Homework 7: ECO220Y – SOLUTIONS

Required Problems:

(1) (a) No, that would be an incorrect interpretation of these conditional probabilities. What we can see from the given probabilities is that among unemployed people more are high school graduates than not. This is not surprising because there are far more high school graduates in the population than non-high school graduates: see the marginal probabilities in the table. Hence even if conditional on having a high school degree you are less likely to be unemployed, the number of unemployed people who have high school degrees can be big just because there are so many high school graduates (even if only a very small fraction of them are unemployed).

(b) No, that would again be a mistaken interpretation of the conditional probabilities. These conditional probabilities show that those who complete high school have less chance of being unemployed than those who do not complete high school. That does not mean that of unemployed people fewer are high school graduates. In fact we see from part (a) that more of the unemployed people are high school graduates than non-high school graduates. There is nothing illogical about this and the probabilities given in 6.45 and the derived conditional probabilities are reasonable. The important thing is to get their interpretation correct.

(c)

Education	**Employed**	**Unemployed**	**Total**
Not HS graduate	0.0709	0.0095	0.0804
HS graduate	0.1689	0.0120	0.1809
Some post-sec.	0.0447	0.0032	0.0479
Post-sec. degree	0.3639	0.0207	0.3846
University degree	0.2915	0.0146	0.3061
Total	0.9399	0.0601	1.0000

This type of question often confuses people, but is VERY important. The reason it is important is because all probabilities can be thought of as conditional probabilities. What? I thought we learned *three* kinds of probabilities: joint, marginal and conditional? Yes, we did. Looking back at the table given with this question (and discussed in Lecture 6): we talk about it showing joint and marginal probabilities. HOWEVER, we all understand that this is conditional on being in Canada (i.e. this table would look different for the U.S., China, or another country) AND conditional on being 25 – 54 years old. Obviously if we did not condition on age by including kids and seniors, the chance of not being in the labor force would be *much* higher. Hence we often have to redraw tables conditioning on various factors. In this case you were asked to redraw the table conditioning on being in the labor force. So how do we get the numbers in the table above? Let's do it two ways: the intuitive way we discussed in Lecture 6 when first thinking about conditional probabilities and with formal notation.

Intuitive way: Like in Lecture 7, Slide 11, imagine there are 10,000 people. Of those, 1,341 would not be in the labor force. Ok, so throw those people out. (Remember the new table is supposed to be *only* about those that *are* in the labor force.) How many people are left? 8,659 (=10,000 – 1,341). How many of the remaining people are not a HS graduate and employed? 614. So, the number in the table for not a HS graduate and employed should be 614/8,659 = 0.0709.

Formal way: The original table gives P(E & NHS) = 0.0614. We need to find P(E & NSH | LF). Use the definition of a conditional probability: P(E & NHS | LF) = P(E & NHS & LF)/P(LF). The numerator is the same as P(E & NHS) because if you are employed then you are in the labor force. However, we do not have P(LF). P(LF) = P(E) + P(U) = 0.8139 + 0.0521 = 0.8659. Hence, P(E & NHS | LF) = P(E & NHS & LF)/P(LF) = 0.0614/0.8659 = 0.0709.

Look back and forth between the "intuitive way" and the "formal way": they are clearly the same.

(2) If customers are indifferent to discounts then the discount offered is *independent* of whether or not the customer renews. Hence use the simple multiple rule for independent events to fill in the table: find the marginal probabilities for the original table and multiply. For example, the probability of the joint event of renewed and no discount would be 0.259 = P(renewed)*P(no discount) = (0.37)*(0.7) = (0.20+0.10+0.07)* (0.20+0.50). Compared to the original table this means that the chance of renewing is higher if offered no discount but lower if offered a large discount.

	No disc.	Modest disc.	Large disc.
Renewed	0.259	0.074	0.037
Did not renew	0.441	0.126	0.063

(3) (a) Yes, because events are independent.
(b) No, because events are not independent.

(4) (a) Define the event O as: a financial adviser has a prior offense. Define the event M as: engaging in misconduct. The given sentence can be translated into this formal comparison of two conditional probabilities: P(M | O) = 5*P(M | O').

(b) Define the event M as: engaging in misconduct. Define the event R as: remain with the firm. The first sentence says: P(R' | M) ≈ 0.5. Further, define the event J as: join a different firm within 1 year. The second sentence says: P(J | (M & R')) = 0.44. Further, this can be calculated from Table 8a:

P(J | (M & R'))
= P(J & M & R') / P(M & R') *Use the definition of a conditional probability*
= P(J & M) / P(M & R') *Logic is same as: P(Rose & Red & Yellow') = P(Rose & Red)*
= (P(J | M)*P(M)) / (P(R' | M)*P(M)) *We use the multiple rule for non-independent events in reverse*
= P(J | M) / P(R' | M) *We cancel P(M) from numerator and denominator*
= 0.2105/0.4801 = 0.4385 ≈ 0.44.

(5) (a) Negative autocorrelation means that you are biased against making a string of decisions in the same direction. For example if a judge decided to grant asylum in each of the past three cases, negative autocorrelation means they are less likely to grant asylum in the current case (due to this human bias, which thinks it is really surprising if you had four of the same outcomes in a row). No autocorrelation would mean that current decisions are not influenced by past decisions. In the context of the judge deciding asylum cases, no autocorrelation would be consistent with a judge fairly deciding each case on its merits (and not allowing decisions in unrelated cases to bias her/his judgement).

(b) This conversation *does* reflect the point of the NBER paper: the professor (a human) is subject to the gambler's fallacy. A couple of finer points, though: if this is an experienced professor with lots of marking experience, this bias is likely to be less strong. Also, we can consider the strength of the incentives for accuracy (regrade requests? petitions?). Probably those incentives (especially if we're talking about small biases like giving an A- instead of an A), are pretty weak. However, overall, even in the circumstances that generate them (a chance sequence of groups that have all done well or all done poorly), these biases are likely to be fairly small. Time is better spent working on the project itself.

(6) (a) Conditional. P(work | being a woman) = 0.65
(b) Conditional. P(women | being a worker) = 0.45
(c) Conditional. P(English not first | Commerce) = 0.66
(d) Conditional. P(minor in Philosophy | major in Economics) = 0.25
(e) Marginal. P(part-time) = 0.15
(f) Conditional. P(part-time | U of T) = 0.15
(g) Conditional. P(male | tenured faculty member in economics) = 0.90
(h) Marginal. P(dislikes green) = 0.10
(i) Conditional. P(dislikes green | dislike red) = 0.05
(j) Conditional. P(ECO204 | ECO220) = 0.60
(k) Joint. P(ECO204 and ECO220) = 0.70