Prerequisite Review with a Diagnostic Quiz

ECO220Y1Y: 2018/19; Written by Jennifer Murdock

Contents

1 Overview

This supplement reviews some prerequisite math and economics skills necessary for ECO220Y1Y. It places particular emphasis on skills we actually use in ECO220Y1Y: it is not a generic review. **Because the applications and exercises in this supplement use prerequisite skills in the specific context of our course, working with this supplement will actually help you learn some of the curriculum in ECO220Y1Y.**

Start with the diagnostic quiz in Section 2. Working through it will help you refresh your skills and start applying them in contexts relevant for our course. Section 10 gives the answers for you to grade yourself. It also directs you to specific sections of this document (where applicable). In addition to working through the diagnostic quiz, review each section of this supplement, making sure to carefully study anything that is not easy for you. Most sections include a few practice exercises with answers given in Section 9. If you are having particular difficulty, you may also need to consult your course materials from your prerequisite courses.

2 Diagnostic Quiz: Review, Refresh, and Update Prerequisite Skills

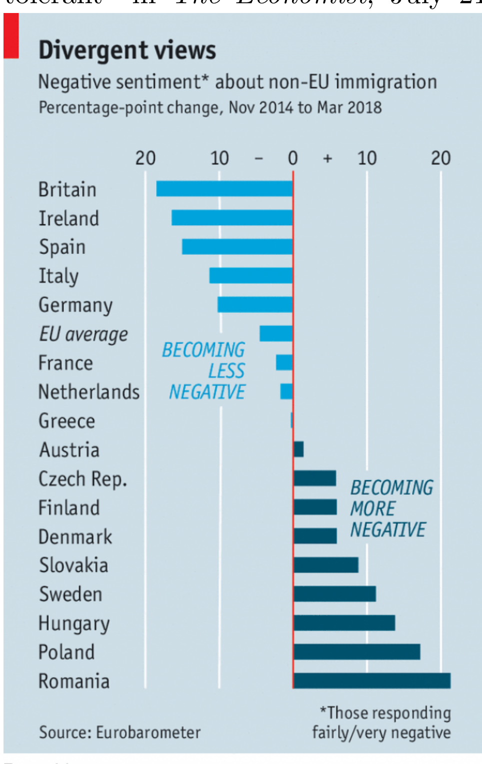
1. If an interest rate is 10 percent and rises by 5 *percentage points* then what is the new rate?

   (a) 10.05
   (b) 10.50
   (c) 10.75
   (d) 12.50
   (e) 15.00

2. If an interest rate is 10 percent and rises by 5 *percent* then what is the new rate?

   (a) 10.05
   (b) 10.50
   (c) 10.75
   (d) 12.50
   (e) 15.00

3. See the figure from "Xenophobia's ups and downs: Europeans, on the whole, are becoming more positive about foreigners, but those from the north and the east tend to be less tolerant" in *The Economist*, July 21, 2018 (link). (Next are some open-ended questions.)



   (a) If, in November 2014, 59% of Germans had negative sentiments about non-EU immigration, then *approximately* what percent had negative sentiments in March 2018?
   (b) If, in November 2014, 54% of Poles (i.e. people in Poland) had negative sentiments about non-EU immigration, then *roughly* what percent had negative sentiments in March 2018?

(c) Consider this quote from the article: “In November 2014 Eurobarometer began asking citizens of EU countries about their sentiments towards immigrants. Since then, the overall share of people who have negative feelings about arrivals from outside the bloc has fallen from 57% to 52%. Different regions, however, have been pulling in opposite directions.” What is the percent change? What is the percentage point change?

(d) Consider adding major cities to the figure (e.g. Paris). Suppose, in a particular city, 46% had negative sentiments about non-EU immigration in November 2014 but only 33% did in March 2018. This city would be between which two countries in the figure?

4. Consider the abstract for a 2016 NBER working paper “Does ‘Ban the Box’ Help or Hurt Low-Skilled Workers? Statistical Discrimination and Employment Outcomes When Criminal Histories are Hidden.” What is the employment rate for young, low-skilled black men?

   ABSTRACT: Jurisdictions across the United States have adopted “ban the box” (BTB) policies preventing employers from conducting criminal background checks until late in the job application process. Their goal is to improve employment outcomes for those with criminal records, with a secondary goal of reducing racial disparities in employment. However, removing information about job applicants’ criminal histories could lead employers who don’t want to hire ex-offenders to try to guess who the ex-offenders are, and avoid interviewing them. In particular, employers might avoid interviewing young, low-skilled, black and Hispanic men when criminal records are not observable. This would worsen employment outcomes for these already-disadvantaged groups. In this paper, we use variation in the details and timing of state and local BTB policies to test BTB’s effects on employment for various demographic groups. We find that BTB policies decrease the probability of being employed by 3.4 percentage points (5.1%) for young, low-skilled black men, and by 2.3 percentage points (2.9%) for young, low-skilled Hispanic men. These findings support the hypothesis that when an applicant’s criminal history is unavailable, employers statistically discriminate against demographic groups that are likely to have a criminal record.

   (a) 66.7%
   (b) 72.4%
   (c) 77.8%
   (d) 78.1%
   (e) 86.5%

5. Continuing with the “Ban the Box” research paper, what is the employment rate for young, low-skilled Hispanic men? (An open-ended question, not multiple choice.)

6. A receipt including taxes is $22.00. If the tax rate is 10%, what is the amount due before taxes?

   (a) 19.60
   (b) 19.80
   (c) 20.00
   (d) 20.20
   (e) 20.40

7. A receipt including taxes is \$X. If the tax rate is $r\%$, what is amount due before taxes?

   (a) X/r

   (b) $X * (100 - r)$

   (c) $X - (r/100) * X$

   (d) $\frac{X}{1+r/100}$

   (e) $\frac{X}{1-r/100}$

8. In a 2012 report "The Demand for Disaggregated Food-Away-From-Home and Food-at-Home Products in the United States," the USDA estimates the own-price elasticity of demand for many composite goods.

   (a) The own-price elasticity of dairy is -0.05 (Table 5, p. 18). Interpret -0.05.

   (b) The own-price elasticity of cereals and bakery is -0.58 (Table 5, p. 18). Interpret -0.58.

   (c) The own-price elasticity of white bread is -1.54 (Table 6, p. 20). Interpret -1.54.

9. When interpreting lines, we choose easy-to-understand units of measurement. For students, if $y = 25.3 + 0.002 * x$, where y is a quiz percentage mark and x is seconds spent studying, we say each extra *hour* of study is associated with a quiz score that is 7.2 percentage points higher ($= 0.002 * 60 * 60$). Each case below gives context, but the numbers are hypothetical. For now, do not worry about how these lines are obtained and how that affects the interpretation. Interpret the *slope*, carefully choosing the units of measurement for the x and y variables.

   (a) For Canadian workers, $y = 43.0283 + 6.1884 * x$ where y is annual salary measured in 1,000s of Canadian dollars (e.g. 76.022) and x is years of education (e.g. 13).

   (b) For people participating in a sleep study, $y = 6.4021 + 0.1562 * x$ where y is hours of sleep in a night (e.g. 6.541) and x is milligrams of a sleeping drug taken at bedtime (e.g. 10).

   (c) For corporate boards of publicly traded companies in North America, $y = 0.2452 - 0.0052 * x$ where y is the fraction of a corporate board that is female (e.g. 0.33) and x is the number of years since the corporation was founded (e.g. 10).

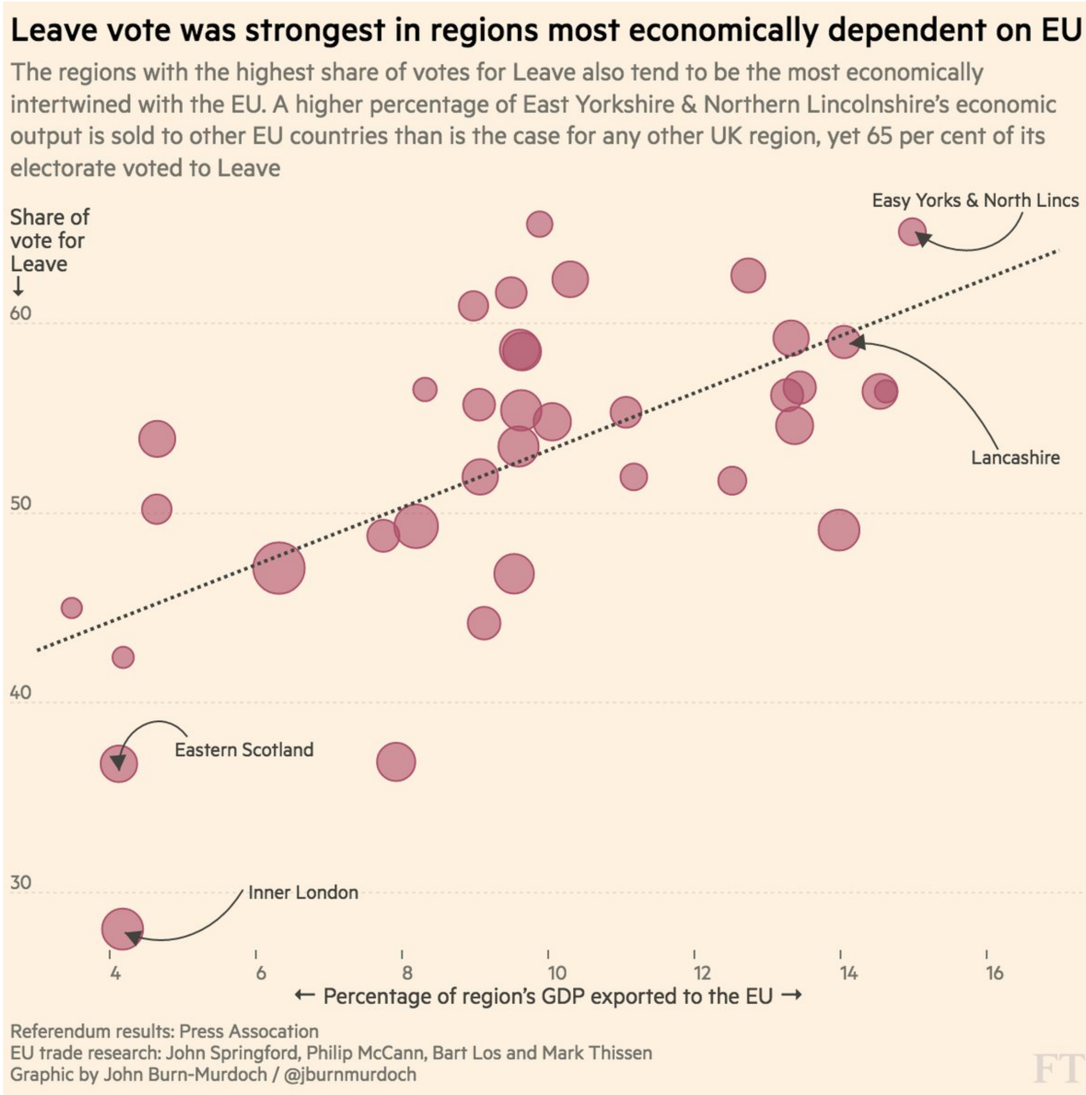
   (d) For the 500 most popular bottles of wine sold by the LCBO in 2016, $y = -64.0105 + 0.9681 * x$ where y is the price of a bottle of wine in dollars (e.g. \$16.99) and x is its rating (by a wine critic) on a scale of 0 to 100 (e.g. 89).

   (e) For 100 large retail firms operating in North America, $y = 0.2419 + 2.0585 * x$ where y is the fraction of employees that leave a company within one year of hire (e.g. 0.28) and x is the fraction of employees paid the minimum wage (e.g. 0.15).

   (f) For colonies of bacteria being exposed to varying intensities of heat in a lab, $y = 30,854,882,104 + 1,612,115,885 * x$ where y is the number of bacteria that die within five minutes and x is the temperature in Fahrenheit (e.g. 110). (Assume your readers are most comfortable with Celsius and recall $T_{Celsius} = (T_{Fahrenheit} - 32) * \frac{5}{9}$.)

   (g) For ten years of annual observations of household shopping trips in the European Union, $y = 0.8199 - 0.0104 * x$ where y is the share of shopping trips where the total purchase amount is less than 10 euro (e.g. 0.76) and x is the year (e.g. 2010).

10. Consider the figure below posted on Twitter by *The Financial Times* (FT) on June 25, 2016 https://twitter.com/FT/status/746681886131486720.



(a) Approximately, what is the equation of the dashed line? (Make sure to clearly define x and y and be careful with the units.)

(b) What would the (approximate) equation of the line be if the horizontal axis were the fraction of the region's GDP exported to the EU (instead of the percentage) but the other variable continued to be measured as in the FT figure?

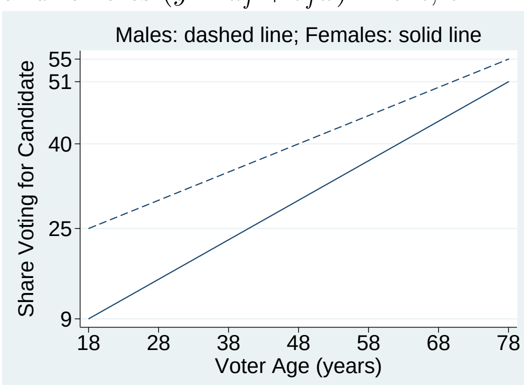
(c) What would the (approximate) equation of the line be if the vertical axis were the fraction voting for "Leave" (instead of the percentage) but the other variable continued to be measured as in the FT figure?

(d) What would the (approximate) equation of the line be if the horizontal axis were the fraction of the region's GDP exported to the EU (instead of the percentage) and the vertical axis were the fraction voting for "Leave" (instead of the percentage)?

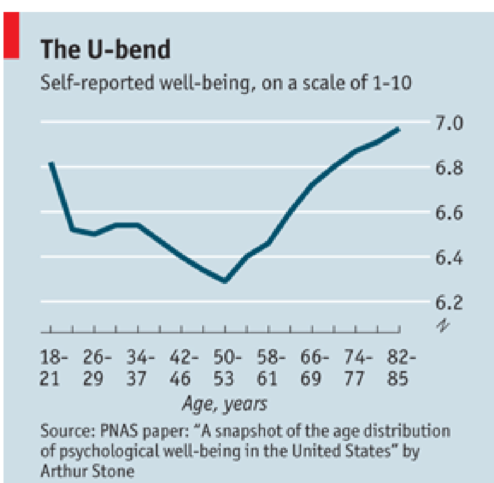
(e) Consider the original FT figure and how this relationship may differ by sex: instead of a single dashed line, two lines (one for female voters and one for male voters). (Of course, such an analysis would need to be based on exit polls, not actual voting data. Voting is anonymous and hence cannot be analyzed by sex or any other voter-specific information, only by geographic area.) Define a variable m to be equal to 1 if the voter is male and zero otherwise. Suppose that male voters are 5 percentage points more likely than female voters to vote “Leave” regardless of export dependency on the EU, which means that the dashed lines for males and females are parallel. Consider this equation $y = 35.7 + 1.5x + d * m$ where y is the percent voting for “Leave”, x is the percent of exports to the EU, m is equal to 1 if the voter is male and zero otherwise, and d is a constant.

i. Given the presented facts, what is the implied value of the constant d?
ii. What is the meaning of 35.7?
iii. If you used a variable f that is equal to 1 if the voter is female and zero otherwise, then what would be the values of the constants a, b and e in $y = a + bx + e * f$?

11. Consider a political candidate whose support varies depending on voters’ age and sex: s/he has higher support among older voters and among males. Let the variable x be voter age and y be the percent intending to vote for the candidate. Define m to be 1 for male voters and 0 otherwise. For the figure below, what are the values of the constants a, b, c and d in $y = a + bx + cm + d(x * m)$? Hint: First, *separately* find the equation for males ($y = a_m + b_m x$) and females ($y = a_f + b_f x$). Next, think about how the intercepts and slopes differ.



12. See the figure from “Age and happiness: The U-bend of life, why, beyond middle age, people get happier as they get older” in *The Economist*, December 16, 2010 (link).



A 2012 paper “The mystery of the U-shaped relationship between happiness and age” is the source of the numeric examples below. Define y to be happiness on a ten-point scale (where 10 is happiest and 0 is most miserable) and define x to be age in years.

(a) Consider $y = 8.4890 - 0.0417x + 0.0006x^2$ for ages 22 to 80. Draw a fully-labeled graph.

i. What is the slope when age is 25?

ii. What is the slope when age is 40?

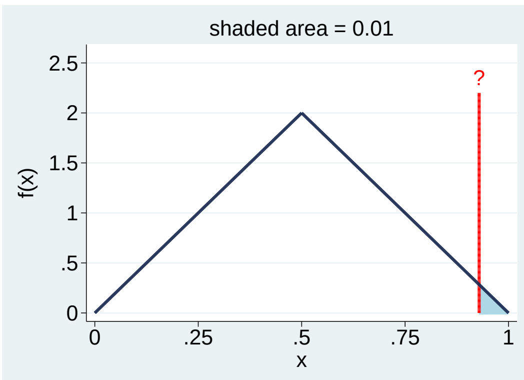
iii. What is the slope when age is 60?

iv. Why are your answers to the previous three parts different?

(b) How would the graph look different if the function were $y = 8.4890 - 0.0617x + 0.0006x^2$ instead of $y = 8.4890 - 0.0417x + 0.0006x^2$?

(c) How would the graph look different if the function were $y = 8.4890 - 0.0417x + 0.00075x^2$ instead of $y = 8.4890 - 0.0417x + 0.0006x^2$?

13. Given that the shaded area in the figure below is equal to 0.01, what is the value of "?"?



14. What is the slope of the function $ln(y) = 2.1 + 1.1x$ when $x = 1$?

15. Consider the statement: "as few as 15 murders." Letting x be the number of murders, which is a correct translation into a precise mathematical statement?

(a) $x > 15$

(b) $x < 15$

(c) $x <= 15$

(d) $x >= 15$

(e) $x >= 16$

16. Consider the statement: "at least half the votes." Letting x be the number of "yes" votes in a random sample of 10 voters, which is a correct translation into a precise mathematical statement?

(a) $x > 1/2$

(b) $x > 4$

(c) $x > 5$

(d) $x <= 5$

(e) $x >= 6$

17. Consider the statement: "at most 160 pounds." Letting x be the allowable weight of a horse-rider in pounds, which is a correct translation into a precise mathematical statement?

    (a) $x <= 159$

    (b) $x <= 160$

    (c) $x < 161$

    (d) $x >= 160$

    (e) $x >= 161$

18. Suppose $\sum_{k=1}^{50} x_k^2 = 258251$ and $\sum_{k=1}^{50} x_k = 3565$. Also, $\sum_{k=1}^{50} y_k = 3469$

    (a) Find $\sum_{k=1}^{50}(x_k - \bar{X})^2$.

    (b) Find $\sum_{k=1}^{50}(2 + 0.4 * x_k + 0.6 * y_k)$.

    (c) Suppose x_k is in thousands of dollars (i.e. 70 means $70,000). The variable z_k measures the same thing as x_k except that z_k is measured in dollars. Find $\sum_{k=1}^{50} z_k$ and $\sum_{k=1}^{50} z_k^2$.

19. Assume a, b, c, d are real, non-zero numbers. Assume that n, x, and y are also real numbers and that x and y are non-negative and n is positive.

    (a) Is it true or false that $\frac{1}{c/d} = \frac{d}{c}$?

    (b) Is it true or false that $\frac{c}{a+b} = \frac{c}{a} + \frac{c}{b}$?

    (c) Is it true or false that $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$?

    (d) Is it true or false that $\sqrt{x} = x^{0.5}$?

    (e) Is it true or false that $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$?

    (f) Is it true or false that $\sqrt{x * y} = \sqrt{x} * \sqrt{y}$?

    (g) Is it true or false that $\sqrt{x/n^2 + y/n^2} = \frac{\sqrt{x+y}}{n}$?

    (h) Is it true or false that $a^0 = 1$?

3 Percent changes versus percentage point changes

Is there a difference between a *percent change* and a *percentage point change*? Yes, these are very different. Percent changes make sense regardless of the units of measurement of the original variable. For example, if a city had 26 smog-alert days in 2013 and 32 in 2014, then it is up by 6 days, which is a 23 percent increase ($= 100 * (32 - 26)/26$). However, percentage point changes only make sense if a variable is measured as a percent: percentage points are the units of measurement for a variable measured as a percent. For example, if a city had an unemployment rate of 7.2 percent in 2013 and 8.1 percent in 2014, then unemployment is up by 0.9 percentage points.

Consider this sentence: "Lottery winners are 14.1 percentage points more likely to enroll in college the fall after their senior year, a 49.0 percent increase from the control mean of 28.8 percent."[1] What does this mean exactly? This means that 28.8 percent of lottery losers (the control group)

[1] This appears on page 16 of a 2013 NBER working paper entitled "The Medium-Term Impacts of High-Achieving Charter Schools on Non-Test Score Outcomes" by Will Dobbie and Roland Fryer http://www.nber.org/papers/w19581.

enroll in college while 42.9 percent of lottery winners (the treatment group) enroll. There are two different ways to describe this (large) difference in these percents. You could say that the lottery winners are 14.1 *percentage points* more likely to enroll (= 42.9 − 28.8). Alternatively, you could say that the lottery winners are 49.0 *percent* more likely to enroll ($100 * (42.9 - 28.8)/28.8$). Obviously 14.1 and 49.0 are very different numbers: percent and percentage point mean *different* things.

Suppose the sentence were shorter: "Lottery winners are 14.1 percentage points more likely to enroll in college the fall after their senior year, a 49.0 percent increase." Given just that information can you figure out what percent of the control group (lottery losers) enrolled in college? Yes. Define x to be the percent of the control group that enroll in college the fall after their senior year. We know that $\frac{x+14.1}{x} = (1 + 0.49)$. We can solve to obtain $x = 28.8$ percent. Hence, in the original sentence, the last phrase ("from the control mean of 28.8 percent") is redundant. However, given that the authors are interpreting the results, it is a great idea to include that extra phrase to make sure the reader understands.

Section 3 Exercises:

1. Suppose that the interest rate is currently 2.76%.
   (a) What would the new interest rate be if the bank increased it by 1 percentage point?
   (b) What would the new interest rate be if the bank increased it by 1 percent?

4 Functions

A function f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B. It is common to write $y = f(x)$. For example, $f(x) = 2x + 1$ can be written $y = 2x + 1$. This section reviews linear and nonlinear functions. For nonlinear functions, Section 5 discusses slopes.

4.1 Linear Functions

A line can be written in terms of the y-intercept, a, and the slope, b, where a and b are constants.

$$y = a + bx \tag{1}$$

The intercept, a, is the value of y when x is equal to zero. The slope, b, can be found by using any two points, (x_1, y_1) and (x_2, y_2), that a line passes through (so long as the line is not perfectly vertical, in which case the slope is not defined: it is infinitely steep).

$$b = \frac{y_2 - y_1}{x_2 - x_1} \tag{2}$$

One way to find the equation of a line is to use the "point-slope" formula, which requires that you know the slope of the line, b, and one point the line passes through (x_1, y_1):

$$y - y_1 = b(x - x_1) \tag{3}$$

Section 4.1 Exercises:

1. Consider a line that passes through the points (0,0) and (10,10).
   (a) What is the equation of the line? (Write it in the same format as Equation 1.)
   (b) According to the line, what happens to the value of y when x increases from 4 to 5?
   (c) According to the line, what happens to the value of y when x increases from 6 to 7?
   (d) Why did you get the same answer for (b) and (c)?
   (e) According to the line, what happens to the value of y when x decreases from 10 to 9?
   (f) According to the line, what is the value of y when x equals 0?
2. Consider a line that passes through the points (1,2) and (2,2).
   (a) What is the equation of the line? (Write it in the same format as Equation 1.)
   (b) According to the line, what happens to the value of y when x increases from 1 to 100?
3. Consider a line that passes through the points (1,2) and (2,0).
   (a) What is the equation of the line? (Write it in the same format as Equation 1.)
   (b) According to the line, what happens to the value of y when x increases from 1 to 2?
4. Consider the line $y = 10 - x$. All of the question subparts are relative to this "original" line.
   (a) Imagine a sketch of the line. What is the equation of new line that is parallel to the original but is everywhere two units higher?
   (b) What is the equation of a line that is parallel to the original but is everywhere three units lower?
   (c) What is the equation of a line that has a slope twice as steep but the same y-intercept?
   (d) What is the equation of a line that has a slope twice as steep but the same x-intercept?

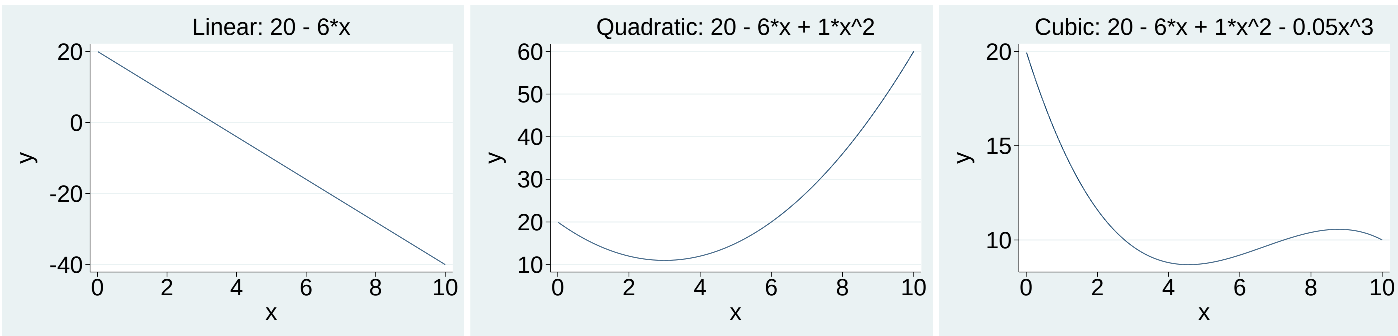
4.2 Power Functions and Polynomials

One type of nonlinear function is power functions: $f(x) = x^a$, where a is a constant. Recall the rules:

- $f(x) = x^a x^b = x^{a+b}$
- $f(x) = (x^a)^b = x^{ab}$
- $f(x) = \frac{x^a}{x^b} = x^{a-b}$
- $f(x) = x^{1/a} = \sqrt[a]{x}$ (which implies $f(x) = x^{0.5} = \sqrt{x}$)
- $f(x) = x^0 = 1$
- $f(x) = \frac{1}{x^a} = x^{-a}$ (which implies $f(x) = \frac{1}{x} = x^{-1}$)

A polynomial function is of the form $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{n-1}x^{n-1} + a_nx^n$, where a_i for $i = 1, 2, 3, ..., n$ are $n+1$ constants. The numbers $a_0, a_1, ..., a_n$ are the coefficients and n is the degree. For example, if $n = 3$ and $a_i = 2$ for all $i = 1, 2, 3$ then we'd have $f(x) = 2 + 2x + 2x^2 + 2x^3$, which is a third degree polynomial, which is also called a cubic function. If $n = 2$ then we have a second degree polynomial, which is also called a quadratic function or a parabola. If $n = 1$ then we have first degree polynomial, which is also called a line.

The higher the degree of the polynomial the more twists and turns it allows: it becomes more *flexible*. When researchers talk about a *flexible functional form*, it means they have included terms in their model (such as higher order polynomials) that allow the model to fit the twists and turns in the data. However, researchers also value *parsimonious* models: in other words, use the most simple possible.



For a quadratic function, recall the quadratic formula that allows you solve quadratic equations (even if you cannot factor): if $f(x) = ax^2 + bx + c$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Section 4.2 Exercises:

1. Consider $w = 23.1 + 2.4z + 5.9z^2 - 6.2z^3$. What is the degree of this polynomial? The coefficients?

2. Solve $5x^2 + 3x - 3 = 0$.

4.3 Exponential and Logarithmic Functions

An exponential function is of the form $f(x) = a^x$, where a is a positive constant. (A common value of the constant a is e: $f(x) = e^x$. Recall, $e \approx 2.71828$.)

- If $x = n$, n a positive integer, then $a^n = a * a * ... * a$
- If $x = 0$ then $a^0 = 1$
- If $x = -n$, n a positive integer, then $a^{-n} = \frac{1}{a^n}$
- If x is rational, $x = m/n$, where m and n are integers and $n > 0$, then $a^x = x^{m/n} = \sqrt[n]{a^m}$

Laws of Exponents:

- $a^{x+y} = a^x a^y$
- $a^{x-y} = \frac{a^x}{a^y}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$

The inverse of the exponential function $f(x) = a^x$ is the logarithmic function to the base a. (The symbol $\Leftrightarrow$ is read "is equivalent to.")

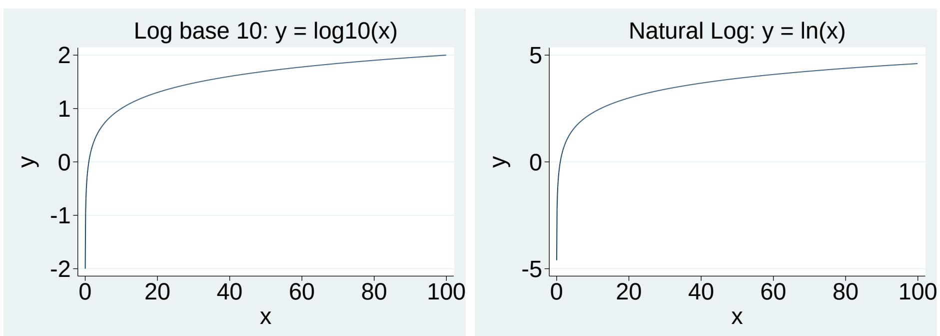
$$log_a x = y \Leftrightarrow a^y = x$$

The logarithmic function, log_a, has a domain (possible inputs: x values) of $(0, \infty)$. The logarithm of 0 or a negative number does not exist. Its range (possible outputs: y values) is all real numbers.

Rules for Logarithms:

- For all x: $log_a(a^x) = x$
- For $x > 0$: $a^{log_a x} = x$

A logarithm with base e is the natural logarithm, $log_e(x) = ln(x)$, where e is a constant equal to ≈ 2.71828 and $ln(e) = 1$. Among economists, "logarithm" means "natural logarithm." It is common to write $log()$ even when talking about a natural logarithm. Popular software reflects this convention. In STATA both the function log and ln return the natural logarithm: you must type log10 to get the base 10 logarithm. However, Excel is an exception: log means a base 10 logarithm.



Rules for Natural Logarithms:

- For all x: $ln(e^x) = x$
- For $x > 0$: $e^{ln(x)} = x$
- $ln(xy) = ln(x) + ln(y)$
- $ln\left(\frac{x}{y}\right) = ln(x) - ln(y)$
- $ln(x^y) = yln(x)$
- $ln(e) = 1$
- $ln(1) = 0$

5 Derivatives

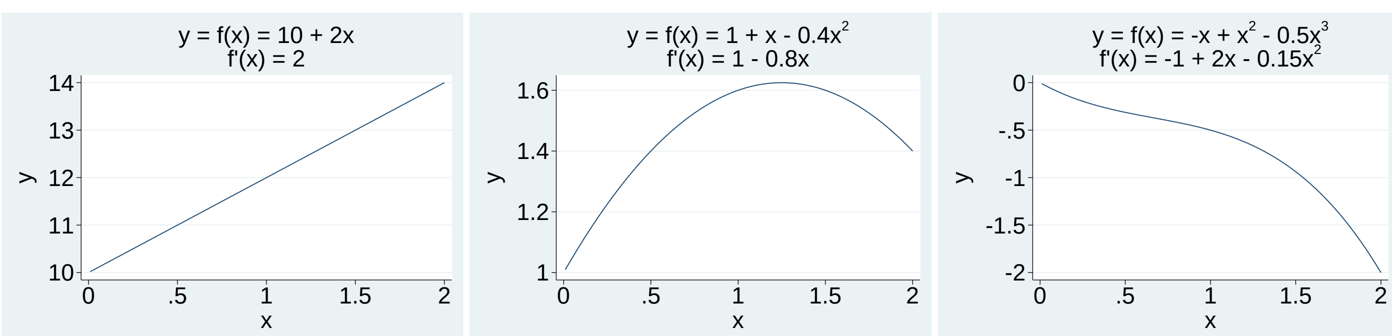
To find the rate of change, slope, at a particular point, take the derivative of a function. It common to write the derivative of a function as: $f'(x)$, y' and $\frac{dy}{dx}$. Derivatives are useful in economics. For example, to obtain MC (marginal cost) from TC (total cost), take the derivative of TC. While we only need derivatives for part of ECO220Y, we work with functions – both linear and nonlinear – extensively, which adds much value to reviewing derivatives.

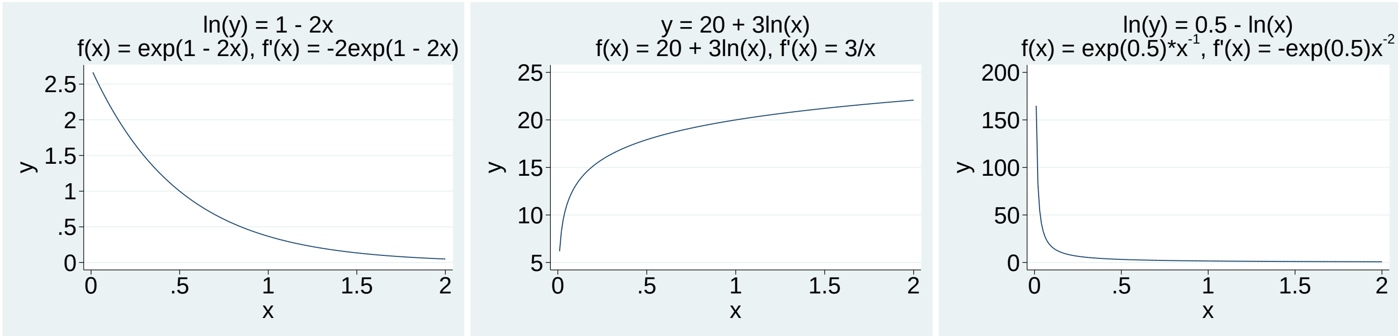
Rules of Differentiation: (Suppose c is a constant and $f'(x)$ and $g'(x)$ exist)

- If the function is constant, $f(x) = c$, then $f'(x) = 0$
- If n is any real number and $f(x) = x^n$, then $f'(x) = nx^{n-1}$ ("Power Rule")
- If the function is exponential, $f(x) = e^x$, then $f'(x) = e^x$
- If the function is exponential, $f(x) = a^x$, then $f'(x) = a^x ln(a)$
- If the function is a natural log, $f(x) = ln(x)$ then $f'(x) = \frac{1}{x}$
- If the function is a log base a, $f(x) = log_a(x)$ then $f'(x) = \frac{1}{xln(a)}$
- If $h(x) = cf(x)$ then $h'(x) = cf'(x)$
- If $h(x) = f(x) + g(x)$ then $h'(x) = f'(x) + g'(x)$
- If $h(x) = f(x) - g(x)$ then $h'(x) = f'(x) - g'(x)$
- If $h(x) = f(x)g(x)$ then $h'(x) = f'(x)g(x) + f(x)g'(x)$ ("Product Rule")
- If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ ("Quotient Rule")
- If F is the composite function defined by $F(x) = f(g(x))$, then $F'(x) = f'(g(x))g'(x)$ ("Chain Rule": In words, take the derivative of the outside function, $f(x)$, keeping the inside function, $g(x)$, and then multiply by the derivative of the inside function, $g'(x)$.)

For example, what is the derivative of $ln(y) = a + bx + cx^2$ with respect to x? To simplify, rewrite as $y = e^{a+bx+cx^2}$. Use the Chain Rule to obtain $y' = e^{a+bx+cx^2}(b + 2cx)$.

Next, consider six examples, illustrated with graphs, that highlight some specific types of functions highly relevant for ECO220Y1Y.





Notice that, except for the linear function with a constant slope, the slope depends on the value of x (i.e. where you are on the curve). In some cases the sign of the slope depends on the value of x.

- A *monotonic function* is one that is either always increasing or decreasing: in other words, the *sign* of the slope does not vary. Examples of monotonic functions: $y = a + bx$, $ln(y) = a + bx$, $y = a + bln(x)$ and $ln(y) = a + bln(x)$ where a and b are constants.
- A *non-monotonic function* is one that is sometimes increasing and sometimes decreasing: in other words, the *sign* of the slope varies. Examples: quadratic and higher degree polynomials.

5.1 Partial Derivatives

There is an important moment in ECO220Y requiring partial derivatives: multiple regression. Consider a function of two variables, x and w, such that $y = f(x, w)$. It common to write the partial derivative with respect to x as $f_x(x, w)$, $\frac{\partial f}{\partial x}$, $\frac{\partial}{\partial x} f(x, w)$, and $\frac{\partial y}{\partial x}$ and likewise for w.

Rules of Partial Differentiation:

- To find the partial derivative with respect to x, treat w as a constant and differentiate $f(x, w)$ with respect to x
- To find the partial derivative with respect to w, treat x as a constant and differentiate $f(x, w)$ with respect to w

For example, consider $y = f(x, w) = a + bx + cw$, where a, b, and c are constants. The partial derivative with respect to x is $\frac{\partial y}{\partial x} = b$ and the partial derivative with respect to w is $\frac{\partial y}{\partial w} = c$. Remember that this means b is the change in y given a change in x while *holding w constant.* Similarly, c is the change in y given a change in w while *holding x constant.*

As another example, consider $y = f(x, w) = a + bx + cw + d(x * w)$, where a, b, c, and d are constants. $\frac{\partial y}{\partial x} = b + dw$ and $\frac{\partial y}{\partial w} = c + dx$. In other words, the slope of the relationship between y and x depends on the level of w. These kinds of interactions are very common. A classic example in economics are the complementarities between labor and capital in producing output: the productivity of labor often depends on capital. For example, the value of a farmer spending an extra hour tilling his field greatly depends on whether or not he can use a tractor. In this example the farm output is the y variable and level of labor and capital are the x and w variables.

Section 5 Exercises:

1. What is the slope of $y = 22.1 + 0.8x - 0.05x^2$ when $x = 2$?

2. What is the slope of $ln(y) = 4.2 - 2.1ln(x)$ when $x = 10$?

3. What is the effect on y of increasing x by one unit if $y = 10 + 1x + 3w + 4(x * w)$ and $w = 2$?

6 Inequalities, Intervals, and Absolute Values

Inequalities

Consider any two real numbers a and b:

$a < b$	a is less than b	a is *strictly* less than b
$a \leq b$	a is less than or equal to b	a is *weakly* less than b
$a > b$	a is greater than b	a is *strictly* greater than b
$a \geq b$	a is greater than or equal to b	a is *weakly* greater than b
$a \neq b$	a is not equal to b	

An important skill is being able to translate from plain English statements about inequalities to the corresponding precise mathematical statement. Consider these illustrations, remembering that there are many ways to precisely state something in plain English: these are not a comprehensive list.

- "At least four people" translates into $x \geq 4$ where x is the integer number of people. This is a weak inequality because the statement implies that exactly four people is a possibility (four is at least four). It would also be correct to write $x > 3$, which is equivalent to $x \geq 4$ when x is an integer.

- "More than one defective item" translates into $x > 1$ where x is the integer number of defective items. This is a strict inequality because the statement implies that exactly one is excluded (one is not more than one). It would also be correct to write $x \geq 2$, which is equivalent to $x > 1$ when x is an integer.

- "Fewer than 10 heat alert days" translates into $x < 10$ where x is the integer number of days. This is a strict inequality because the statement implies that exactly ten is excluded (ten is not fewer than ten). It would also be correct to write $x \leq 9$, which is equivalent to $x < 10$ when x is an integer.

- "As few as 3 complaints" translates into $x \leq 3$ where x is the integer number of complaints. This is a weak inequality because the statement implies that exactly three complaints is a possibility. It would also be correct to write $x < 4$, which is equivalent to $x \leq 3$ when x is an integer. (Note: If you are confused about the direction of the inequality, "as few as" means that few or fewer: "as few as" implies that the amount is surprisingly small.)

- "No more than 2 pieces of carry-on luggage" translates into $x \leq 2$ where x is the integer number of pieces. This is a weak inequality because the statement implies that exactly two pieces is a possibility (two is no more than two). It would also be correct to write $x < 3$, which is equivalent to $x \leq 2$ when x is an integer.

- "As few as 10 percent of people exit during a fire alarm" translates into $x \leq 10$ where x is the percent that exit. x is *not* logically restricted to integer values (9.25 percent is possible).

- "Weigh less than 25 kg" translates into $x < 25$ where x is weight. This is a strict inequality because the statement implies that exactly 25 is excluded (25 is not less than 25).
- "Growth of 2.5 percent at a minimum" translates into $x \geq 2.5$ where x is the percentage growth rate. This is a weak inequality because the statement implies that exactly 2.5 is a possibility.

Rules for Inequalities:

- If $a < b$, then $a + c < b + c$
- If $a < b$ and $c < d$, then $a + c < b + d$
- If $a < b$ and $c > 0$, then $ac < bc$
- If $a < b$ and $c < 0$, then $ac > bc$
- If $0 < a < b$, then $\frac{1}{a} > \frac{1}{b}$

Intervals

Intervals are sets of real numbers. For real numbers a and b where $a < b$, possible intervals are:

(a, b)	$\{x \mid a < x < b\}$	Open interval of all values of x such that $x > a$ and $x < b$
$[a, b]$	$\{x \mid a \leq x \leq b\}$	Closed interval of all values of x such that $x \geq a$ and $x \leq b$
$[a, b)$	$\{x \mid a \leq x < b\}$	Interval of all values of x such that $x \geq a$ and $x < b$
$(a, b]$	$\{x \mid a < x \leq b\}$	Interval of all values of x such that $x > a$ and $x \leq b$
(a, ∞)	$\{x \mid x > a\}$	Infinite interval of all values of x such that $x > a$
$[a, \infty)$	$\{x \mid x \geq a\}$	Infinite interval of all values of x such that $x \geq a$
$(-\infty, b)$	$\{x \mid x < b\}$	Infinite interval of all values of x such that $x < b$
$(-\infty, b]$	$\{x \mid x \leq b\}$	Infinite interval of all values of x such that $x \leq b$
$(-\infty, \infty)$	All Real numbers	Infinite interval of all values of x

Again, some examples to illustrate translating between plain English and precise statements.

- "Two to four requests are anticipated" translates into $2 \leq x \leq 4$ where x is the integer number of requests. It would also be correct to write $1 < x < 5$, which is equivalent to $2 \leq x \leq 4$ when x is an integer.
- "Concentrations must not exceed 6 ppm (parts per million) but must be at least 3 ppm" translates into $3 \leq x \leq 6$ where x is the concentration. x is *not* logically restricted to integer values (3.6 ppm is possible).
- "Exposure should exceed 30 minutes but not surpass 60 minutes" translates into $30 < x \leq 60$ where x is minutes. x is *not* logically restricted to integer values (40.75 minutes is possible).

Other useful symbols:

- $\approx$, $\simeq$ or $\cong$ is read as "approximately equal to." For example, $27.8889 \approx 28$.
- $\pm$ is read as "plus or minus." For example 4 ± 3 means [1, 7].

Absolute Values

The absolute value of a number a, denoted by $|a|$, is the distance from a to 0 on the real number line. Distances are always positive or 0, so $|a| \geq 0$ for every number a. $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$. (Recall: If a is negative then $-a$ is positive.)

7 Areas

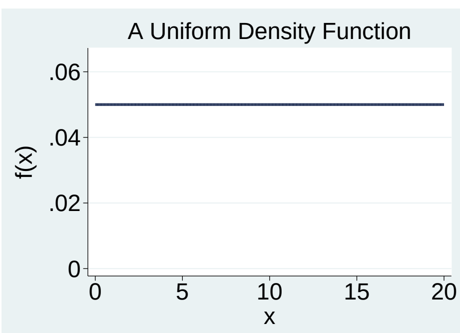
Finding areas is important in statistics. This may suggest that integration (calculus) is necessary. However, the types of functions we work with are either too simple or too complex to use integration. That means you will either be using simple formulas for finding areas or relying on statistical tables.

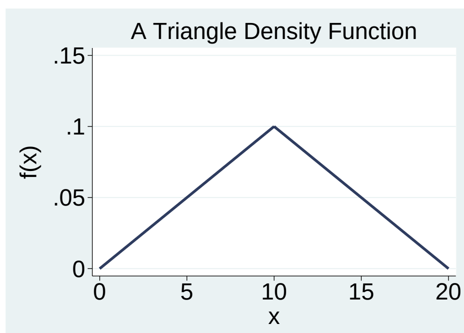
- Area of a rectangle $= b * h$ where b is the length of the base and h is the height.
- Area of a triangle $= \frac{1}{2}b * h$ where b is the length of the base and h is the height.

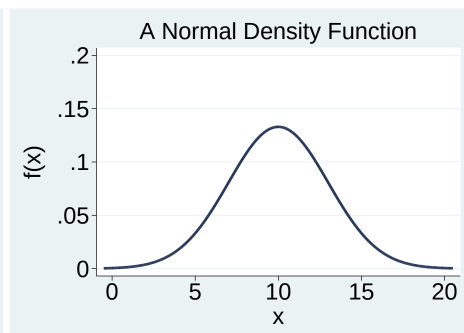
Whether using simple formulas or statistical tables, you will need to also use some basic logic. For example, if the total area is 5 and a sub-part has an area of 3 then the non-sub-part area is 2. The exercises below are specifically constructed for practice on commonly needed skills in ECO220Y. In particular, all of the figures show *density functions*. Density functions are always non-negative. Further, the total (positive) area under a density function is exactly 1. The *support* of the density function is the range of x-axis values that have a (non-zero) line above them. To summarize:

- A density function is non-negative: $f(x) >= 0$
- The total (positive) area under a density function is exactly 1

Here are some sample density functions. Finding areas for the first two is simple. Finding areas under the Normal density function is harder: a statistical table is required as there is no formula for the area.

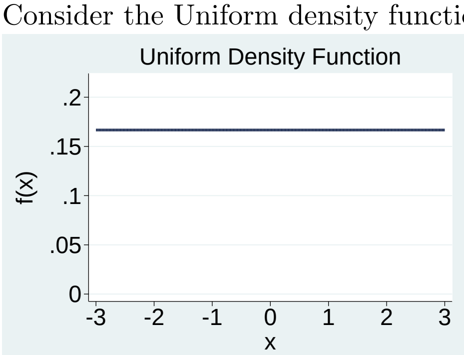






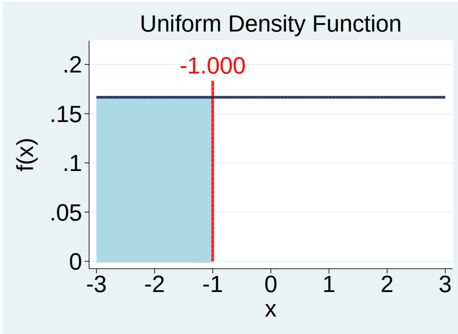
Section 7 Exercises:

1. Consider the Uniform density function given below. Its support is $[-3, 3]$.

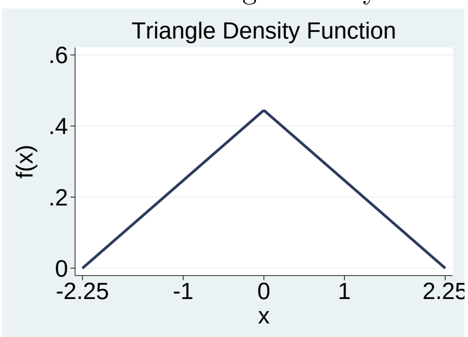


   (a) What is the *exact* height of the density function? In other words, what is the constant a in $f(x) = a$? (Hint: Remember that the total area under a density function is exactly 1.)

(b) What is the area under the density function between -3 and -1 (i.e. shaded area below)?

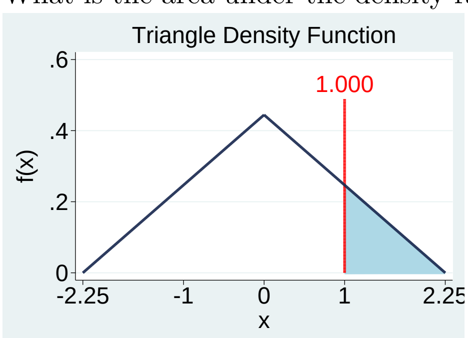


2. Consider the Triangle density function given below. Its support is $[-2.25, 2.25]$.

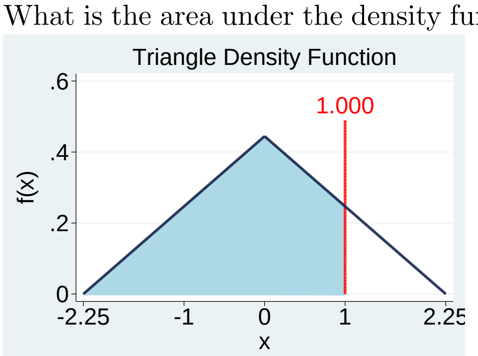


(a) What is the *exact* maximum height of the density function? In other words, what is the value of $f(x)$ at the peak of the triangle? (Hint: Again, remember total area is 1.)

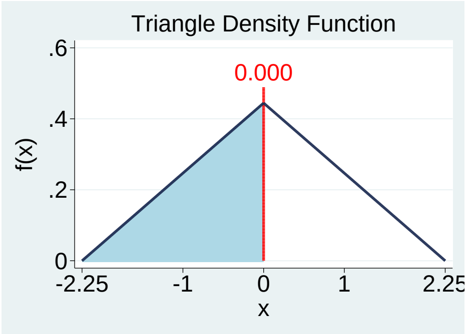
(b) What is the area under the density function between 1 and 2.25 (i.e. shaded area below)?



(c) What is the area under the density function between -2.25 and 1 (i.e. shaded area below)?

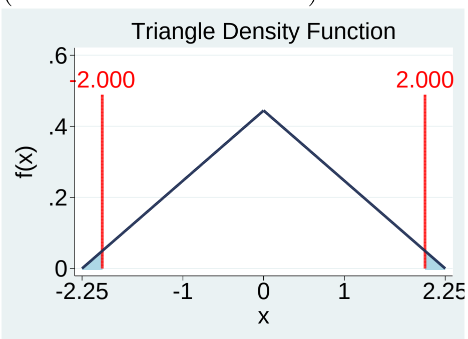


(d) What is the area under the density function between -2.25 and 0 (i.e. shaded area below)?

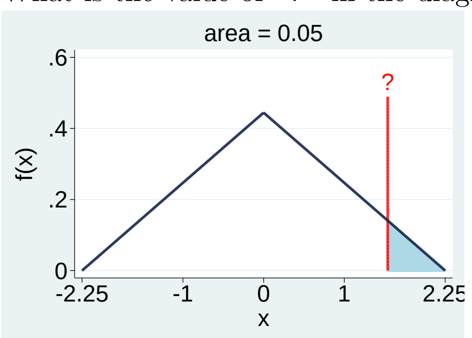


(e) What is the area under the density function between 0 and 1? (Draw a graph.)

(f) What is the area under the density function between -2.25 and -2 and between 2 and 2.25 (i.e. shaded areas below)?



(g) What is the value of "?" in the diagram below if the shaded area shown is exactly 0.05?



8 Summation

A quick browse through the formulas in ECO220Y reveals many occurrences of summation in sigma notation: Σ. For example, the mean is obtained by added up all values and dividing by the number of observations.

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + ... + x_n}{n} \tag{4}$$

Consider a general definition of summation. If $a_m, a_{m+1}, ..., a_n$ are real numbers and m and n are integers such that $m \leq n$, then

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + ... + a_{n-1} + a_n \tag{5}$$

The *index* of summation is i. As a simple example, write 2 + 2 = 4 in sigma notation.

$$\sum_{i=1}^{2} 2 = 4 \tag{6}$$

In this example $a_1 = a_2 = 2$. The index i takes two values (1 and 2) because there were two terms in the original sum. Write 2 + 2 + 2 + 2 = 8 in sigma notation.

$$\sum_{i=1}^{4} 2 = 8 \tag{7}$$

In this example $a_1 = a_2 = a_3 = a_4 = 2$. The index i takes four values (1, 2, 3 and 4) because there were four terms in our original sum.

As another example, consider annual compensation for 10 randomly selected CEO's in Fortune 500 companies measured in millions of dollars. Name the variable x_i. Hence, x_1 records the first CEO's annual compensation in millions (14.2 million) and x_{10} records the tenth CEO's (22.4 million).

$$\sum_{i=1}^{10} x_i = 14.2 + 8.8 + 25.6 + 22.7 + 4.1 + 3.0 + 9.5 + 9.1 + 50.1 + 22.4 = 169.5 \tag{8}$$

$$\bar{X} = \frac{169.5}{10} = 16.95 \tag{9}$$

Laws of Summation: where c is a constant and n is a positive integer

- $\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i$
- $\sum_{i=m}^{n}(a_i + b_i) = \sum_{i=m}^{n} a_i + \sum_{i=m}^{n} b_i$
- $\sum_{i=m}^{n}(a_i - b_i) = \sum_{i=m}^{n} a_i - \sum_{i=m}^{n} b_i$
- $\sum_{i=1}^{n} 1 = n$
- $\sum_{i=1}^{n} c = nc$
- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

For example, suppose x_i records days to complete stage 1 of a production process for each of 25 items and $\sum_{i=1}^{25} x_i = 23.14$ days. Similarly, y_i records hours to complete stage 2 and $\sum_{i=1}^{25} y_i = 114.81$ hours. For stage 3, every item needs exactly 105 minutes of drying time (no variation). What is the total time it takes to complete all three stages for all 25 items? To answer, choose common units, say hours. Then, find $\sum_{i=1}^{25}(x_i * 24 + y_i + 105/60)$. Using the laws of summation, $\sum_{i=1}^{25}(x_i * 24 + y_i + 105/60) = 24 * \sum_{i=1}^{25} x_i + \sum_{i=1}^{25} y_i + \sum_{i=1}^{25} 105/60 = 24 * 23.14 + 114.81 + 25 * 105/60 = 713.92$ hours.

Section 8 Exercises:

1. Consider these data with two variables and 30 observations. *None* of the questions require a lot of computation. Make sure to use all given information and the laws of summation.

i	x	y
1	18.1	3.8
2	32.8	7.5
3	4.9	4.2
4	22.1	6.6
5	18.7	4.0
6	44.7	5.6
7	21.6	4.1
8	10.8	6.3
9	47.2	6.4
10	10.9	4.2
11	21.2	3.3
12	8.0	2.6
13	24.5	5.4
14	32.8	1.8
15	3.2	7.9
16	28.7	1.0
17	33.4	1.1
18	41.7	5.5
19	1.9	0.3
20	46.9	8.2
21	27.2	3.6
22	0.7	8.1
23	20.4	0.5
24	47.9	2.1
25	40.7	5.9
26	35.8	9.8
27	28.0	6.7
28	39.9	1.9
29	36.1	2.5
30	45.2	7.2
Total	796.0	138.1

(a) What is $\sum_{i=1}^{30} x_i$?

(b) What is $\sum_{i=1}^{30} y_i$?

(c) What is $\sum_{i=1}^{30}(x_i + y_i)$?

(d) What is $\sum_{i=1}^{30}(x_i - 3y_i)$?

(e) What is $\sum_{i=1}^{3} x_i y_i$? (Note it says 3, not 30.)

(f) What is $(\sum_{i=1}^{3} x_i)(\sum_{i=1}^{3} y_i)$? (Note it says 3, not 30.)

(g) Are your answers equal in (e) and (f)? Why or why not?

(h) What is $\sum_{i=29}^{30} x_i^2$? (Note it says 29, not 1.)

(i) What is $(\sum_{i=29}^{30} x_i)^2$? (Note it says 29, not 1.)

(j) Are your answers equal in (h) and (i)? Why or why not?

(k) What is $\sum_{i=1}^{30}(10 + 22x_i - 30y_i)$?

(l) What is $\sum_{i=1}^{30}((x_i - 30) + (y_i - 2))$?

(m) Suppose the observations represent a cross-section of small businesses. Further, suppose x is reported in the above table in 1000's of dollars and is a measure of profits for the entire year (12 months). This means that for Firm 1 (the first observation) x is 18.1, which corresponds to profits of \$18,100. Suppose we create a new variable z that is x measured in dollars (hence, z_1 is \$18,100). What is $\sum_{i=1}^{30} z_i$?

(n) Suppose y represents 1000's of dollars lost to theft over two years. Suppose we create a new variable w that is an annualized value of y (average loss per year) in 1000's of dollars. What is $\sum_{i=1}^{30} w_i$?

(o) Suppose that the government is considering charging a lump-sum profit tax of \$1,000 per year per small business. If x measures each firm's profits before this profit tax, what is $\sum_{i=1}^{30} q_i$ where q measures profits in 1000's of dollars after this profit tax?

(p) Suppose that the government is considering charging a profit tax of 10% of profits per year per small business. If x measures each firm's profits before this profit tax, what is $\sum_{i=1}^{30} q_i$ where q measures profits in 1000's of dollars after this profit tax ?

2. Suppose $\sum_{k=1}^{1000} x_k^2 = 427744.2$ and $\sum_{k=1}^{1000} x_k = 20051.8$.

   (a) Find $\sum_{k=1}^{1000}(x_k - 10)^2$.

   (b) Find $\sum_{k=1}^{1000}(x_k - \bar{X})^2$.

   (c) Find $\sum_{k=1}^{1000}(x_k - 4)^2$.

   (d) In which case is the sum the smallest? Intuition for that finding?

9 Answers to Exercises that Appear in Sections 3 - 8

Section 3 Exercises, Answers:

1. (a) 3.76

   (b) 2.7876

Section 4.1 Exercises, Answers:

1. (a) $y = x$

   (b) Increases by 1 unit

   (c) Increases by 1 unit

   (d) Because the slope of a line is constant

   (e) Decreases by 1 unit

   (f) 0, this is the definition of the intercept

2. (a) $y = 2$

(b) No change: this is a horizontal line

3. (a) $y = 4 - 2x$

   (b) Decreases by 2 units

4. (a) $y = 12 - x$

   (b) $y = 7 - x$

   (c) $y = 10 - 2x$

   (d) $y = 20 - 2x$

Section 4.2 Exercises, Answers:

1. This is a third degree polynomial (cubic). $a_0 = 23.1, a_1 = 2.4, a_2 = 5.9, a_3 = -6.2$

2. x = 0.5307 and x = -1.1307

Section 5 Exercises, Answers:

1. 0.6

2. -0.1112

3. 9

Section 6 Exercises, Answers:

1. $(-2/3, \infty)$

2. $[2, 5)$

3. $[2, 10/3]$

Section 7 Exercises, Answers:

1. (a) $\frac{1}{6}$

   (b) $\frac{1}{3}$

2. (a) $\frac{2}{4.5} = \frac{4}{9}$

   (b) The shaded area is $\frac{25}{162}$. To find the area, requires first finding the equation of the line that connects the triangle peak $(0, \frac{4}{9})$ to the right corner $(\frac{9}{4}, 0)$. That line is $f(x) = \frac{4}{9} - \frac{16}{81}x$.

   (c) The shaded area is $\frac{137}{162}$, which is most easily found as $1 - \frac{25}{162}$.

   (d) The shaded area is $\frac{1}{2}$, which is most easily found by noting that that is exactly half of the total area and the total area is 1.

   (e) The area is $\frac{56}{162}$, which is most easily found as $\frac{1}{2} - \frac{25}{162}$.

   (f) The shaded area is $\frac{2}{162}$. Note that you may find the area in one of the tails and multiple it by two: the two shaded areas are exactly the same.

   (g) $\frac{90-9\sqrt{10}}{40}$. To find this is not hard but requires some work. You will need to use the quadratic formula, reviewed in Section 4.2 to solve.

Section 8 Exercises, Answers:

1. (a) 796.0
   (b) 138.1
   (c) 796.0 + 138.1 = 934.1
   (d) 796.0 - 3*138.1 = 381.7
   (e) 18.1*3.8 + 32.8*7.5 + 4.9*4.2 = 335.36
   (f) (18.1 + 32.8 + 4.9)*(3.8 + 7.5 + 4.2) = 864.9
   (g) No. No rule that says they should be equal and in general they are not.
   (h) $36.1^2 + 45.2^2 = 3346.25$
   (i) $(36.1 + 45.2)^2 = 6609.69$
   (j) No. No rule that says they should be equal and in general they are not.
   (k) 30*10 + 22*796.0 - 30*138.1 = 13,669
   (l) 796.0 - 30*30 + 138.1 - 2*30 = -25.9
   (m) 796,000
   (n) 138.1 / 2 = 69.05
   (o) 796.0 - 30*1 = 766.0
   (p) 796.0*(1 - 0.10) = 716.4

2. (a) 126708.2
   (b) 25669.5
   (c) 283329.8
   (d) The sum is smallest when computing the squared difference between each observation and the mean. The mean is a measure of the center of the data, so it makes sense that the data are "closer" to the center of the data than some other arbitrary point. For example, if x_i recorded the cGPA of each student in ECO220Y1Y, we would expect $\sum_{k=1}^{850}(x_k - \bar{X})^2$ would be smaller than $\sum_{k=1}^{100}(x_k - 4.0)^2$: there is going to be more observations close to average cGPA than a 4.0 cGPA.

10 Diagnostic Quiz Answers

1. (e) 15.00 (See Section 3)

2. (b) 10.50 (See Section 3)

3. (Another application of Section 3)

   (a) The figure clearly indicates that it shows the "percentage-point change" for each country from 2014 to 2018. Further, it says that, in Germany, the percent with negative sentiments declined by *about* 10 percentage points. Hence, if it had been 59% in 2014, then about 49% of Germans have negative sentiments about non-EU immigration in 2018 (just three and a half years later).

   (b) The figure shows that it increased by *about* 17 percentage points. Hence, if it had been 54% in 2014, then about 71% of Poles have negative sentiments about non-EU immigration in 2018 (just three and a half years later).

   (c) The change from 57% to 52% is a 5 percentage point decline and a 8.77 percent decline.

   (d) That is a 13 percentage point decline, which would put this city between Italy and Spain in the figure.

4. (a) 66.7% (See Section 3)

5. 79.3% (See Section 3)

6. (c) 20.00 (Solve with algebra)

7. (d) $\frac{X}{1+r/100}$ (Solve with algebra)

8. Recall from your principles of economics course, that the elasticity of demand is $\frac{\%\Delta Q_D}{\%\Delta P}$.

   (a) When the price of dairy increases by 10%, the quantity demanded of dairy declines by 0.5%. The demand for dairy is extremely inelastic.

   (b) When the price of cereals and bakery products increases by 1%, the quantity demanded of cereals and bakery products decreases by 0.6%. The demand for cereals and bakery products is inelastic. Generally the more you aggregate up products the more inelastic demand becomes. Cereals and bakery products is a highly aggregate group that includes a range of things such as flour, breakfast cereals, rice, pasta, bread, muffins, cakes, etc. There are not a lot of good substitutes for this broad group, which explains why demand is inelastic.

   (c) When the price of white bread increases by 1 percent, the quantity demanded of white bread decreases by 1.55 percent. The demand for white bread is elastic.

9. (a) For Canadian workers, we observe annual salaries increase by $6,188 Canadian with each additional year of education. (We tend to think about salaries in dollars. If we are talking about CEOs, celebrities, or athletes, we may think in millions of dollars.)

   (b) For people participating in a sleep study, an extra milligram of the drug yields an extra 9 minutes of sleep. (If you tell someone to meet you is 0.1562 hours, they will not understand you. For odd fractions of an hour, we tend to think in minutes.)

(c) For corporate boards of publicly traded companies in North America, for each additional year older the company is we see a slightly lower percentage of the board being female, 0.5 percentage points lower. An extra 10 years of age is associated with a decline of female representation on the board of 5 percentage points. (We tend to think in percentages, not fractions.)

(d) For the 500 most popular bottles of wine sold by the LCBO in 2016, each additional rating point (on a 100-point scale) is associated with the price of the bottle being nearly $1 (Canadian) higher. (In this case the original units are good. We can just round 0.9681 to 1. Alternatively, it would have also been acceptable to say "being 97 cents (Canadian) higher.")

(e) For 100 large retail firms operating in North America, for every additional percentage point of the workforce making the minimum wage, we observe annual turnover that is about two percentage points higher.

(f) For colonies of bacteria being exposed to varying intensities of heat in a lab, increasing the temperature by one degree Celsius increase the number of bacteria killed by about 2.9 billion. (For huge numbers, we usually upgrade the units of measurement. For example, nobody talks about the Canadian GDP in dollars, but rather trillions of dollars. Also, if you answered 0.9 billion instead of 2.9 billion you got mixed up on the temperature conversion. An extra degree Celsius corresponds to an extra $\frac{9}{5}$ degrees Fahrenheit.)

(g) For ten years of annual observations of household shopping trips in the European Union, the share of shopping trips with a small purchase (less than 10 euro) has been declining by about one percentage point per year.

10. (a) Define y as the share (a percent) that voted for "Leave" and define x as the percentage of the region's GDP exported to the EU. Two points determine a line. We can very accurately approximate when the y value is either 50 or 60 as the figure clearly marks those: we just need to find (approximately) the corresponding x values. Hence, (x_1, y_1) is (about) $(7.85, 50)$ and (x_2, y_2) is (about) $(14.5, 60)$. Hence, after finding the slope and then using the point-slope formula, the approximate equation of the line is $y = 38.2 + 1.5x$. Thus, when x is 1 percentage point higher, y is 1.5 percentage points higher and the base value (intercept) is 38.2 percentage points, which is outside the range of the data and hence not shown in the figure (no region has zero EU exports). (See Section 4.1)

(b) $y = 38.2 + 150x$. This still says that when x is 0.01 higher (which means one percentage point higher), y is 1.5 percentage points higher. (If you are confused, redraw the graph with the new units and/or re-do the point-slope formula calculations with the new units.) (See Section 4.1)

(c) $y = 0.382 + 0.015x$. This still says that when x is 1 higher (which means one percentage point higher), y is 0.015 higher, which is 1.5 percentage points. (If you are confused redraw the graph with the new units and/or re-do the point-slope formula calculations with the new units.) (See Section 4.1)

(d) $y = 0.382 + 1.5x$. This still says that when x is 0.01 higher (which means one percentage point higher), y is 0.015 higher, which is 1.5 percentage points. (If you are confused redraw the graph with the new units and/or re-do the point-slope formula calculations with the new units.) (See Section 4.1)

(e) i. $d = 5$ because we are told that males are 5 percentage points more likely to vote "Leave" regardless of exports. (See Section 4.1)

ii. 35.7 is the intercept for the line for female voters (for whom $m = 0$), whereas the line for male voters has an intercept 5 units higher ($40.7 = 35.7 + 5$). (See Section 4.1)

iii. $y = 40.7 + 1.5x - 5f$, which, just like in the previous part, implies that the male-voter line is $y = 40.7 + 1.5x$ (because for males $f = 0$) and the parallel female-voter line is $y = 35.7 + 1.5x$ (because for females $f = 1$). (See Section 4.1)

11. The line for male voters is $y = 16 + 0.5x$ and the line for female voters is $y = -3.6 + 0.7x$. Hence, the intercept for male voters is 19.6 percentage points higher compared to female voters and the slope for male voters is 0.2 less steep: $y = -3.6 + 0.7x + 19.6m - 0.2(x * m)$. (See Sections 4.1 and 5.1)

12. (a) 

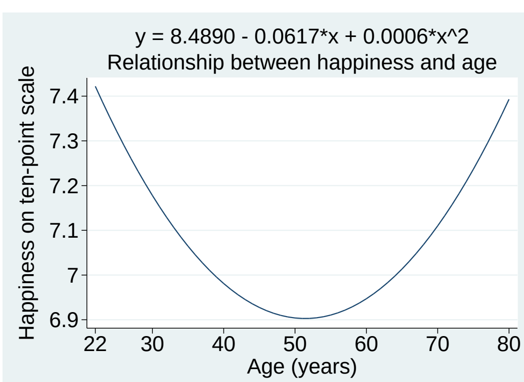
(See Sections 4.2 and 5)

i. -0.0117 (See Section 5)

ii. 0.0063 (See Section 5)

iii. 0.0303 (See Section 5)

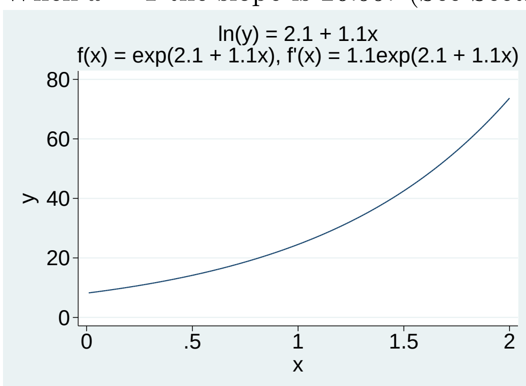
iv. Because a quadratic function does not have a constant slope. This function is non-monotonic: sometimes it is decreasing and sometimes increasing. (See Section 5)

(b) 

(c) 

13. 0.929. If you cannot solve this, make sure to work through Sections 4.1, 4.2 and 7.

14. When $x = 1$ the slope is 26.99. (See Sections 4.3 and 5)



15. (c) $x <= 15$ (See Section 6)

16. (b) $x > 4$. Note that $x >= 5$ would have also been correct had it been one of the choices. $x > 4$ is equivalent to $x >= 5$ because in this example x is an integer. (See Section 6)

17. (b) $x <= 160$ (See Section 6)

18. (a) 4066.5 (See Section 8)
    (b) 3607.4 (See Section 8)
    (c) Note that $z_k = 1000 * x_k$. $\sum_{k=1}^{50} z_k = \sum_{k=1}^{50} 1000 * x_k = 1000 \sum_{k=1}^{50} x_k = 3565000$ and $\sum_{k=1}^{50} z_k^2 = \sum_{k=1}^{50} (1000 * x_k)^2 = 1000^2 \sum_{k=1}^{50} x_k^2 = 258251000000$ (See Section 8)

19. (a) True
    (b) False
    (c) True
    (d) True
    (e) False
    (f) True
    (g) True
    (h) True (See Section 4.2)