Randomness and Probability

Lecture 7

Reading: Chapter 8

Human Nature versus Probability & Statistics

- TK71: "People have erroneous intuitions about the laws of chance. In particular, they regard a sample randomly drawn from a population as highly representative, that is, similar to the population in all essential characteristics."

  "Big Die" Rolls:

  "Casino Die" Rolls: 3, 2, 6, 1, 5, 4, 3, 4, 1, 6, 5, 2

Tversky, Amos and Daniel Kahneman (1971) "Belief in the Law of Small Numbers" p.p. 105 - 110 of Volume 76(2) of the *Psychological Bulletin* (TK71)

Law of Large Numbers

- TK71, ¶10: "The law of large numbers guarantees that very large samples will indeed be highly representative of the population from which they are drawn."
  - Sampling error decreases as n increases
  - There is <u>no</u> law of small numbers
    - Humans tend to wrongly believe in a small # law
    - Sampling error *is* a major factor when n is small
    - <u>False</u> laws: "Law of Small Numbers" ("Law of Averages")

All of these Mutual Fund Mangers are Equally Skilled

Manager	1	2	3	Mean
Yu	6.0	-0.1	5.7	3.87
Rajiv	-0.7	6.8	8.9	5.00
Erik	0.1	7.9	-0.2	2.60
John	1.7	3.9	-7.9	-0.77
Xin	1.1	9.6	6.8	5.83
Shanshan	-0.8	3.5	6.1	2.93
Ellen	3.4	2.6	4.2	3.40
Joshua	8.6	2.5	8.5	6.53
Fatima	2.2	12.4	9.7	8.10

Would you infer that Fatima is a good analyst? That John is a poor analyst?

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Believers in the Law of Small Numbers

- "Such "fictitious variation" is one of the economically most important implications of the law of small numbers…Because he underestimates how often average analysts will have consecutive successful or unsuccessful years, he interprets what he sees as evidence of the existence of good and bad analysts." Rabin (2002)

Rabin, M., 2002, "Inference by Believers in the Law of Small Numbers," *The Quarterly Journal of Economics*, 117(3): 775-816.

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Catching Cheating Teachers

- Even since Freakonomics, stories of teachers cheating on standardized tests by improving students' papers have continued to show up in the news
  - Analysis of the database with students' answers can detect cheating by teachers, which we suppose is fairly rare (1% of teachers cheat)
    - Suppose a method that's 95% accurate in detecting cheating is applied to all teachers

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Probability Basis of Inference

- Inference about parameters uses statistics affected by sampling noise: probability
  - Random experiment: Process that leads to one of several possible outcomes
    - Is drawing a sample a random experiment?
  - Sample space (S): $S = \{O_1, O_2, \ldots, O_k\}$
    - Exhaustive: List all possible outcomes
    - Mutually exclusive: Outcome can be only one
    - What is the sample space of median course load?

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Events and Probabilities

- Event: Some combination of outcomes
- Probabilities: interpret relative to infinite repetitions of the random experiment
  - E.g. Roll a die twice; find mean; P($\bar{X} > 5$) = 3/36
  - If outcomes mutually exclusive & exhaustive: probabilities between 0 and 1 and sum to 1
- Complement: The event that occurs when A does not occur: A^C (or A'): P(A) + P(A^C) = 1

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Three Types of Probabilities

- Joint: P(two events both occur), P(A and B)
    - P(spade and king) = P(spade ∩ king) = 1/52
- Marginal: P(single event)
    - P(spade) = 13/52 = 1/4; P(king) = 4/52 = 1/13
  - If outcomes exhaustive and mutually exclusive, add all joint probabilities with the single event
- Conditional: P(A given B has occurred), P(A|B)
    - P(king | spade) = 1/13; P(spade | king) = 1/4

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

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Joint Probability Table: 2012, 25 – 54 year olds, Stats Canada web site

Education	Employed	Unemp.	Not in LF	Total
Not HS graduate	0.0614	0.0082	0.0292	0.0988
HS graduate	0.1463	0.0104	0.0312	0.1879
Some post-sec.	0.0387	0.0028	0.0080	0.0495
Post-sec. degree	0.3151	0.0180	0.0377	0.3707
University degree	0.2524	0.0127	0.0280	0.2931
Total	0.8139	0.0521	0.1341	1.0000

How do you interpret these numbers?

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For an unemployed person, what is probability that s/he has a University degree? What kind of probability is the answer?

Education		Unemp.		
Not HS graduate		0.0082		
HS graduate		0.0104		
Some post-sec.		0.0028		
Post-sec. degree		0.0180		
University degree		0.0127		
Total		0.0521		

Imagine 10,000 people: 521 would be unemployed and 127 of the 521 would have University degrees.

P(Univ. degree | Unemp.) = 127/521 = 0.24

P(Not HS degree | Unemp.) = 82/521 = 0.16

People with more education have a *higher* chance of being unemployed?

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How does the chance of being unemployed vary by educational achievement?

Education		Unemp.		Total
Not HS graduate		0.0082		0.0988
HS graduate		0.0104		0.1879
Some post-sec.		0.0028		0.0495
Post-sec. degree		0.0180		0.3707
University degree		0.0127		0.2931

P(unemp. | not a HS graduate) = 0.0082/0.0988 = 0.083

P(unemp. | HS graduate) = 0.0104/0.1879 = 0.055

P(unemp. | some post-sec.) = 0.0028/0.0495 = 0.056

P(unemp. | post-sec. degree) = 0.0180/0.3707 = 0.048

P(unemp. | university degree) = 0.0127/0.2931 = 0.043

Contradicts previous slide?

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Joint Probability Table: 2012, 25 – 54 year olds, Stats Canada web site

Place of birth	Employed	Unemp.	Not in LF	Total
Canada	0.6136	0.0351	0.0885	0.7373
Not Canada	0.2002	0.0169	0.0455	0.2627
Total	0.8139	0.0521	0.1341	1.0000

Review pp. 62 – 68 of textbook

Independence

- Two events are independent if and only if P(A | B) = P(A) (or equivalently P(B | A) = P(B))
  - Chance of A not affected by occurrence of B
  - Ex: Is being unemployed independent of having a University degree (UD)?
    - P(Unemp | UD) = 0.0127 / 0.2931 = 0.043
    - P(Unemp) = 0.052
  - Ex: Tossed 10 heads in a row with fair coin
    - P(11th toss H | 10 H's) = P(11th toss H)?

Probability Rules

- Complement Rule: P(A^C) = 1 – P(A)
  - E.g. If P($\bar{X}$ > 5) = 3/36 then P($\bar{X} \leq 5$) = ?
- Multiplication Rule: P(A & B) = P(A | B) * P(B)
  - Special case: P(A & B) = P(A) * P(B) if independent
  - Special case: P(A_1 and A_2 … and A_N) = P(A_1) * P(A_2) * … * P(A_N) if all events independent
  - E.g. tossing a fair coin (independent events)
    - P(HHHHHHHHH) = P(HTTHHTHHT) = $(0.5)^9$ = 0.0020

TK71 (portal): "Even the fairest of coins, however, given the limitations of its memory and moral sense, cannot be as fair as the gambler expects it to be."

Be Precise: Illustration with an Example from Chapter 8

- The probability a called customer qualifies for a platinum card is 0.35: P(Q) = 0.35
  - Considering the next 3 calls what is the probability that customer 1 qualifies, customer 2 does not qualify, and customer 3 qualifies?
  - $P(QQ^{C}Q) = 0.35*0.65*0.35 = 0.08$ (reasonably assumes independence for this context)
  - But, P(2 out of 3 qualify) = 0.24
    - You will learn how to find this in Lecture 8/Chapter 9

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Resale Housing Example

- Marginal probabilities:
  P(Bought ≤ 5) = 0.3
  P(For sale) = 0.3
- Probability house for sale and bought within last 5 yrs?
  - 0.3*0.3 = 0.09?
  - Are events independent?

	For sale (FS)	Not For sale (FS')
Bought ≤ 5 yrs ago (BR)	0.2	0.1
Bought > 5 yrs ago (BR')	0.1	0.6

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Addition Rules

- Addition Rule mutually exclusive (disjoint) events: P(A or B) = P(A) + P(B)
  - Mutually exclusive if both cannot occur
  - Used to compute marginal probabilities
  - Ex: P(FS') = P(FS' and BR) + P(FS' and BR')
- Addition Rule: Probability that either event A or event B occurs is

  P(A or B) = P(A) + P(B) – P(A and B)

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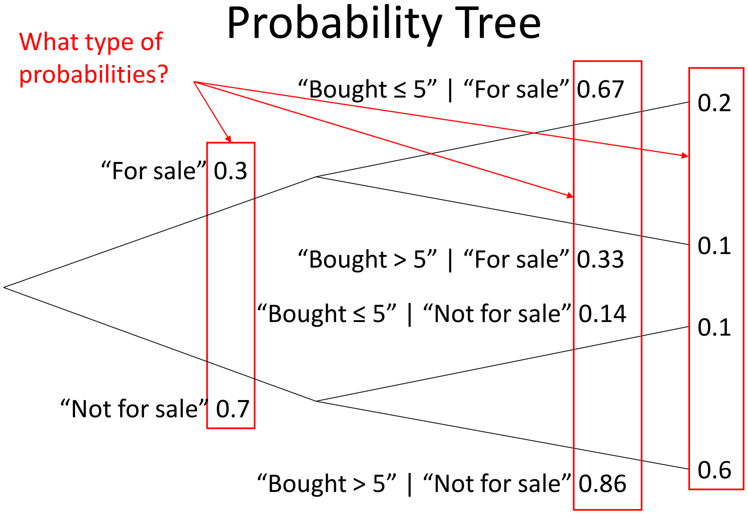
Why Subtract Joint Probability?

- Probability house is either for sale or bought within 5 years:
  0.2 + 0.1 + 0.1 = 0.4
- P(For sale) = 0.3
  P(Bought ≤ 5) = 0.3
- P(For sale OR Bought ≤ 5) ≠ 0.3 + 0.3
  - Why not?

	For sale (FS)	Not For sale (FS’)
Bought ≤ 5 yrs ago (BR)	0.2	0.1
Bought > 5 yrs ago (BR’)	0.1	0.6

Probability Tree



Union: Either A or B or both occur (A or B) aka (A U B)

Place of birth	**Employed**	**Unemp.**	**Not in LF**	**Total**
Canada	0.6136	0.0351	0.0885	0.7373
Not Canada	0.2002	0.0169	0.0455	0.2627
Total	0.8139	0.0521	0.1341	1.0000

Place of birth	**In LF**	**Not in LF**	**Total**
Canada	0.6487	0.0885	0.7373
Not Canada	0.2171	0.0455	0.2627
Total	0.8660	0.1341	1.0000

Which addition rule used in this example?

Mutually Exclusive (Disjoint) ≠ Independent

Place of birth	Employed	Unemp.	Not in LF	Total
Canada	0.6136	0.0351	0.0885	0.7373
Not Canada	0.2002	0.0169	0.0455	0.2627
Total	0.8139	0.0521	0.1341	1.0000

The events "Canada" and "Not Canada" are mutually exclusive (i.e. disjoint): P(Canada and Not Canada) = 0

The events "Canada" and "Not Canada" are certainly not independent:
P(Canada) = 0.7373 but P(Canada | Not Canada) = 0

Apply Concepts to Current Research

Table 8a. Consequences of Misconduct: Industry and Firm Discipline

	No Misconduct	Misconduct
Remain with the Firm	81.29%	51.99%
Leave the Firm	18.71%	48.01%
Leave the Industry	8.92%	26.96%
Join a Different Firm (within 1 year)	9.79%	21.05%

Note: Table 8a displays the average annual job turnover among financial advisers over the period 2005-2015. The table shows, on average, the percentage of advisers that remain with their firm, leave the industry (for at least one year) or join a new firm (within a year). The job transitions are broken down by whether or not the advisor was disciplined for misconduct in the previous year.

Egan, Mark, Gregor Matvos and Amit Seru (2016) "The Market for Financial Advisor Misconduct," *NBER Working Paper*, http://www.nber.org/papers/w22050

Cheating Teachers, Worked Out