# ECO 2901
# EMPIRICAL INDUSTRIAL ORGANIZATION

## Lecture 2:
## Price competition with differentiated product

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# Today's Lecture

**[Topic 2]. Price competition with differentiated product**

1. **Model**
2. **Estimating marginal costs based on assumption on the form of competition**
3. **Testing for the form of competition [Bertrand vs. Collusion]**
4. **Empirical application: Bresnahan (1987) on automobiles**
5. **Empirical application: Nevo (2001) on cereals**

# 1. Model

# Model

- Consider an industry with $J$ differentiated products (e.g., automobiles) indexed by $j \in \mathcal{J} = \{1, 2, ..., J\}$.
- Consumer demand for each of these products can be represented using the demand system:

$$q_j = D_j\left(\mathbf{p}, \mathbf{x}\right) \qquad \text{for } j \in \mathcal{J}$$

  $\mathbf{p} = \left(p_1, p_2, ..., p_J\right)$ is the vector of product prices;
  $\mathbf{x} = \left(x_1, x_2, ..., x_J\right)$ is a vector of other product attributes.

- There are $F$ firms in the industry, indexed by $f \in \{1, 2, ..., F\}$.
- Each firm $f$ owns a subset $\mathcal{J}_f \subset \mathcal{J}$ of the brands.

# Model [2]

- The profit of firm $f$ is:

$$\Pi_f = \sum_{j \in \mathcal{J}_f} p_j \; q_j - C_j(q_j)$$

  Firms compete in prices. For the moment, we assume Nash-Bertrand competition: each firm chooses its own prices to maximize profits under the conjecture that $\dfrac{\partial p_k}{\partial p_j} = 0$ for $j \in \mathcal{J}_f$ and $k \notin \mathcal{J}_f$.

- First order conditions for firm $f$: for $j \in \mathcal{J}_f$

$$q_j + \sum_{k \in \mathcal{J}_f} \left[ p_k - MC_k(q_k) \right] \; \frac{\partial D_k}{\partial p_j} = 0$$

# Model [3]

- We can write this system in vector form for each firm:

$$\mathbf{q}^f + \Delta\mathbf{D}^f \left[\mathbf{p}^f - \mathbf{MC}^f\right] = 0$$

$\mathbf{q}^f$ = column vector of $q_j'$s for every $j \in \mathcal{J}_f$
$\mathbf{p}^f$ = column vector of $p_j'$s for every $j \in \mathcal{J}_f$
$\mathbf{MC}^f$ = column vector of $MC_j'$s for every $j \in \mathcal{J}_f$
$\Delta\mathbf{D}^f$ = matrix of demand-price derivatives $\dfrac{\partial D_k}{\partial p_j}$ for every $j, k \in \mathcal{J}_f$

# Model [4]

- Solving for price-cost margins in this system:

$$\mathbf{p}^f - \mathbf{MC}^f = -\left[\Delta\mathbf{D}^f\right]^{-1}\mathbf{q}^f$$

- The RHS of this equation depends only on demand parameters, not costs. Given an estimated demand system, the vector of Price-Cost Margins under Nash-Bertrand competition (and a particular ownership structure of brands), $-\left[\Delta\mathbf{D}^f\right]^{-1}\mathbf{q}^f$, is known to the researcher.

$$p_j - MC_j = \phi_j$$

where $\phi_j$ is known given demand, prices, and quantities.

# Example: Single product firms & Logit model

- For single product firms, we have:

$$p_j - MC_j = - \left[ \frac{\partial D_j}{\partial p_j} \right]^{-1} q_j$$

- Logit demand system. $q_j = H \ s_j$, where $H$ is market size, $s_j$ is the market share of product $j$ and:

$$s_j = \frac{\exp \left\{ x_j' \beta - \alpha \ p_j \right\}}{1 + \sum_{k=1}^{J} \exp \left\{ x_k' \beta - \alpha \ p_k \right\}}$$

where $\beta$ and $\alpha$ are parameters.

- This demand system implies that $\dfrac{\partial D_j}{\partial p_j} = -\alpha \ q_j (1 - s_j)$. Therefore,

$$PCM_j \equiv p_j - MC_j = \frac{1}{\alpha (1 - s_j)}$$

# Example: Logit model: Multi-product firm

$$q_j + \sum_{k \in \mathcal{J}_f} PCM_k \frac{\partial D_k}{\partial p_j} \quad = \quad 0$$

- In Logit demand system: $\dfrac{\partial D_j}{\partial p_j} = -\alpha \; q_j (1 - s_j)$ and $\dfrac{\partial D_j}{\partial p_k} = \alpha \; q_j \; s_k$. And this implies:

$$s_j - \alpha \; PCM_j + \alpha \; s_j \sum_{k \in \mathcal{J}_f} PCM_k \; s_k \quad = \quad 0$$

- And, defining $\overline{PCM}_f \equiv \sum_{k \in \mathcal{J}_f} PCM_k \; s_k$, we have that:

$$PCM_j = \overline{PCM}_f = \frac{1}{\alpha \left( 1 - \sum_{k \in \mathcal{J}_f} s_k \right)}$$

# Logit model: Multi-product firm [2]

$$PCM_j = \frac{1}{\alpha \left(1 - \sum_{k \in \mathcal{J}_f} s_k\right)} \qquad \text{for any } j \in \mathcal{J}$$

- For the Logit demand model, a multi-product firm charges the same price-cost margin to all its products.
- This prediction does not extend to more general/flexible demand systems.
- Note also that a multi-product firm charges higher prices than a single-product firm:

$$\frac{1}{\alpha \left(1 - \sum_{k \in \mathcal{J}_f} s_k\right)} > \frac{1}{\alpha \left(1 - s_j\right)}$$

- This prediction is robust and it extends to Bertrand competition when products are substitutes.

# Multiproduct as source of market power

- We can write F.O.C. for firm $f$ product $j$ as:

$$
\begin{aligned}
PCM_j &= \left[\frac{-\partial D_j}{\partial p_j}\right]^{-1} q_j \\
&+ \left[\frac{-\partial D_j}{\partial p_j}\right]^{-1} \left[\sum_{k \in J_f;\ k \neq j} PCM_k \frac{\partial D_k}{\partial p_j}\right]
\end{aligned}
$$

- With substitutes, $\dfrac{\partial D_k}{\partial p_j} > 0$ for $k \neq j$, and the second term is positive.
- Selling multiple products contribute to increase the price-cost margin of each of the products.

# Collusion and other ownership structures

- Suppose that there is collusion between some of all the firms.
- We can represent a "collusion rink" using the following indicator variables:

$$\Theta_j^{R(f)} = \begin{cases} 1 & \begin{array}{l} \text{if product } j \text{ is owned by firm } f \\ \text{or by other firm in the collusion rink of firm } f \end{array} \\ 0 & \text{otherwise} \end{cases}$$

- For instance:
  - No collusion implies: $\Theta_j^{R(f)} = 1\{j \in \mathcal{J}_f\}$
  - Collusion of all firms: $\Theta_j^{R(f)} = 1$ for every $f$ and $j$

# Collusion [2]

- Firm $f$ maximizes its collusion rink profit:

$$\sum_{j=1}^{J} \Theta_{j}^{R(f)} \; [p_j \; q_j - C_j(q_j)]$$

- The F.O.C.s for firm $f$: for $j \in \mathcal{J}_f$

$$q_j + \sum_{k=1}^{J} [p_k - MC_k] \; \Theta_{k}^{R(f)} \; \frac{\partial D_k}{\partial p_j} = 0$$

- In vector form, using all the products that belong to the collusion rink $R(f)$

$$\mathbf{q}^{R(f)} + \left[\Delta\mathbf{D}^{R(f)}\right] \; \left[\mathbf{PCM}^{R(f)}\right] = 0$$

$\Delta\mathbf{D}^{R(f)} =$ matrix of demand-price derivatives $\dfrac{\partial D_k}{\partial p_j}$ for every $j, k$ in the collusion rink of firm $f$.

# Collusion [3]

- Such that:

$$\mathbf{PCM}^{R(f)} = - \left[ \Delta \mathbf{D}^{R(f)} \right]^{-1} \mathbf{q}^{R(f)}$$

# 2. Estimating MCs based on assumption of form of competition

# Estimation of MCs: Bertrand competition

- The researcher has data from $J$ products over $T$ markets, and knows the ownership structure:

$$\text{Data} = \{p_{jt},\, q_{jt},\, x_{jt} :\ j = 1, ..., J;\ t = 1, 2, ..., T\}$$

- Suppose that the demand function has been estimated in a fist step, such that there is a consistent estimator of the demand system $D_j(\mathbf{p}_t, \mathbf{x}_t)$.

- For every firm $f$, the research has an estimate of vector $-\left[\Delta \mathbf{D}_t^f\right]^{-1} \mathbf{q}_t^f$ for every firm $f$. Therefore, under the assumption of Bertrand competition she has consistent estimates of the vectors of MCs:

$$\mathbf{MC}_t^f = \mathbf{p}_t^f + \left[\Delta \mathbf{D}_t^f\right]^{-1} \mathbf{q}_t^f$$

# Estimation of MCs: Collusion

- Similarly, given an hypothetical collusion rink $R(f)$ represented by the indicators $\Theta_j^{R(f)}$, the researcher can construct $\left[\Delta \mathbf{D}^{R(f)}\right]^{-1} \mathbf{q}^{R(f)}$ and obtain the estimate of marginal costs:

$$\mathbf{MC}_t^{R(f)} = \mathbf{p}_t^{R(f)} + \left[\Delta \mathbf{D}_t^{R(f)}\right]^{-1} \mathbf{q}_t^{R(f)}$$

- Different hypothesis about collusion, or ownership structures of products (e.g., mergers), imply different Price-Cost margins and different estimates of marginal costs.

# Estimation the MC function

- After estimating the realized values of MCs, we can estimate the marginal cost function.

- Consider the following cost function:

$$C(q_{jt}) = \frac{1}{\gamma + 1} q_{jt}^{\gamma+1} \ \exp\{x'_{jt}\alpha + \omega_{jt}\}$$

- Such that:

$$MC_{jt} = q_{jt}^{\gamma} \exp\{x'_{jt}\alpha + \omega_{jt}\}$$

where $\omega_{jt}$ is unobservable to the researcher.

- The econometric model is:

$$\ln\left(MC_{jt}\right) = \gamma \ \ln\left(q_{jt}\right) + x'_{jt}\alpha + \omega_{jt}$$

# Estimation the MC function [2]

$$\ln\left(MC_{jt}\right) = \gamma \ \ln\left(q_{jt}\right) + x_{jt}'\alpha + \omega_{jt}$$

- We are interested in the estimation of the parameters $\alpha$ and $\gamma$.
- **Endogeneity:** The equilibrium model implies that $E(\ln\left(q_{tj}\right) \ \omega_{jt}) \neq 0$.
- Firms/products with larger $\omega_{jt}$ are less efficient in terms of costs (or products are more costly to produce), and this, all else equal, implies a smaller amount of output.

# Estimation the MC function [3]

$$\ln\left(MC_{jt}\right) = \gamma \ \ln\left(q_{jt}\right) + x_{jt}'\alpha + \omega_{jt}$$

- **Instrumental variables.** Suppose that $E(x_{kt} \ \omega_{jt}) = 0$ for any $(k, j)$.
- We can use as instruments for $\ln\left(q_{jt}\right)$ the characteristics of other firms/products.

$$E\left( \begin{bmatrix} x_{jt} \\ \sum_{k \neq j} x_{kt} \end{bmatrix} \ \left[\ln\left(MC_{jt}\right) - \gamma \ \ln\left(q_{jt}\right) - x_{jt}'\alpha\right] \right) = \mathbf{0}$$

- We could also use demand shifters as instruments.

# 3. Testing the Nature of Competition

# Testing the form of competition: With information on MCs

- Suppose that the researcher observes the true $MC_{jt}$. Or more realistically, observes a measure of costs, $S_{obs}^{MC}$, e.g., the mean value of the MCs of all products and firms in the industry; the mean value of the MC of one particular firm.

- Given an estimated demand system and an hypothesis about collusion, represented by a matrix of collusion rink dummies $\hat{}^{R} = \{\Theta_{j}^{R(f)}\}$, we can obtain the MCs under this hypothesis: $MC_{j}(\hat{}^{R})$.

- Let $S^{MC}(\hat{}^{R})$ the value of the statistic (e.g., mean value of all MCs) under the hypothesis $\hat{}^{R}$.

- We can use $S^{MC}(\hat{}^{R})$ and $S_{obs}^{MC}$ to construct a test of the null hypothesis $\hat{}^{R}$. For instance, if $S^{MC}$ is a vector of sample means, we could use a Chi-square test.

- This is the approach in Nevo (2001).

# Testing form of competition: Without info on MCs

- It is possible to consider $\Theta_j^{R(f)}$ as parameters to estimate, similarly as the conjectural variation parameters in the homogeneous product case.
- Using the estimated demand, our specification of the MC function, and the F.O.C.s of profit maximization, it is possible to jointly identify $\Theta_j^{R(f)}$ and parameters in MCs.
- We need similar rotation demand variables as in the homogeneous demand case (Nevo, 1998).

# Testing form of competition: Without info on MCs [2]

- Instead of estimating $\Theta_j^{R(f)}$ some papers have used non-nested hypothesis tests to test null hypothesis of Collusion against the alternative of Bertrand (or viceversa).

- The most commonly used non-nested tests procedures are: Cox-Test and Vuong-Test.

- Davidson & McKinnon provide an intuitive interpretation of these tests:

    - Obtain residuals from the model under $H_0$
    - Run regression of the residuals on variable in the model under $H_1$
    - Under null, #obs $\times$ R-square of this regression is Chi-square.

# 4. Bresnahan (1987) on automobiles

# Bresnahan (1987) : Descriptive Stats

TABLE I

| *Year* | (1) *Auto Production*[a] | (2) *Real Auto Price-CPI*[b] | (3) *% Change Auto Price-Cagan*[c] | (4) *Auto Sales*[d] | (5) *Auto Quantity Index*[e] |
|---|---|---|---|---|---|
| *1953* | 6.13 | 1.01 | NA | 14.5 | 86.8 |
| *1954* | 5.51 | 0.99 | NA | 13.9 | 84.9 |
| *1955* | 7.94 | 0.95 | −2.5 | 18.4 | 117.2 |
| *1956* | 5.80 | 0.97 | 6.3 | 15.7 | 97.9 |
| *1957* | 6.12 | 0.98 | 6.1 | 16.2 | 100.0 |

*Notes:* [a] Millions of units over the model year. [Source: *Automotive News*.]
[b] (CPI New automobile component)/CPI. [Source: *Handbook of Labor Statistics*.]
[c] Adjusted for quality change. [See Cagan (1971), especially pp. 232–3.]
[d] Auto output in constant dollars, *QIV* of previous year through *QIII* of named year, in billions of 1957 dollars. [Source: *National Income and Product Accounts*.]
[e] (4)/(2), normalized so 1957 = 100.

# Bresnahan (1987): Non-nested tests of conduct

TABLE III
COX TEST STATISTICS

| *Hypotheses* | *C* | *N–C* | *'p'* | *H* |
|---|---|---|---|---|
| a—1954 | | | | |
| *Collusion* | — | 0.8951 | 0.9464 | −1.934 |
| *Nash-Competition* | −2.325 | — | −0.8878 | −2.819 |
| *"Products"* | −3.978 | 3.029 | — | −1.604 |
| *Hedonic* | −12.37 | −10.94 | −13.02 | — |
| b—1955 | | | | |
| *Collusion* | — | −10.36 | −9.884 | −13.36 |
| *Nash-Competition* | −1.594 | — | 1.260 | 0.6341 |
| *"Products"* | −0.7598 | −4.379 | — | −1.527 |
| *Hedonic* | −3.353 | −8.221 | −5.950 | — |
| c—1956 | | | | |
| *Collusion* | — | 1.227 | 0.8263 | 1.629 |
| *Nash-Competition* | −2.426 | — | −4.586 | 0.8314 |
| *"Products"* | −3.153 | 0.9951 | — | 4.731 |
| *Hedonic* | −5.437 | −9.671 | −11.58 | — |

# Estimates Demand & MCs: Collusion 1954 & 1956, Collusion 1955

TABLE IV
PARAMETER ESTIMATES 1954–56, MAINTAINED SPECIFICATION

| *Parameters* | *1954*[a] | *1955*[b] | *1956*[a] |
|---|---|---|---|
| Physical Characteristics | | | |
| Quality Proxies | | | |
| *Constant* | 47.91 | 48.28 | 50.87 |
| | (32.8) | (43.2) | (29.4) |
| *Weight #/1000* | 0.3805 | 0.5946 | 0.5694 |
| | (0.332) | (0.145) | (0.374) |
| *Length "/1000* | 0.1819 | 0.1461 | 0.1507 |
| | (0.128) | (0.059) | (0.146) |
| *Horsepower/100* | 2.665 | 3.350 | 3.248 |
| | (0.692) | (0.535) | (0.620) |
| *Cylinders* | −0.7387 | −0.9375 | −0.9639 |
| | (0.205) | (0.115) | (0.186) |
| *Hardtop Dummy* | 0.9445 | 0.4531 | 0.4311 |
| | (0.379) | (0.312) | (0.401) |
| Demand/Supply | | | |
| $\mu$—*Marginal Cost* | 0.1753 | 0.1747 | 0.1880 |
| | (0.024) | (0.020) | (0.035) |
| $\gamma$—*Lower Endpoint* | 4.593 | 3.911 | 4.441 |
| | (1.49) | (2.08) | (1.46) |
| $V_{max}$—*Upper Endpoint* | $1.92E+7$ | $2.41E+7$ | $2.83E+7$ |
| | $(8.44E+6)$ | $(9.21E+6)$ | $(7.98E+6)$ |
| $\delta$—*Taste Density* | 0.4108 | 0.4024 | 0.4075 |
| | (0.138) | (0.184) | (0.159) |

# Estimates Demand & MCs: Bertrand 1954, 1955, 1956

- The estimated structural model under the maintained assumption of collusion in years 1954 & 1956 and Bertrand competition in1955 implies very stable coefficient estimates over the three years.
- That is, the observed changes in quantity and prices in 1955 can be fully explained by the change in conduct, and not by a change in demand or costs parameters.
- Instead, the models that impose Collusion over the three years, or Bertrand over the three years imply estimates of structural parameters with strong and implausible changes in demand and costs in year 1955.

# 5. Nevo (2001) on cereals

# Nevo (2001) on Cereals

- Ready-to-Eat (RTE) cereal market: highly concentrated; many apparently similar products, and yet price-cost margins (PCM) are high.
- What are the sources of market power? Product differentiation? Multi-product firms? Collusion?
- Nevo: (1) estimates a demand system of differentiated products for this industry; (2) recovers PCMs and compare them to rough/aggregate estimates of PCM at the industry level; (3) based on this comparison, tests Bertrand vs (full) Collusion [and rejects collusion]; (4) Under Bertrand, compares estimated PCMs with the counterfactual with single-product firms.

# Nevo (2001): Data

- A market is a city-quarter. IRI data on market shares and prices.
- 65 cities x 20 quarters [Q188-Q492] x 25 brands [total share is 43-62%].
- Most of the price variation is cross-brand (88.4%), the remainder is mostly cross-city, and a small amount is cross-quarter.
- Relatively poor brand characteristics so model includes brand fixed effects.

TABLE I

VOLUME MARKET SHARES

| | 88Q1 | 88Q4 | 89Q4 | 90Q4 | 91Q4 | 92Q4 |
|---|---|---|---|---|---|---|
| Kellogg | 41.39 | 39.91 | 38.49 | 37.86 | 37.48 | 33.70 |
| General Mills | 22.04 | 22.30 | 23.60 | 23.82 | 25.33 | 26.83 |
| Post | 11.80 | 10.30 | 9.45 | 10.96 | 11.37 | 11.31 |
| Quaker Oats | 9.93 | 9.00 | 8.29 | 7.66 | 7.00 | 7.40 |
| Ralston | 4.86 | 6.37 | 7.65 | 6.60 | 5.45 | 5.18 |
| Nabisco | 5.32 | 6.01 | 4.46 | 3.75 | 2.95 | 3.11 |
| C3 | 75.23 | 72.51 | 71.54 | 72.64 | 74.18 | 71.84 |
| C6 | 95.34 | 93.89 | 91.94 | 90.65 | 89.58 | 87.53 |
| Private Label | 3.33 | 3.75 | 4.63 | 6.29 | 7.13 | 7.60 |

*Source:* IRI Infoscan Data Base, University of Connecticut, Food Marketing Center.

TABLE III

DETAILED ESTIMATES OF PRODUCTION COSTS

| Item | \$/lb | % of Mfr Price | % of Retail Price |
|---|---|---|---|
| Manufacturer Price | 2.40 | 100.0 | 80.0 |
| Manufacturing Cost: | 1.02 | 42.5 | 34.0 |
| Grain | 0.16 | 6.7 | 5.3 |
| Other Ingredients | 0.20 | 8.3 | 6.7 |
| Packaging | 0.28 | 11.7 | 9.3 |
| Labor | 0.15 | 6.3 | 5.0 |
| Manufacturing Costs (net of capital costs)[a] | 0.23 | 9.6 | 7.6 |
| Gross Margin | | 57.5 | 46.0 |
| Marketing Expenses: | 0.90 | 37.5 | 30.0 |
| Advertising | 0.31 | 13.0 | 10.3 |
| Consumer Promo (mfr coupons) | 0.35 | 14.5 | 11.7 |
| Trade Promo (retail in-store) | 0.24 | 10.0 | 8.0 |
| Operating Profits | 0.48 | 20.0 | 16.0 |

[a] Capital costs were computed from ASM data.
*Source:* Cotterill (1996) reporting from estimates in CS First Boston Reports "Kellogg Company," New York, October 25, 1994.

TABLE VI

RESULTS FROM THE FULL MODEL[a]

| Variable | Means ($\beta$'s) | Standard Deviations ($\sigma$'s) | Interactions with Demographic Variables: Income | Income Sq | Age | Child |
|---|---|---|---|---|---|---|
| Price | −27.198 | 2.453 | 315.894 | −18.200 | — | 7.634 |
| | (5.248) | (2.978) | (110.385) | (5.914) | | (2.238) |
| Advertising | 0.020 | — | — | — | — | — |
| | (0.005) | | | | | |
| Constant | −3.592[b] | 0.330 | 5.482 | — | 0.204 | — |
| | (0.138) | (0.609) | (1.504) | | (0.341) | |
| Cal from Fat | 1.146[b] | 1.624 | — | — | — | — |
| | (0.128) | (2.809) | | | | |
| Sugar | 5.742[b] | 1.661 | −24.931 | — | 5.105 | — |
| | (0.581) | (5.866) | (9.167) | | (3.418) | |
| Mushy | −0.565[b] | 0.244 | 1.265 | — | 0.809 | — |
| | (0.052) | (0.623) | (0.737) | | (0.385) | |
| Fiber | 1.627[b] | 0.195 | — | — | — | −0.110 |
| | (0.263) | (3.541) | | | | (0.0513) |
| All-family | 0.781[b] | 0.1330 | — | — | — | |
| | (0.075) | (1.365) | | | | |
| Kids | 1.021[b] | 2.031 | — | — | — | |
| | (0.168) | (0.448) | | | | |
| Adults | 1.972[b] | 0.247 | — | — | — | |
| | (0.186) | (1.636) | | | | |
| GMM Objective (degrees of freedom) | | | 5.05 (8) | | | |
| MD $\chi^2$ | | | 3472.3 | | | |
| % of Price Coefficients > 0 | | | 0.7 | | | |

[a] Based on 27,862 observations. Except where noted, parameters are GMM estimates. All regressions include brand and time dummy variables. Asymptotically robust standard errors are given in parentheses.
[b] Estimates from a minimum-distance procedure.

Direct measure of mean value of the price-cost margin in the industry: 31%

TABLE VIII

MEDIAN MARGINS[a]

| | Logit (Table V column ix) | Full Model (Table VI) |
|---|---|---|
| Single Product Firms | 33.6% (31.8%–35.6%) | 35.8% (24.4%–46.4%) |
| Current Ownership of 25 Brands | 35.8% (33.9%–38.0%) | 42.2% (29.1%–55.8%) |
| Joint Ownership of 25 Brands | 41.9% (39.7%–44.4%) | 72.6% (62.2%–97.2%) |
| Current Ownership of All Brands | 37.2% (35.2%–39.4%) | — |
| Monopoly/Perfect Price Collusion | 54.0% (51.1%–57.3%) | — |

[a] Margins are defined as $(p - mc)/p$. Presented are medians of the distribution of 27,862 (brand-city-quarter) observations. 95% confidence intervals for these medians are reported in parentheses based on the asymptotic distribution of the estimated demand coefficients. For the Logit model the computation is analytical, while for the full model the computation is based on 1,500 draws from this distribution.