

MAT344 Midterm, March 6, 2018
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Section

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1. Consider the following function

$$f(z) = (e^x + e^{-x}) \cos y + i(e^x - e^{-x}) \sin y ,$$

where $x = \operatorname{Re} z$ and $y = \operatorname{Im} z$.

(a) Verify that $f(z)$ is entire.

(b) Verify that $f''(z) = f(z)$

2. Let γ be the oriented counterclockwise circle of radius 2 centered at 3. Find

$$\int_{\gamma} \frac{1}{z-2} \cos\left(\frac{1}{z}\right) dz .$$

3. Find the radius of convergence of the Taylor series of $\frac{z^2}{\cos(z)-1}$ at $z = i$ and at $z = 0$.

4. Find

$$\int_{\gamma} \frac{3}{1+z^2} dz ,$$

where γ is the circle of radius 2 centered at $2i$ oriented counterclockwise.

5. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \sin\left(\frac{\pi n}{3}\right) z^n .$$

(Hint: write a few first terms of the series.)

6. Pick a branch of \sqrt{z} for $z \in \{\operatorname{Re} z \geq 0, \operatorname{Im} z \geq 0\}$ and find

$$\int_{\gamma} \sqrt{z} dz ,$$

where γ is the arc of the circle $|z| = 4$ that lies in the first quadrant (with counterclockwise orientation).