ECO 2901
EMPIRICAL INDUSTRIAL ORGANIZATION

Lecture 1: Introduction
Cournot and Conjectural Variations models

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Organization of the Course

- **Class Meetings:** Thursdays, 9:00-11:00am; room WO 25
- **Office hours:** Tuesdays and Thursdays 2:00-3:00pm
- **Evaluation:** Problem Set (50%); Final Exam (50%)
- **I expect that you:** (1) attend every class meeting; (2) read the papers/material before each lecture; (3) participate in class; (4) go through class notes and understand them; (5) do the problem set on time; (6) prepare for the final exam.

A General Description of this Course

- This courses deals with **models, methods, and empirical applications** in Industrial Organization (IO).
- IO deals with the behavior and competition of firms in markets. In Empirical IO, we use data and models to understand the factors that determine firm behavior in markets.
- This course emphasizes the need to combine **data, models, and econometric techniques** to understand how markets operate**.**

Topics

- The topics covered in the course are divided in three parts that correspond to three general forms of competition between firms.

- **PART I: STATIC MODELS OF COMPETITION IN PRICES OR QUANTITIES**

  Competition at the intensive margin between incumbent firms.

- **PART II: STATIC MODELS OF MARKET ENTRY AND SPATIAL COMPETITION**

  Competition at the extensive margin: entry and adoption decisions.

- **PART III: DYNAMIC GAMES OF OLIGOPOLY COMPETITION**

  Competition on investment decisions that affect future profits

Topics: Part I

PART I: STATIC MODELS OF COMPETITION IN PRICES OR QUANTITIES

- 1. Cournot competition and Conjectural Variations with homogeneous product
- 2. Bertrand competition with differentiated product
- 3. Conjectural Variations with differentiated product
- 4. Empirical models of price / quantity competition with incomplete information

Topics: Part II

PART II: STATIC MODELS OF MARKET ENTRY AND SPATIAL COMPETITION

- 5. Estimation of market entry models: complete and incomplete information
- 6. Market entry and spatial competition
- 7. Relaxing assumptions on information structure in discrete choice games
- 8. Static games of incomplete information with non-equilibrium beliefs

Topics: Part III

PART III: DYNAMIC GAMES OF OLIGOPOLY COMPETITION

- 9. The structure and estimation of dynamic games of oligopoly competition
- 10. Dynamic games of innovation
- 11. Dynamic games with non-equilibrium beliefs
- 12. Dynamic games with firms' learning

Today's Lecture

- **1. Some Basic Ideas in IO**
- **2. Topic 1: Cournot competition and Conjectural Variations with homogeneous product**

  2.1. Estimating Marginal Costs given a form of competition

  2.2. Estimating the form of competition when MCs are observed

  2.3. Estimating the form of competition & MCs

1. Some Basic Ideas in IO

Some Basic Ideas in IO (1)

- IO studies the behavior of firms in markets, their strategic interactions, and the implications on profits and consumer welfare.

- Some examples of type of firm decisions that we study in IO are:

  - Price and Quantity choice;
  - Investment in capacity, inventories, physical capital, ...;
  - R&D, patents;
  - Advertising;
  - Geographic location of plants and stores;
  - Product design;
  - Entry in new markets;
  - Adoption of new technologies;
  - Vertical relationships;

Specification of a Structural Model in EIO (1)

- To study competition in an industry, EIO researchers propose and estimate **structural models of demand and supply**.
- **What is an structural model in empirical IO?**
- Models of behavior where each agent (consumer, firm) maximizes a criterion function (expected utility, profit) given her information and resources.
- The **parameters are structural** in the sense that they describe **consumer preferences, firms' technology / costs, and institutional constraints**.
- Under the principle of **revealed preference**, these parameters are estimated using micro data on consumers' and firms' choices and outcomes.

Specification: Typical Structure of IO Models

1. **Model of consumer behavior (Demand)**

   - Product differentiation?

2. **Model for firms' costs**

   - Economies of scale; Economies of scope? Entry costs? Investment costs?

3. **Equilibrium model of static competition**
   - Price (Bertrand), Quantity (Cournot).

4. **Equilibrium model of market Entry-Exit**

5. **Equilibrium model of dynamic competition**

- Investment, advertising, quality, product characteristics, stores, etc.

2. Cournot competition and Conjectural Variations with homogeneous product

Introduction

- Firms' decisions of how much to produce (or sell) and the price to charge are fundamental determinants of firms' profits.
- These decisions are main sources of strategic interactions between firms.
- In the market for an homogeneous good, the price depends on the total quantity produced by all the firms in the industry.
- With differentiated products, demand for a firm's product depends on the prices of products sold by other firms in the industry.
- These **strategic interactions** have first order importance to understand competition and outcomes in most industries.
- For this reason, models of competition where firms choose prices or quantities are at the core of Industrial Organization.

Equilibrium model of competition

- The answer to many economy questions require not only the estimation of demand and cost functions but also the explicit specification of an equilibrium model of competition.
- For instance, suppose that we are interested in measuring the effects of:

  - a merger

  - a sales tax

  - firms' collusion

  - the entry of a new firm or product in the market

  - ...
- Answering these questions requires the explicit specification of a model of competition.

Empirical models of competition in quantities with homogeneous product

- We can distinguish three classes of applications of empirical models of competition in prices or quantities.
- [1] Estimation of firms' marginal costs.
- [2] Identification of the "form of competition".
- [3] Joint identification of marginal costs and "form of competition"

Estimation of firms' marginal costs

- In many empirical applications, the researcher has information on firms' prices and quantities sold, but **information on firms' costs is not always available**.
- In this context, empirical models of competition in prices or quantities may provide an approach to obtain estimates of firms' marginal costs, and of the structure of these costs.
- Given an assumption about competition (e.g., Cournot, Bertrand, Stackelberg, Collusion), the model predicts that for every firm i, $MR_i = MC_i$, where the concept of MR_i depends on the assumption of the model of competition.
- Based on a estimation of demand, we can construct estimates of firms' MR. Then, the equilibrium conditions of the model imply and estimate of MCs.

Identification of the "Nature of competition"

- Suppose that the researcher has data to estimate separately the demand function and firms' marginal costs (e.g., from the production function and firms' input prices).
- Given an assumption about the form or nature of competition in this industry (e.g., Perfect competition, Cournot, Collusion), the researcher can use the demand to obtain firms' marginal revenues, MR_i, and check if they are equal to the observed marginal costs, MC_i
- That is, the researcher can test if a particular form of competition is consistent with the data.
- In this way, the researcher can find the form of competition that is more consistent the data, **e.g., identify if there is evidence of firms' collusion**.

Joint identification of MCs and Nature of competition

- Suppose that the researcher does not have data on firms' MCs (or estimates of these MCs from production function).
- We will see that, under some conditions, it is still possible to use the estimated demand and equilibrium conditions **to jointly identify firms' marginal costs and the form of competition in the market**.
- This is the purpose of the **conjectural variation approach**.

2.1. Estimating marginal costs given a form of competition

Estimating MCs: Perfect competition

- We first illustrate this approach in the context of a **perfectly competitive industry for an homogeneous product**.
- The research has data on the market price and on firms' output for T periods of time (or geographic markets):

$$\text{Dataset } = \{p_t,\ q_{it} : \text{ for } i = 1, 2, ..., N_t \ \&\ t = 1, 2, ..., T\}$$

where N_t is the number of firms active at period t.

- The variable profit of firm i is:

$$\Pi_{it} = p_t\ q_{it} - C_i(q_{it})$$

- Under perfect competition, the marginal revenue of any firm i is the market price, p_t. Profit maximization implies:

$$p_t = MC_i(q_{it}) \quad \text{for every firm } i$$

where $MC_{it} \equiv C_i'(q_{it})$.

Estimating MCs: Perfect competition [2]

- Suppose that:

$$MC_i(q_{it}) = q_{it}^{\theta} \quad \exp\{\varepsilon_{it}^{MC}\}$$

where θ is a technological parameter and ε_{it}^{MC} is an unobservable that captures the cost efficiency of a firm.

- (i) Constant Returns to Scale (CRS), i.e., constant marginal cost or $\theta = 0$;
(ii) Decreasing Returns to Scale (DRS), i.e., increasing marginal cost or $\theta > 0$;
(iii) Increasing Returns to Scale (IRS), i.e., decreasing marginal cost or $\theta < 0$.

- Using the equilibrium condition, we can estimate θ and the cost efficiency ε_{it}^{MC} of every firm i.

Estimating MCs: Perfect competition [3]

- The equilibrium condition $p_t = MC_i(q_{it})$ implies the following regression model in logarithms:

$$\ln(p_t) = \theta \ \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- Using data on prices and quantities, we can estimate the slope parameter θ in this regression equation.

- Given an estimate of θ, we can estimate ε_{it}^{MC} as a residual from this regression, i.e., $\varepsilon_{it}^{MC} = \ln(p_t) - \theta \ \ln(q_{it})$.

- Therefore, we can estimate the marginal cost function of each firm, $MC_i(q_{it}) = q_{it}^{\theta} \ \exp\{\varepsilon_{it}^{MC}\}$.

Estimating MCs: Perfect competition [4]

$$\ln(p_t) = \theta \ \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- Estimation of this equation by OLS suffers of an **Endogeneity problem**.
- The equilibrium condition implies that the less efficient firms (with larger value of ε_{it}^{MC}) have a lower level of output.
- Therefore, the regressor $\ln(q_{it})$ is negatively correlated with the error term ε_{it}^{MC}.
- This negative correlation between the regressor and the error term implies that the OLS estimator provides a downward biased estimate of the true θ, e.g., the OLS estimate can show IRS (i.e., $\theta < 0$) when the true technology has DRS (i.e., $\theta > 0$).

Estimating MCs: Perfect competition [5]

$$\ln(p_t) = \theta \ \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- We can deal with this endogeneity problem by using **instrumental variables**.
- Suppose that X_t^D is a vector of observable variables that affect demand. These variables should be correlated with $\ln(q_{it})$ because demand shocks affect firms' output decisions.
- Under the assumption that these observable demand variables X_t^D are not correlated with ε_{it}^{MC}, we can use these variables as instruments for $\ln(q_{it})$ for the consistent estimation of θ.

Estimating MCs: Cournot competition

- We still have an homogeneous product industry and a researcher with data on quantities and prices over T periods of time: $\{p_t, q_{it}\}$ for $i = 1, 2, ..., N_t$ and $t = 1, 2, ..., T$.
- But now, the researcher assumes that the market is not perfectly competitive and that firms compete a la Nash-Cournot.
- The variable profit of firm i is $\Pi_{it} = p_t \, q_{it} - C_i(q_{it})$.
- The demand can be represented using the inverse demand function,

$$p_t = P\left(Q_t, X_t^D\right)$$

where $Q_t \equiv \sum_{i=1}^{N} q_{it}$ is the market total output, and X_t^D is a vector of exogenous market characteristic that affect demand.

Estimating MCs: Cournot competition [2]

- Each firm chooses its own output q_{it} to maximize profit.
- Since profit is equal to revenue minus cost, profit maximization implies the condition of **marginal revenue equal to marginal cost**.
- The marginal revenue function is:

$$
\begin{aligned}
MR_{it} &= \frac{d(p_t \; q_{it})}{dq_{it}} = p_t + \frac{dp_t}{dq_{it}} q_{it} \\
&= p_t + P'_Q\left(Q_t, X_t^D\right) \left[1 + \frac{dQ_{(-i)t}}{dq_{it}}\right] q_{it}
\end{aligned}
$$

where:
$P'_Q\left(Q_t, X_t^D\right)$ is the derivative of the inverse demand function with respect to total output;
$Q_{(-i)t}$ is the aggregate output of firms other than i.

Estimating MCs: Cournot competition [3]

$$MR_{it} = p_t + P'_Q\left(Q_t, X_t^D\right) \left[1 + \frac{dQ_{(-i)t}}{dq_{it}}\right] q_{it}$$

- $\dfrac{dQ_{(-i)t}}{dq_{it}}$ represents the **belief** or **conjecture** that firm i has about how other firms will respond by changing their output when this firm changes marginally its own output.

- Under the assumption of Nash-Cournot competition, this *belief* or *conjecture is zero:*

$$Nash - Cournot \Leftrightarrow \frac{dQ_{(-i)t}}{dq_{it}} = 0$$

- Firm i takes as fixed the quantity produced by the rest of the firms, $Q_{(-i)t}$, and chooses his own output q_{it} to maximize his profit.

Estimating MCs: Cournot competition [4]

- Therefore, the first order condition of optimality under Nash-Cournot competition is:

$$MR_{it} = p_t + P'_Q\left(Q_t, X_t^D\right)\ q_{it} = MC_i(q_{it})$$

- Since $P'_Q\left(Q_t, X_t^D\right) < 0$ (downward sloping demand curve), it is clear that $MR_{it} < p_t$.

- Therefore, if the marginal cost $MC_i(q_{it})$ is a non-decreasing function, we have that the optimal amount of output q_{it} under Cournot is smaller than under perfect competition.

- Oligopoly competition reduces output and consequently increases price.

Estimating MCs: Cournot competition [5]

- Consider the same specification of the cost function as before, with $MC_i(q_{it}) = q_{it}^{\theta} \exp\{\varepsilon_{it}^{MC}\}$.
- Suppose that the demand function has been estimated in a fist step, such that there is a consistent estimate of the demand function.
- The researcher can construct consistent estimates of marginal revenues $MR_{it} = p_t + P_Q' \left(Q_t, X_t^D\right) q_{it}$ for every firm i.
- Then, the econometric model can be described in terms of the following linear regression model in logarithms:

$$\ln\left(MR_{it}\right) = \theta \ \ln(q_{it}) + \varepsilon_{it}^{MC}$$

Estimating MCs: Cournot competition [6]

$$\ln\left(MR_{it}\right) = \theta \; \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- OLS estimation of this regression function suffers of the same endogeneity problem as in the perfect competition case.
- To deal with this endogeneity problem, we can use instrumental variables.
- As in the case of perfect competition, we can use observable variables that affect demand but not costs, X_t^D, as instruments.
- In the case of Cournot competition we can have additional types of instruments.

Estimating MCs: Cournot competition [7]

- Suppose that the researcher observes also some exogenous characteristics of firms that affect the marginal cost.
- For instance, suppose that there is information at the firm level on the firm's wage rate, or its capital stock, or its installed capacity.
- Let us represent these variables using the vector Z_{it}.
- Therefore, the marginal cost function is now $MC_i(q_{it}) = q_{it}^{\theta}$ $\exp\{Z_{it}\gamma + \varepsilon_{it}^{MC}\}$, where γ is a vector of parameters. The marginal condition of optimality, in logarithms, becomes:

$$\ln(MR_{it}) = \theta \ \ln(q_{it}) + Z_{it} \ \gamma + \varepsilon_{it}^{MC}$$

Estimating MCs: Cournot competition [8]

$$\ln\left(MR_{it}\right) = \theta\ \ln(q_{it}) + Z_{it}\ \gamma + \varepsilon_{it}^{MC}$$

- Note that the characteristics Z_{jt} of firms j other than i have an effect on the equilibrium amount of output of a firm i.
- The smaller Z_{jt} the more cost efficient firm j, the larger its output, the smaller price p_t and the marginal revenue MR_{it}, and the smaller q_{it} for any firm i other than j.
- Under the assumption that the vector of firm characteristics in Z are exogenous, i.e., $E(Z_{jt}\ \ \varepsilon_{it}^{MC}) = 0$ for any (i, j), we can use the characteristics Z_{jt} of other firms as instrumental variables.

Estimating MCs: Cournot competition [9]

- For instance, we can use $\sum_{j \neq i} Z_{jt}$ as an instrumental variables, and estimate θ and γ using the moment conditions:

$$E \left(\left[\begin{array}{c} Z_{it} \\ \sum_{j \neq i} Z_{jt} \end{array} \right] \left[\ln \left(MR_{it} \right) - \theta \ \ln(q_{it}) - Z_{it} \ \gamma \right] \right) = \mathbf{0}$$

- Or equivalently, using a 2SLS estimator.

2.2. Estimating the form of competition when marginal costs are known:

Conjectural variations model

Conjectural Variation Model: Homogeneous product

- Consider an industry where, at period t, the inverse demand curve is $p_t = P\left(Q_t, X_t^D\right)$, and firms, indexed by i, have cost functions $C_i(q_{it})$.

- Every firm i chooses q_{it} to maximize its profit, $p_t \; q_{it} - C_i(q_{it})$.

- Without further assumptions, the marginal condition for the profit maximization of a firm is **marginal revenue = marginal cost**, where the marginal revenue of firm i is:

$$MR_{it} = p_t + P'_Q\left(Q_t \; X_t^D\right) \; \left[1 + \frac{\partial Q_{(-i)t}}{\partial q_{it}}\right] \; q_{it}$$

- The term $\dfrac{\partial Q_{(-i)t}}{\partial q_{it}}$ represents the **belief** that firm i has about how the other firms in the market will respond if he changes its own amount of output marginally. We denote this **conjecture** or **belief** as the **conjectural variation of firm** i, CV_i.

Conjectural Variations: Nash-Cournot equilibrium

- **Nash conjecture** implies that:

$$CV_{it} \equiv \frac{\partial Q_{(-i)t}}{\partial q_{it}} = 0$$

- This conjecture implies the **Cournot equilibrium** (or Nash-Cournot equilibrium).

- For every firm i, the "perceived" marginal revenue is:

$$MR_{it} = p_t + P'_Q \left(Q_t \ X_t^D \right) \ q_{it}$$

and the condition $p_t + P'_Q \left(Q_t \ X_t^D \right) \ q_{it} = MC_i(q_{it})$ implies the Cournot equilibrium.

Other Conjectural Variations: Perfect Competition

- **Perfect competition**. For every firm i, $CV_{it} = -1$.
- Note that this conjecture implies that:

$$MR_{it} = p_t + P'_Q\left(Q_t\ X_t^D\right)\ \ [1 - 1]\ \ q_{it} = p_t$$

and the condition $p_t = MC_i(q_{it})$ implies the perfect competition equilibrium.

Other Conjectural Variations: Collusion

- **Collusion (Monopoly)**. For every firm i, $CV_{it} = N_t - 1$. This conjecture implies:

$$MR_{it} = p_t + P'_Q\left(Q_t \ X_t^D\right) \ N_t \ q_{it}$$

- This conjecture implies the equilibrium conditions:

$$p_t + P'_Q\left(Q_t \ X_t^D\right) \ N_t \ q_{it} = MC_i(q_{it})$$

- When firms have constant and homogeneous MCs, this condition implies:

$$p_t + P'_Q\left(Q_t \ X_t^D\right) \ Q_t = MC_t$$

  which is the equilibrium condition for the Monopoly (collusive or cartel) outcome.

Conjectural Variations: Nature of Competition

- The value of the beliefs CV are related to the "nature of competition", i.e., Cournot, Perfect Competition, Cartel (Monopoly).

$$\begin{array}{rl} \text{Perfect competition:} & CV_{it} = -1; \quad MR_{it} = p_t \\ \text{Nash-Cournot:} & CV_{it} = 0; \quad MR_{it} = p_t + P'_Q\left(Q_t\right)\ q_{it} \\ \text{Cartel all firms:} & CV_{it} = N_t - 1; \quad MR_{it} = p_t + P'_Q\left(Q_t\right)\ Q_t \end{array}$$

- CV is related to the **nature of competition**.
- If CV is negative, the degree of competition is stronger than Cournot. The closer to -1, the more competitive.
- If CV is positive, the degree of competition is weaker than Cournot. The closer to $N_t - 1$, the less competitive.

Conjectural Variation: Estimation

- Suppose a researcher has data on firms' quantities and marginal costs, and market prices over T periods of time:

$$\text{Data} = \{p_t,\ MC_{it},\ q_{it}\} \text{ for } i = 1, 2, ..., N_t \ \&\ t = 1, 2, ..., T$$

- Under the assumption that every firm chooses the amount of output that maximizes its profit given its belief CV_{it}, the following condition holds:

$$p_t + P'_Q\left(Q_t\ X_t^D\right)\ [1 + CV_{it}]\ q_{it} = MC_{it}$$

- And solving for the conjectural variation,

$$CV_{it} = \frac{p_t - MC_{it}}{-P'_Q\left(Q_t\ X_t^D\right)\ q_{it}} - 1 = \left[\frac{p_t - MC_{it}}{p_t}\right]\ \left[\frac{1}{q_{it} / Q_t}\right] |\eta_t| - 1$$

where η_t is the demand elasticity.

2.3. Estimating CV parameters without data on MCs

Estimating CV parameters without data on MCs

- So far, we have considered the estimation of CV parameters when the researcher knows both demand and firms' marginal costs.
- We now consider the case where the researcher **knows the demand, but it does not know firms' marginal costs**.
- Identification of CVs requires also de identification of MCs.
- Under some conditions, we can **jointly identify CVs and MCs** using the marginal conditions of optimality and the demand.

Data

- Researcher observes data:

$$\text{Data} = \left\{ P_t, \;\; q_{it}, \, X_t^D, \, X_{it}^{MC} \; : \;\; i = 1, ...N_t; \; t = 1, ..., T \right\}$$

- X_t^D are variables affecting consumer demand, e.g., average income, population.
- X_{it}^{MC} are variables affecting marginal costs, e.g., some input prices.

Model: Demand and MCs

- Consider the linear (inverse) demand equation:

$$P_t = \alpha_0 + \alpha_1 \ X_t^D - \alpha_2 \ Q_t + \varepsilon_t^D$$

with $\alpha_2 \geq 0$, and ε_t^D is unobservable to the researcher.

- Consider the marginal cost function:

$$MC_{it} = \beta_0 + \beta_1 \ X_{it}^{MC} + \beta_2 \ q_{it} + \varepsilon_{it}^{MC}$$

with $\beta_2 \geq 0$, and ε_{it}^{MC} is unobservable to the researcher.

Model: Profit maximization

- Profit maximization implies $MR_{it} = MC_{it}$, or equivalently:

$$P_t + \frac{dP_t}{dQ_t} \left[1 + CV_{it}\right] \; q_{it} = MC_{it}$$

- In the model above, $\dfrac{dP_t}{dQ_t} = -\alpha_2$. Therefore,

$$P_t - \alpha_2 \left[1 + CV_{it}\right] \; q_{it} = \beta_0 + \beta_1 \; X_{it}^{MC} + \beta_2 \; q_{it} + \varepsilon_{it}^{MC}$$

- Or equivalently,

$$P_t = \beta_0 + \beta_1 \; X_{it}^{MC} + \left[\beta_2 + \alpha_2 (1 + CV_{it})\right] \; q_{it} + \varepsilon_{it}^{MC}$$

- We assume now that $CV_{it} = CV$ for every observation i, t in the data.

Complete structural model

- The structural equations of the model are:

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t + \varepsilon_t^D$$

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \; X_t^{MC} + [\beta_2 + \alpha_2 (1 + CV)] \; q_{it} + \varepsilon_{it}^{MC}$$

- Using this model and data, **can we identify (estimate consistently, without asymptotic bias) the CV parameter?**
- First, we will see that NO. In this model we cannot separately identify CV and β_2.
- Second, we will see that a simple (and testable) modification of this model implies separate identification of CV and β_2.

Identification of demand parameters

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t + \varepsilon_t^D$$

- Endogeneity problem: in equilibrium, $cov(Q_t, \varepsilon_t^D) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, Q_t depends on X_t^{MC}. Note that X_{it}^{MC} does not enter in demand. If X_{it}^{MC} is not correlated with ε_t^D, then X_{it}^{MC} satisfies all the conditions for being a valid instrument (or the mean value of X_{it}^{MC} over the N_t firms at period t, $\overline{X}_t^{MC}$).
- Parameters α_0, α_1, and α_2 are identified using this IV estimator.

Identification of CV and MCs

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \, X_{it}^{MC} + [\beta_2 + \alpha_2(1 + CV)] \; q_{it} + \varepsilon_{it}^{MC}$$

- Endogeneity problem: in equilibrium, $cov(q_{it}, \varepsilon_{it}^{MC}) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, q_{it} depends on X_t^D. Note that X_t^D does not enter enter in the F.O.C. If X_t^D is not correlated with ε_{it}^{MC}, then X_t^D satisfies all the conditions for being a valid instrument.
- Parameters β_0, β_1, and $\gamma \equiv \beta_2 + \alpha_2(1 + CV)$ are identified using this IV estimator.

The identification problem

$$\text{F.O.C.:} \quad P_t = \quad \beta_0 + \beta_1 \; X_t^{MC} + \gamma \; q_{it} + \varepsilon_{it}^{MC}$$

- Note that we can identify the parameter γ, where $\gamma \equiv \beta_2 + \alpha_2(1 + CV)$, and the slope of inverse demand function, α_2.
- However, knowledge of γ and α_2 is not sufficient to identify separately CV and the slope of the MC, β_2.
- Suppose that $\gamma = 1$ and $\alpha_2 = 0.4$, such that we have the constraint:

$$1 = \beta_2 + 0.4 \; (1 + CV)$$

- This equation is satisfied by any of the following:
  [Perfect competition] $CV = -1$ and $\beta_2 = 1.0$
  [Cournot] $CV = 0$ and $\beta_2 = 0.6$
  [Cartel, with $N = 3$] $CV = N - 1 = 2$ and $\beta_2 = -0.2$

Identification problem: Graphical representation

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \, X_t^{MC} + \gamma \, q_{it} + \varepsilon_{it}^{MC}$$

- Define two hypothetical "marginal cost" functions (lines), MC_c and MC_m:

$$MC_c \equiv \beta_0 + \beta_1 X^{MC} + \gamma \, q$$

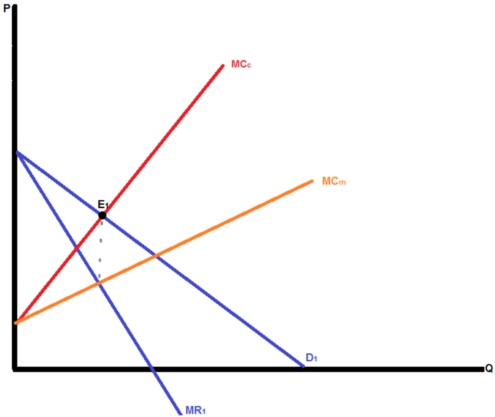
$$MC_m \equiv \beta_0 + \beta_1 X^{MC} + [\gamma - \alpha_2] \, q$$

- MC_c is the marginal cost function that rationalizes observed (P, q) under the hypothesis of Perfect Competition.

- MC_m is the marginal cost function that rationalizes observed (P, q) under the hypothesis of Monopoly (Collusion).

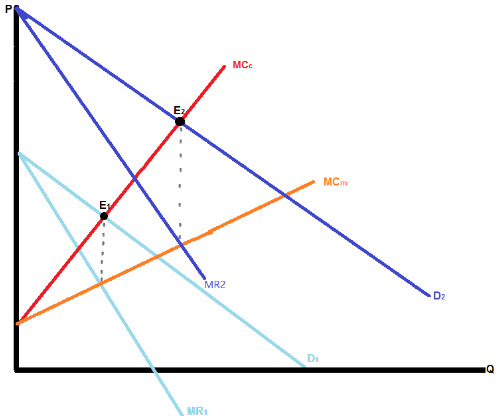
Identification problem: Graphical representation [2]

- Observed (P_1, q_1) can be explained by: Perfect competition with MC_c, or by Monopoly and MC_m.



Identification problem: Graphical representation [3]

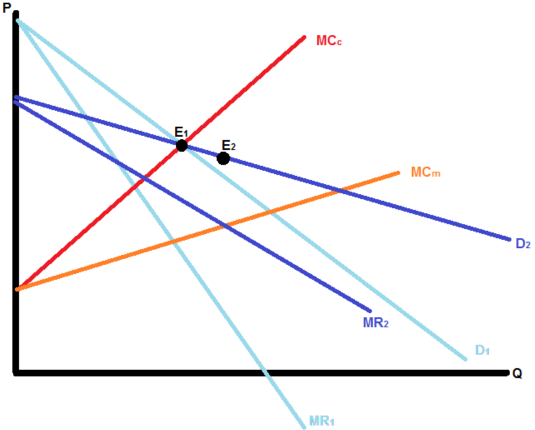
- Changes in X_t^D that move demand and MR curves horizontally do not help.



Solving the identification problem

- Solving the identification problem involves generalizing demand so that changes in exogenous variables do **more than just parallel shift** the demand curve and MR.
- In particular, we need to allow for additional exogenous variables that are capable of **rotating** the demand curve as well.
- "Demand Rotators" are exogenous variables affecting the slope of the demand curve:

Solving the identification problem [2]



- Under perfect competition we should remain in E_1.
- Under monopoly we should move from E_1 to E_2.

Solving the identification problem [3]

- Consider now the following demand equation:

$$P_t = \alpha_0 + \alpha_1 \ X_t^D - \alpha_2 \ Q_t - \alpha_3 \ [R_t \ Q_t] + \varepsilon_t^D$$

- R_t is an observable variable that affects the slope of the demand, i.e., the price of a substitute or complement product.
- Key condition: $\alpha_3 \neq 0$.
- That is, when R_t varies, there should be rotation (i.e., change in the slope of the demand curve).

Solving the identification problem [4]

- Given this demand model, we have that:

$$\frac{dP_t}{dQ_t} = -\alpha_2 - \alpha_3 \ R_t$$

- And the F.O.C. for profit maximization

$$P_t + \frac{dP_t}{dQ_t} \left[1 + CV\right] \ q_{it} = MC_{it}$$

become:

$$P_t + \left(-\alpha_2 - \alpha_3 \ R_t\right) \ \left[1 + CV\right] \ q_{it} = MC_{it}$$

or equivalently:

$$P_t = MC_{it} + \left(\alpha_2 + \alpha_3 \ R_t\right) \ \left[1 + CV\right] \ q_{it}$$

Solving the identification problem [5]

- Combining this F.O.C. with the MC function, $MC_{it} = \beta_0 + \beta_1 X_t^{MC} + \beta_2 \ q_{it} + \varepsilon_{it}^{MC}$, we have:

$$P_t = \beta_0 + \beta_1 X_{it}^{MC} + \beta_2 \ q_{it} + (\alpha_2 + \alpha_3 \ R_t) \ \ [1 + CV] \ \ q_{it} + \varepsilon_{it}^{MC}$$

- That we can represent using the following regression model:

$$P_t = \beta_0 + \beta_1 X_{it}^{MC} + \gamma_1 \ q_{it} + \gamma_2 \ \ (R_t \ q_{it}) + \varepsilon_{it}^{MC}$$

with $\gamma_1 \equiv \beta_2 + \alpha_2 \ [1 + CV]$ and $\gamma_2 \equiv \alpha_3 \ [1 + CV]$.

Solving the identification problem [6]

- The structural equations of the model are:

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t - \alpha_3 \; [R_t \; Q_t] + \varepsilon_t^D$$

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \; X_t^{MC} + \gamma_1 \; q_{it} + \gamma_2 \; (R_t \; q_{it}) + \varepsilon_{it}^{MC}$$

- Using this model and data, **we can identify separately CV and MC parameters.**

Identification of demand parameters

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t - \alpha_3 \; [R_t \; Q_t] + \varepsilon_t^D$$

- Endogeneity problem: in equilibrium, $cov(Q_t, \varepsilon_t^D) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, Q_t depends on X_t^{MC}. Note that X_t^{MC} does not enter in demand. If X_t^{MC} is not correlated with ε_t^D, then X_t^{MC} satisfies all the conditions for being a valid instrument.
- Parameters α_0, α_1, α_2, and α_3 are identified using this IV estimator.

Identification of CV and MCs

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \, X_{it}^{MC} + \gamma_1 \, q_{it} + \gamma_2 \, (R_t \, q_{it}) + \varepsilon_{it}^{MC}$$

- Endogeneity problem: in equilibrium, $cov(q_{it}, \varepsilon_{it}^{MC}) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, q_{it} depends on X_t^D and R_t. Note that X_t^D does not enter enter in the F.O.C. If X_t^D is not correlated with ε_{it}^{MC}, then X_t^D satisfies all the conditions for being a valid instrument.
- Parameters β_0, β_1, γ_1, and γ_2 are identified.

Identification of CV and MCs [2]

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \; X_t^{MC} + \gamma_1 \; q_{it} + \gamma_2 \; (R_t \; q_{it}) + \varepsilon_{it}^{MC}$$

- Note that:

$$\gamma_1 = \beta_2 + \alpha_2 \left[1 + CV\right]$$

$$\gamma_2 = \alpha_3 \left[1 + CV\right]$$

- It is clear that given γ_2 and α_3, we identify CV.
- And given γ_1, α_2, and CV we identify β_2.

Identification of CV and MCs [3]

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \; X_t^{MC} + \gamma_1 \; q_{it} + \gamma_2 \; (R_t \; q_{it}) + \varepsilon_{it}^{MC}$$

- with

$$\gamma_2 = \alpha_3 \left[1 + CV\right]$$

- The identification of CV is very intuitive: $1 + CV = \gamma_2 / \alpha_3$. It measures the ratio between the sensitivity of P_t with respect to $(R_t \; q_{it})$ in the F.O.C. and the sensitivity of P_t with respect to $(R_t \; Q_t)$ in the demand.

- Example: $\alpha_3 = 0.5$ and $N = 3$.
  [Perfect competition] $CV = -1$ such that $\gamma_2/\alpha_3 = 0$
  [Cournot] $CV = 0$ such that $\gamma_2/\alpha_3 = 1/0.5 = 2$
  [Cartel, with $N = 3$] $CV = N - 1 = 2$ such that $\gamma_2/\alpha_3 = 2/0.5 = 4$

3. An Empirical Application
Genesove & Mullin (RAND 1998)

An Application: US sugar industry 1890-1914

- Genesove and Mullin (GM) study competition in the US sugar industry during the period 1890-1914.

- Why this period? High quality information on the value of marginal costs because:
  (1) the production technology of refined sugar during this period was very simple;
  (2) there was an important investigation of the industry by the US anti-trust authority. As a result of that investigation, there are multiple reports from expert witnesses that provide estimates about the structure and magnitude of production costs in this industry.

The industry

- Homogeneous product industry.
- Highly concentrated during 1890-1914. The industry leader, American Sugar Refining Company (ASRC), had more than 65% of the market share during most of these years.
- Refined sugar companies buy "raw sugar" from suppliers in national or international markets, transformed it into refined sugar, and sell it to grocers.

Production technology

- Raw sugar is 96% sucrose and 4% water. Refined sugar is 100% sucrose. Process of transforming raw sugar into refined sugar is called "melting".

- Industry experts reported that the industry is a "fixed coefficient" (Leontiev) production technology.

$$q^{refined} = \min\{\lambda \; q^{raw}, \text{other inputs}\}$$

  where $q^{refined}$ is refined sugar output, q^{raw} is the input of raw sugar, and $\lambda \in (0, 1)$ is a technological parameter.

- Cost minimization requires:

$$q^{refined} = \lambda \; q^{raw}$$

Production technology: Costs

- Given this production technology, the marginal cost function is:

$$MC = c_0 + \frac{1}{\lambda} \ P^{raw}$$

- P^{raw} is the price of the input raw sugar (in dollars per pound), and c_0 is a component of the marginal cost that depends on labor and energy.
- Industry experts unanimously report that the value of the parameter λ was close to 0.93, and c_0 was around \$0.26 per pound.
- Therefore, the marginal cost at period (quarter) t, in dollars per pound of sugar, was:

$$MC_t = 0.26 + 1.075 \ P_t^{raw}$$

Data

- Quarterly US data for the period 1890-1914.
- The dataset contains 97 quarterly observations on industry output, price, price of raw sugar, imports of raw sugar, and a seasonal dummy.

$$\text{Data} = \{\ Q_t,\ \ P_t,\ P_t^{raw},\ IMP_t,\ S_t : t = 1, 2, ..., 97\}$$

- IMP_t represents the imports of raw sugar from Cuba.
- And S_t is a dummy variable for the Summer season: $S_t = 1$ is observation t is a Summer quarter, and $S_t = 0$ otherwise.
- The summer was a high demand season for sugar because most the production of canned fruits was concentrated during that season, and the canned fruit industry accounted for an important fraction of the demand of sugar.

Estimates of demand parameters

- GM estimate four different models of demand. The main results are consistent for the four models. Here I concentrate on the linear demand.

$$Q_t = \beta_t \ (\alpha_t - P_t)$$

- GM consider the following specification for α_t and β_t:

$$\begin{aligned}\alpha_t &= \alpha_L \ (1 - S_t) + \alpha_H \ S_t + e_t^D \\ \beta_t &= \beta_L \ (1 - S_t) + \beta_H \ S_t\end{aligned}$$

- α_L and $1/\beta_L$ are the intercept and the slope of the demand during the "Low Season" (when $S_t = 0$).

- And α_H and $1/\beta_H$ are the intercept and the slope of the demand during the "High Season" (when $S_t = 1$).

Estimates of demand parameters [2]

Parameter	Estimate	Standard Error
α_L	5.81	(1.90)
α_H	7.90	(1.57)
β_L	2.30	(0.48)
β_H	1.36	(0.36)

- According to these estimates, in the high season the demand shifts upwards but it also becomes more inelastic.
- The estimated price elasticities of demand in the low and the high season are $\varepsilon_L = 2.24$ and $\varepsilon_H = 1.04$, respectively.
- According to this, any model where firms have some market power predicts that the price cost margin should increase during the price season due to the lower price sensitivity of demand.

Estimates of demand parameters [3]

- Importantly, the seasonality in the demand of sugar introduces a "rotator" in the demand curve.
- The slope of the demand curve is steeper in the high season than in the low season.

Joint estimation of MC and CV

- GM specify a constant-cost Marginal Cost function for US sugar producers

$$MC_t = \beta_0^{MC} + \beta_1^{MC}\ P_t^{RAW} + \beta_2^{MC}\ q_t + \varepsilon_t^{MC}$$

- The $MR = MC$ condition yields:

$$P_t = \beta_0^{MC} + \beta_1^{MC}\ P_t^{RAW} + \gamma_1\ q_t + \gamma_2\ (S_t\ q_t) + \varepsilon_{it}^{MC}$$

- where

$$\begin{aligned} \gamma_1 &= \beta_2^{MC} + \frac{1}{\beta_L}[1 + CV] \\ \gamma_2 &= \left(\frac{1}{\beta_H} - \frac{1}{\beta_L}\right)[1 + CV] \end{aligned}$$

Direct estimate of CV using "true" MC

- The true MC function is $MC_t = 0.26 + 1.075\ P_t^{raw}$, that implies $\beta_0^{MC} = 0.26$, $\beta_1^{MC} = 1.075$, and $\beta_2^{MC} = 0$.
- Given the estimated demand and the true MCs, they obtain the following estimate of the CV parameter $\theta \equiv \dfrac{CV}{N}$.

Estimate of CV		
Parameter	Estimate (s.e.)	
$\theta \equiv \dfrac{CV}{N}$	0.100	(0.020)

- Therefore, we can reject the null hypothesis of Cournot competition in favor of some imperfect collusion.

Joint estimation of MC and CV

	Estimate	Direct Measure
CV/N	0.038 (0.024)	0.100 (0.020)
β_0	0.466 (0.285)	0.260 (-)
β_1	1.052 (0.085)	1.075 (-)

Joint estimation of MC and CV [3]

- Estimated cost parameters not too far from their "direct measures" which seems to validate CV approach.
- Based on the estimates of CV/N, the *predicted values* for the Lerner index in the low and in the high season are:

$$LI_L = 1.7\% \qquad LI_H = 3.6\%$$

- The *observed values* of LI were 3.8% in low season and 6.5% in high. The estimates using the rotation of demand approach under-estimate the actual market power in the industry, but not by much.