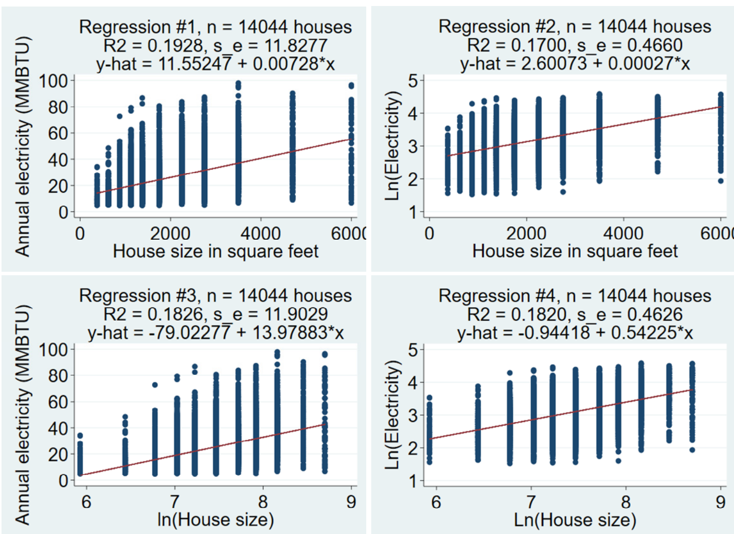
Statistical Inference with Regression

Lecture 19

Reading: Sections 18.3 – 18.6, 19.2

1



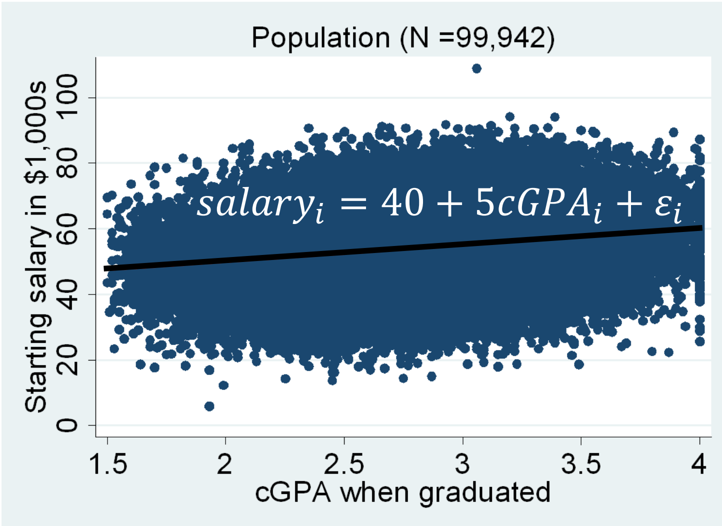
Source: Levinson (2016), calif_energy_regressions.xlxs. Which is best?

2

Inference with Regression

- Linear regression model: $y_i = \alpha + \beta x_i + \varepsilon_i$
  - Which parameters are we interested in?
    - E.g. How much does earning good marks in university affect your starting salary?
- OLS gives a and b: $\widehat{y}_i = a + bx_i$
  - Recall: $b = \frac{s_{xy}}{s_x^2} = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
    - Are these sample statistics or parameters?
    - What do we need to know about a and b so that we may use them for inference?

3



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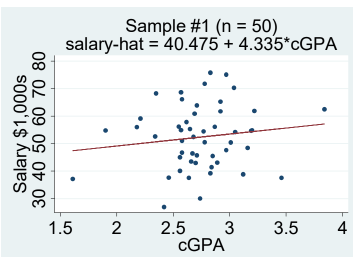

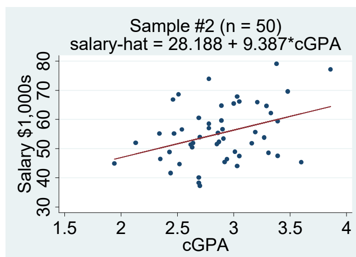

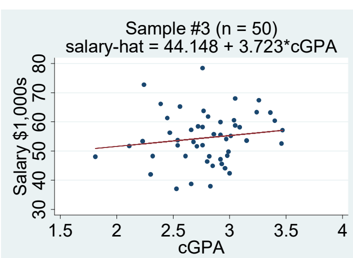

How many samples will you actually have?

What important concept do these graphs illustrate?

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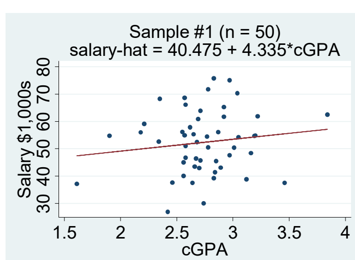
Standard Error of OLS Slope

- $SE(b_1)$ reflects size of sampling error and depends on:
  1) Sample size (n)
  2) Amount of scattering about line (s_e)
  3) How much x-variable varies in the data (s_x)
- $SE(b_1) = s_{b_1} = \frac{s_e}{s_x\sqrt{n-1}}$

Recall: $s_e = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n-2}}$



Often s.e. in parentheses below the point estimate:

Salary-hat = 40.475 + 4.335*cGPA
(4.484)

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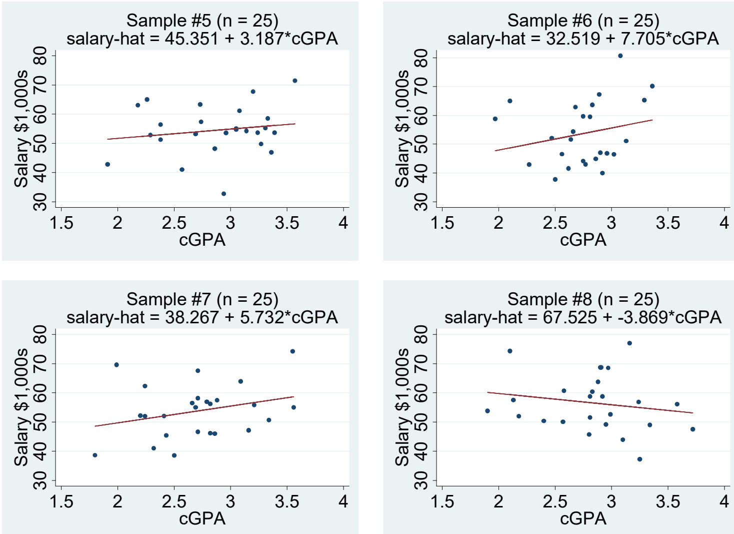

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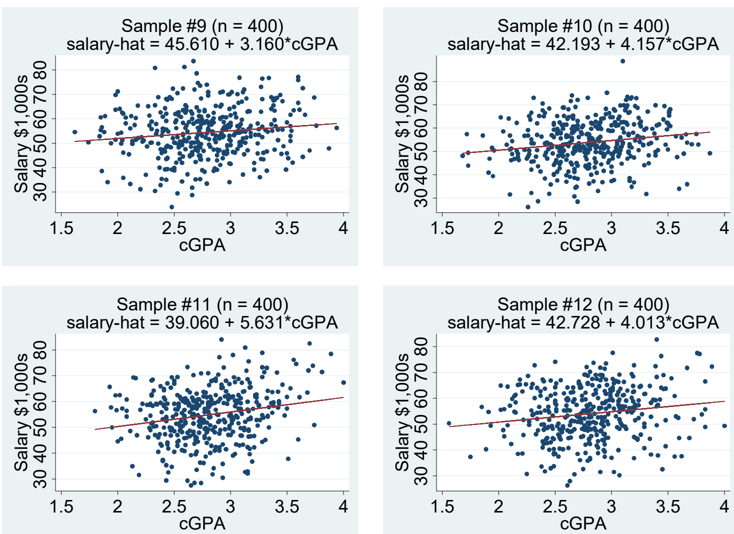

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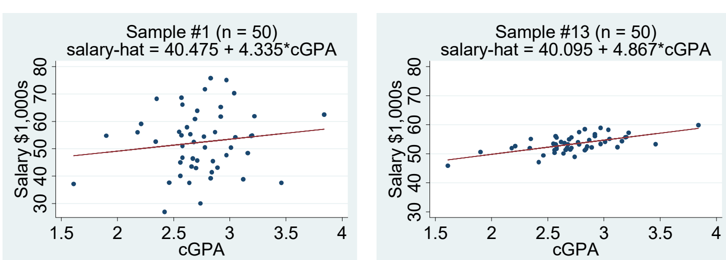
Main Difference? Effect on s.e.?



$$s_{b_1} = \frac{s_e}{s_x\sqrt{n-1}} = \frac{11.487}{0.366\sqrt{50-1}} = 4.484$$

$$s_{b_1} = \frac{s_e}{s_x\sqrt{n-1}} = \frac{2.297}{0.366\sqrt{50-1}} = 0.897$$

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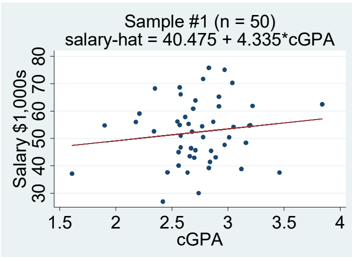
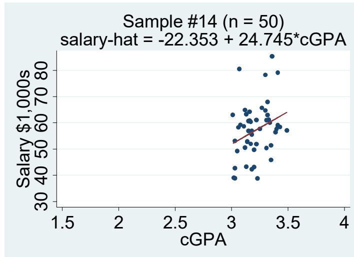
Main Difference? Effect on s.e.?





$$s_{b_1} = \frac{s_e}{s_x\sqrt{n-1}} = \frac{11.487}{0.366\sqrt{50-1}} = 4.484$$

$$s_{b_1} = \frac{s_e}{s_x\sqrt{n-1}} = \frac{9.842}{0.127\sqrt{50-1}} = 11.028$$

Inference about β: $y_i = \alpha + \beta x_i + \varepsilon_i$

- $H_0: \beta = \beta_0; H_1: \beta \neq \beta_0$
  - Test statistic: $t = \frac{b-\beta_0}{s_b}$
  - Student t, $\nu = n - 2$
- "Statistical significance"
  - If $H_0: \beta = 0; H_1: \beta \neq 0$ then $t = \frac{b}{s_b}$
  - *Roughly*, need $t \geq 2$ or $t \leq -2$ at $\alpha = 0.05$

- E.g. Is slope statistically significant in Reg. #1?

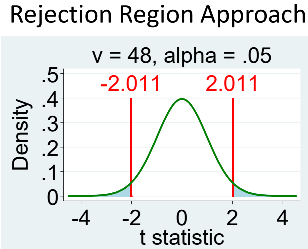
Salary-hat = 40.475 + 4.335*cGPA
(4.484)

- $H_0: \beta = 0$
- $H_1: \beta \neq 0$
- $t = \frac{b-\beta_0}{s_b} = \frac{4.335-0}{4.484} = 0.97$ $(\nu = 50 - 2 = 48)$
- Conclusion?

Note: Section 18.5 "A Hypothesis Test for Correlation" is redundant: same result as a test of the slope coefficient. However, we'll see the t test statistic alternate formula (on p. 617) later when we talk about the F test.

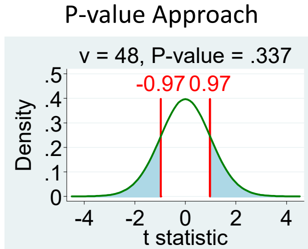
$t = 0.97$, Not Statistically Significant

Rejection Region Approach



Conclusion?

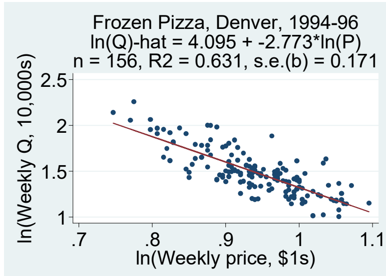
P-value Approach



Conclusion?

Recall Frozen Pizza in Denver

- Statistically significant?
  - $H_0: \beta_1 = 0$
  - $H_1: \beta_1 \neq 0$
    - $t = \frac{-2.773-0}{0.171} = -16.2$
  - Conclusion?
- Is "slope" coefficient less than -1?
  - $H_0: \beta_1 = -1$
  - $H_1: \beta_1 < -1$
    - $t = \frac{-2.773--1}{0.171} = -10.4$



Does *this analysis* imply that the demand for frozen pizza in Denver in the mid-1990s was elastic? Why or why not?

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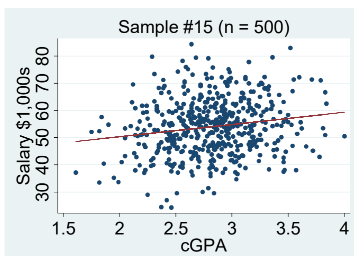
Confidence Interval (CI) Estimate

- CI estimator of β:
  $$b \pm t_{\alpha/2} s_b$$
  - Confidence level: $1 - \alpha$
  - Degrees of freedom: $\nu = n - 2$
- Interpretation?



Sal-hat = 41.376 + 4.479*cGPA
(1.169)

95% CI of slope:
LCL = 2.182 and UCL = 6.776

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Reading STATA Output

```
. regress salary cGPA;

      Source |       SS       df       MS              Number of obs =     500
-------------+------------------------------           F(  1,   498) =   14.68
       Model |  1523.07551     1  1523.07551           Prob > F      =  0.0001
    Residual |   51656.349   498  103.727608           R-squared     =  0.0286
-------------+------------------------------           Adj R-squared =  0.0267
       Total |  53179.4245   499  106.571993           Root MSE      =  10.185

------------------------------------------------------------------------------
      salary |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        cGPA |   4.479379   1.168972     3.83   0.000     2.182653    6.776105
       _cons |   41.37579    3.32467    12.45   0.000     34.84368     47.9079
------------------------------------------------------------------------------
```

Recall: $s_y^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1} = \frac{SST}{n-1}$

Recall: $s_e = \sqrt{\frac{\sum_{i=1}^{n}(e_i-0)^2}{n-2}} = \sqrt{\frac{SSE}{n-2}}$

What does 10.185 mean? Units?

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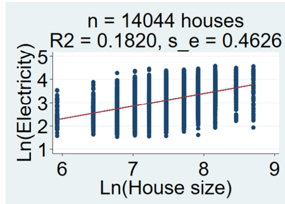
Recap w/ CA Elec. OLS results

ln(elec)-hat = -0.9442 + 0.5423*ln(size)
(0.0097)




```
. regress ln_elec_mmbtu ln_sq_feet;

      Source |       SS           df       MS      Number of obs   =    14,044
-------------+----------------------------------   F(1, 14042)     =   3123.68
       Model |  668.541968         1  668.541968   Prob > F        =    0.0000
    Residual |  3005.32471    14,042  .214023979   R-squared       =    0.1820
-------------+----------------------------------   Adj R-squared   =    0.1819
       Total |  3673.86668    14,043  .261615515   Root MSE        =    .46263

------------------------------------------------------------------------------
ln_elec_mm~u |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  ln_sq_feet |   .5422451    .009702    55.89   0.000     .5232278    .5612624
       _cons |   -.944182   .0724338   -13.04   0.000    -1.086162    -.802202
------------------------------------------------------------------------------
```

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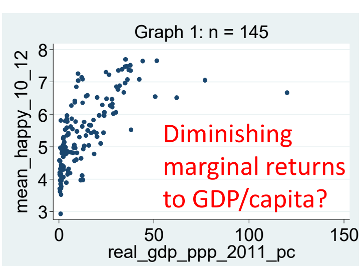
World Happiness Reports (2012, 2013)

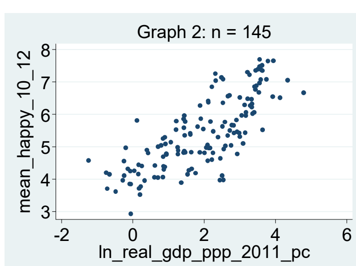
mean_happy_10_12: Mean reply in a country to "Please imagine a ladder with steps numbered from zero at the bottom to 10 at the top. The top of the ladder represents the best possible life for you and the bottom of the ladder represents the worst possible life for you. On which step of the ladder would you say you personally feel you stand at this time?" asked in 2010, 2011, and 2012 Note: "We average the three most recent years (2010-12)." p. 9 (2013)
real_gdp_ppp_2011_pc: Real GDP per capita at current PPP (purchasing power parity) in 2011 $1,000s US
ln_real_gdp_ppp_2011_pc, Natural logarithm of real_gdp_ppp_2011_pc

Variable	obs.	mean	median	s.d.	min.	max.
mean_happy_10_12	145	5.440	5.345	1.119	2.936	7.693
real_gdp_ppp_2011_pc	145	14.169	8.360	16.642	0.288	120.172
ln_real_gdp_ppp_2011_pc	145	1.953	2.123	1.312	-1.245	4.789

http://worldhappiness.report/download/; "One Question" Test #1, Nov. 2013 17

Natural Log: Straighten Scatter Plot





Is there an issue with outliers?

Does Assumption 1 (linearity) hold for Graph 2?

Does Assumption 2 (homoscedasticity) hold for Graph 2?

Does Assumption 6 (exogeneity) $(COV(x_i, \varepsilon_i) = 0)$ hold?

Remember we studied this case with more recent data in Week 6. 18

What do the coefficients mean?

```
. regress mean_happy_10_12 ln_real_gdp_ppp_2011_pc;

      Source |       SS       df       MS              Number of obs =     145
-------------+------------------------------           F(  1,   143) =  213.32
       Model |  107.855278     1  107.855278           Prob > F      =  0.0000
    Residual |  72.3011091   143  .505602162           R-squared     =  0.5987
-------------+------------------------------           Adj R-squared =  0.5959
       Total |  180.156387   144  1.25108602           Root MSE      =  .71106

------------------------------------------------------------------------------
mean_hap_~12 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ln_real_g~pc |   .6597638   .0451723    14.61   0.000     .5704721    .7490556
       _cons |   4.151934   .1061566    39.11   0.000     3.942095    4.361773
------------------------------------------------------------------------------
```

In 2010/12, countries with real GDP per capita that is 10% higher have mean happiness (on a 0-10 scale) that is approximately _____ units higher on average.

In 2010/12, countries with real GDP per capita of $1,000 have mean happiness (on a 0-10 scale) that is on average _____.

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Point Prediction

- Point prediction: Use estimated model to predict y (y-hat) for a given value of x
  - Ex: Salary-hat = 41.376 + 4.479*cGPA
    (3.325) (1.169)
  - If cGPA is 3.2 then salary-hat is 55.709
  - How to interpret 55.709?
- Even with huge n, we cannot precisely predict y given x because $y_i = \alpha + \beta x_i + \varepsilon_i$

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Prediction Interval vs. Confidence Interval

- Prediction Interval: [individual] contains y for a given x_g with confidence $1 - \alpha$

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \text{with } \nu = n - 2$$

- Confidence Interval: [mean] contains E[y] for a given x_g with confidence $1 - \alpha$

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{\frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}} \quad \text{with } \nu = n - 2$$

For alternate (mathematically identical) versions, see textbook

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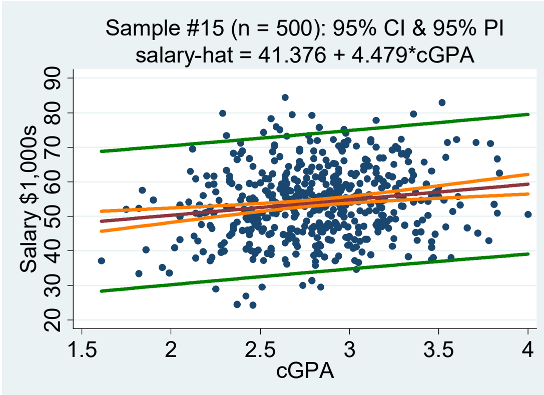
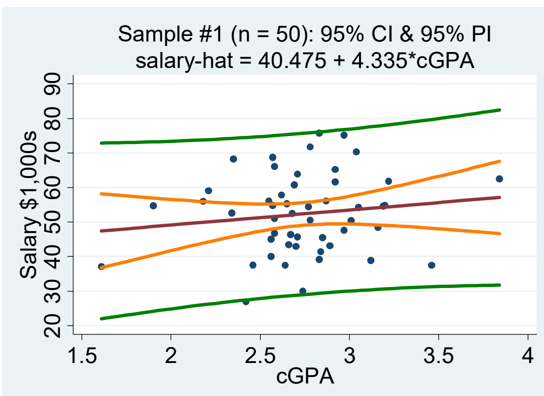


For a cGPA of 3.2 ($x_g = 3.2$) the 95% PI is (36.0, 76.2)

For a cGPA of 3.2 ($x_g = 3.2$) the 95% CI is (54.7, 57.5)

Which should contain ~95% of dots in scatter diagram above?



For a cGPA of 3.2 ($x_g = 3.2$) the 95% PI is (31.6, 81.4)

For a cGPA of 3.2 ($x_g = 3.2$) the 95% CI is (47.2, 65.8)

Recap: Three Kinds of Intervals

- 2010/12 happiness OLS results:

  Happiness-hat = 4.151934 + 0.6597638*ln(GDP per capita)
  (0.0451723)

- Three different intervals:

$$b \pm t_{\alpha/2} s_b$$

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}}$$

$$\hat{y}_{x_g} \pm t_{\alpha/2} s_e \sqrt{\frac{1}{n} + \frac{(x_g - \bar{X})^2}{(n-1)s_x^2}}$$