firm i's information:

$$P_{\sigma_{-i}}(\mathbf{x}_t, q_{it}) \equiv \mathbb{E}_{\varepsilon_{-it}}\left[P\left(x_t^D, q_{it} + \sum_{j \neq i} \sigma_j(\mathbf{x}_t, \varepsilon_{jt}) \right) \right]$$

where $\mathbb{E}_{\varepsilon_{-it}}[.]$ represents expectation over distribution of $\varepsilon_{jt} : j \neq i$.

Econometric model: BNE [2]

- The f.o.c. for the best response function is:

$$P_{\sigma_{-i},t} + \frac{\partial P_{\sigma_{-i},t}}{\partial q_{it}} q_{it} = \exp\{\varepsilon_{it} + w_{it}'\gamma_1\} \ q_{it}^{\gamma_2}$$

- Or in logarithms:

$$\ln\left(P_{\sigma_{-i},t} + \frac{\partial P_{\sigma_{-i},t}}{\partial q_{it}} q_{it}\right) = w_{it}'\gamma_1 + \ \gamma_2 \ \ln(q_{it}) + \varepsilon_{it}$$

- If we knew the expected demand functions $P_{\sigma_{-i}}(\mathbf{x}_t, q_{it})$, then this equation would be a standard regression model with endogeneity (i.e., $\ln(q_{it})$ and ε_{it} are correlated) and we could estimate the model using IV / GMM methods.

- **Note:** vector of "observables" w_{it} can include firm-dummies and time-dummies to account for firm heterogeneity and market-level shocks that are common knowledge to firms but unknown to the researcher.

Estimation: Two-step method

- The key issue for the estimation of this structural model is calculating the expected (inverse) demand functions $P_{\sigma_{-i}}(\mathbf{x}_t, q_{it})$ and their slopes $\dfrac{\partial P_{\sigma_{-i},t}}{\partial q_{it}}$ for every player i.
- We can think in recursive methods that compute a BNE of the model for each trial value of the structural parameters. But this can be computationally very costly.
- Instead, we can exploit the structure of the model to define a two-step method that is much simpler to implement.

Estimation: Two-step method [2]

- The f.o.c. for the best response of firm i implies that:

$$\ln\left[\sigma_i(\mathbf{x}_t, \varepsilon_{it})\right] = f_i(\mathbf{x}_t) + \varepsilon^*_{it}$$

where the functions $f_i(.)$ are unknown to the researcher.

- However, the model implies that $\ln(q_{it}) = \ln\left[\sigma_i(\mathbf{x}_t, \varepsilon_{it})\right]$ and $\mathbb{E}\left[\varepsilon^*_{it} \mid \mathbf{x}_t\right] = 0$ such that:

$$f_i(\mathbf{x}_t) = \mathbb{E}\left[\ln(q_{it}) \mid \mathbf{x}_t\right]$$

and $\mathbb{E}\left[\ln(q_{it}) \mid \mathbf{x}_t\right]$ is a nonparametric regression function that can be identified given the data.

- This property provides the basis for two-step method for the estimation of the model.

Two-step method: Step 1

- We start estimating the nonparametric regression functions $f_1(\mathbf{x}_t)$, $f_2(\mathbf{x}_t)$, ..., $f_N(\mathbf{x}_t)$ from the regressions:

$$\ln[q_{it}] = f_i(\mathbf{x}_t) + \varepsilon_{it}^*$$

  with $\mathbb{E}\left[\varepsilon_{it}^* \mid \mathbf{x}_t\right] = 0$.

- As part of this first stage, we obtain the residuals of ε_{it}^* and use these residuals to estimate nonparametrically the density function of ε_{it}^*, i.e., $g_i^*(.)$.

Two-step method: Step 2

- Suppose that the researcher knows the inverse demand function $P(x_t^D, Q_t)$ because it has been estimated given the data.

- Given the demand function, functions $f_j(\mathbf{x}_t)$ and density functions $g_j^*(.)$, we can obtain consistent estimates of the expected demand function:

$$
\begin{aligned}
P_{\sigma_{-i}}(\mathbf{x}_t, q_{it}) &= \mathbb{E}_{\varepsilon^*_{-it}}\left[P\left(x_t^D,\ q_{it} + \sum_{j \neq i} \exp\left\{f_j(\mathbf{x}_t) + \varepsilon^*_{jt}\right\}\right)\right] \\
&= \int P\left(x_t^D,\ q_{it} + \sum_{j \neq i} \exp\left\{f_j(\mathbf{x}_t) + \varepsilon^*_{jt}\right\}\right) \prod_{j \neq i} g_i^*(\varepsilon^*_{jt})\ d
\end{aligned}
$$

- If N is not small, we can use approximate it by Monte Carlo:

$$
P_{\sigma_{-i}}(\mathbf{x}_t, q_{it}) = \frac{1}{R} \sum_{r=1}^{R} P\left(x_t^D,\ q_{it} + \sum_{j \neq i} \exp\left\{f_j(\mathbf{x}_t) + \varepsilon^{*(r)}_{jt}\right\}\right)
$$

Two-step method: Step 2

- Once we have identified (estimated) $P_{\sigma_{-i}}\left(\mathbf{x}_t, q_{it}\right)$ and its derivative $\dfrac{\partial P_{\sigma_{-i}}\left(\mathbf{x}_t, q_{it}\right)}{\partial q_{it}}$ for each firm i, we can now estimate the costs parameters using the regression function:

$$\ln\left(P_{\sigma_{-i},t} + \frac{\partial P_{\sigma_{-i},t}}{\partial q_{it}} q_{it}\right) = w_{it}'\gamma_1 + \ \gamma_2 \ \ln(q_{it}) + \varepsilon_{it}$$

- As in the model with complete information, this regression function suffers of an endogeneity problem because $\ln(q_{it})$ are (negatively) correlated ε_{it}.

- We can use x_t^D or/and $w_{jt} : j \neq i$ as instruments.

Implications of incomplete information

- The key difference between the Cournot models with complete and incomplete information is in the difference between the demand function $P\left(x_t^D, Q_t\right)$ and derivative $\dfrac{\partial P\left(x_t^D, Q_t\right)}{\partial Q_t}$ and the firm-specific expected demands $P_{\sigma_{-i}}\left(\mathbf{x}_t, q_{it}\right)$ and derivative $\dfrac{\partial P_{\sigma_{-i}}\left(\mathbf{x}_t, q_{it}\right)}{\partial q_{it}}$.
- If $\left|\dfrac{\partial P_{\sigma_{-i}}\left(\mathbf{x}_t, q_{it}\right)}{\partial q_{it}}\right| > \left|\dfrac{\partial P\left(x_t^D, Q_t\right)}{\partial Q_t}\right|$ then incomplete info implies lower quantities and higher prices than complete info.
- If $\left|\dfrac{\partial P_{\sigma_{-i}}\left(\mathbf{x}_t, q_{it}\right)}{\partial q_{it}}\right| < \left|\dfrac{\partial P\left(x_t^D, Q_t\right)}{\partial Q_t}\right|$ then incomplete info implies higher quantities and lower prices than complete info.
- The second case is more common: integrating over unknowns "smooths" the demand function.

4. Cournot competition with private information: Empirical application

Armantier and Richard (RAND, 2003) on airlines

- They study empirically how firms' asymmetric information on marginal costs affects competition and outcomes in the US airline industry.
- They also study how marketing alliances that facilitate information sharing can affect competition.
- Once we relax the assumption of linear demand and/or marginal costs we do not have closed form expressions for equilibrium strategies and the computation of a BNE becomes challenging.
- A methodological contribution of this paper is to propose and algorithm for the solution and structural estimation of this model.

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Model

- N firms indexed by i, and M markets (routes) indexed by m.
- Firms decide simultaneously whether to enter and how much to produce in each of the M markets.
- Marginal costs of production are constant and private information of each firm.
- $MC_{im} = w_{im}'\gamma + \varepsilon_{im}$ is the marginal cost if firm i in route m.
- In market m, ε_{im} is i.i.d. over firms with a density function f_m that is common knowledge.

Demand

- Demand system tries to incorporate the links between the demands for different routes (flight connection), though with strong simplifying assumptions (we will see later in the course a more structural model of airlines demand).

- The inverse demand curve for airline i in route m is:

$$p_{im} = \alpha_m + \beta_m \left[\sum_{m' \neq m} q_{im'} \right] + \lambda_m \left[\sum_{m' \neq m} \sum_{j \neq i} q_{jm'} \right] - \gamma_m \left[\sum_{j=1}^{N} q_{jm} \right]$$

- $-\gamma_m \left[\sum_{j=1}^{N} q_{jm} \right]$ with $\gamma_m > 0$: Downward sloping demand and substitution between the products of the different airlines in the same route.
- $\beta_m \left[\sum_{m' \neq m} q_{im'} \right]$ with $\beta_m > 0$: Complementarity between the airline operation in different markets [i.e., capturing flight connections with the same airline].
- $\lambda_m \left[\sum_{m' \neq m} \sum_{j \neq i} q_{jm'} \right]$ with $\lambda_m > 0$: Complementarity between the

Multi-market Bayesian [Cournot] Nash equilibrium

- Given demand and costs, firms decide simultaneously their respective vectors of quantities at every market, $\mathbf{q}_i = (q_{i1}, ..., q_{iM})$.

- A BNE is an N-tuple of strategy functions, $(\sigma_1(\varepsilon_1), ..., \sigma_N(\varepsilon_N))$, such that each firm maximizes its total profits over the different markets:

$$\sigma_i(\boldsymbol{\varepsilon}_i) = \arg\max_{\mathbf{q}_i \geq 0} \sum_{m=1}^{M} \left[\mathbb{E}_{\varepsilon_{-i}} \left[P_m \left(q_i, \ \sigma_j(\boldsymbol{\varepsilon}_j) : j \neq i \right) \right] - w_{im}' \gamma - \varepsilon_{im} \right] \ q_{im}$$

- Problem in this paper: Assume that fixed costs are zero, and endogenize entry decision but as part of the intensive margin decision of output, i.e., no entry of firm i in market m is just a corner solution $q_{it} = 0$. Quite restrictive and unrealistic.

Empirical Application

- American Airlines and United Airlines at Chicago O'Hare. Both airlines have hubs at this airport.
- A market m is a pair of cities (or airports). Firm i is active in market m if it provides direct flights between the two cities.
- Sample from the 3rd quarter of 1993. $M = 100$ markets, all between Chicago O'Hare and other US cities.
-

Summary statistics: market structure

State of the Market		Number of Markets	Average Quantity per Firm	Average Price	Averag Mileag
Monopoly	American	14	13,586.61	92.74	441.14
(only one firm enters)			(8,055.24)	(62.93)	(463.91
	United	29	19,191.53	138.13	754.13
			(8,699.98)	(39.90)	(385.78
	Total	43	17,366.67	123.35	652.23
			(8,809.43)	(52.44)	(433.42
Duopoly	American	40	31,302.64	127.91	—
(two firms enter)			(20,481.25)	(65.19)	
	United	40	36,635.66	128.71	—
			(26,628.18)	(63.44)	
	Total	40	33,969.15	128.31	698.17
			(22,066.86)	(64.13)	(556.02
Active	American	54	26,709.59	118.79	631.53
(at least one firm enters)			(19,646.95)	(65.88)	(541.49
	United	69	29,304.07	132.67	721.69
			(22,650.92)	(54.64)	(489.25
	Total	83	25,367.86	125.74	674.37
			(18,466.34)	(58.06)	(668.51
Overall		100	—	—	715.28
(100 markets)					(493.74

Standard errors in parentheses.

Estimation

- **Step 1.** Estimation of demand system.
- **Step 2.** Given demand, estimation of the parameters in MCs (including the distribution of private information shocks) from the best response function of the BNE.
- **Counterfactual experiment.** Given the estimated demand and distribution of firms' costs, assume that AA and UA agree to share information on their costs. Compute a new (counterfactual) BNE and compare it to the one from the data.
- **Main results.**
  - (As expected) average profits increase in every market with exchange of cost information.
  - Consumer welfare increases in 61% of the markets. Aggregated over all the markets, consumer welfare declines but moderately.
  - Total welfare increases.

Predictions estimated model

TABLE 3 **Incomplete Information Simulation Results: Average Across All Markets**

	State of the Market			
	Monopoly (Only One Firm Enters)	Duopoly (Two Firms Enter)	Active (At Least One Firm Enters)	Overall (100 market
Probability that the market is	.32	.54	.86	—
	(.19)	(.36)	(.28)	
Expected quantity per firm	27,429.00	27,130.17	27,199.00	18,992.8
	(20,657.04)	(20,772.13)	(20,386.48)	(14,282.9
Expected price	133.56	121.00	123.89	—
	(61.46)	(57.07)	(57.84)	
Expected profit per firm	804,538.52	392,865.68	486,968.57	340,725.4
	(1,184,272.21)	(584,678.93)	(699,237.09)	(488,335.0
Expected consumer surplus	302,607.07	2,014,144.56	1,127,444.41	981,921.0
	(448,104.03)	(3,485,116.57)	(2,572,921.30)	(2,367,420.5

Counterfactual: information sharing AA & UA

TABLE 5 **Complete Information Simulation Results: Average Across All Markets**

	State of the Market			
	Monopoly (Only One Firm Enters)	Duopoly (Two Firms Enter)	Active (At Least One Firm Enters)	Overall (100 markets)
Probability that the market is	.52	.46	.98	—
	(.25)	(.30)	(.21)	
Expected quantity per firm	41,170.47	24,577.39	30,564.647	22,004.37
	(25,847.47)	(19,314.73)	(21,474.42)	(15,452.03)
Expected price	130.16	127.04	128.08	—
	(59.55)	(59.04)	(58.96)	
Expected profit per firm	1,176,839.39	516,927.94	754,974.62	544,199.725
	(1,626,334.05)	(751,433.30)	(1,030,123.59)	(744,184.14)
Expected consumer surplus	333,348.38	1,893,240.42	1,085,103.58	978,405.64
	(450,119.43)	(3,160,894.11)	(2,338,940.18)	(2,149,882.41)

Counterfactual

- Moving from incomplete to complete info has the following effects in the estimated model:

- There is an increase in the probability that a market is active under complete: from 86% to 98%.
- Firms' profits increase.
- Expected aggregated consumer surplus decreases by 3.6%
- Consumer surplus increases in 61% of the markets.
- Small markets benefit the most, since are more likely to have at least one airline active.