ECO 310: Empirical Industrial Organization

Lecture 2: Production Functions: Introduction

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Outline

1. MODEL
2. DATA
3. WHAT DETERMINES PRODUCTIVITY?
4. ESTIMATION: THE SIMULTANEITY PROBLEM

1. MODEL

WHAT IS A PRODUCTION FUNCTION (PF)?

- It is a function that **relates** the **amount of physical output** of a production process (Y) to the **amount of physical inputs** or factors of production (X).
- Estimation of PFs plays a key role in empirical questions such as:
  - Estimation of Firms' Costs.
  - Measurement of Firms' Productivity (and its growth).
  - Misallocation of inputs. Do more productive firms use more inputs?
  - Evaluating the effects of adopting new technologies.
  - Measurement of Learning-by-doing.

PRODUCTION FUNCTIONS

- A standard representation of a PF is:

$$Y = A \times f(X_1, X_2, ..., X_J)$$

- Y is the firm's output (physical units per year).

- X_1, X_2, .., and X_J are measures of J firm inputs.

  Labor, capital, energy, materials, ...

- A represents the firm's **Total Factor Productivity**.

  Anything else affecting output that we do not observe as researchers: managerial ability, organization of production, quality of inputs, quality of land, ...

- The marginal productivity of input j is: $MP_j = \frac{dY}{dX_j} = A \ \frac{df}{dX_j}$.

- Note that TFP increases proportionally the MP of all the inputs.

COBB-DOUGLAS PRODUCTION FUNCTION

- A common specification is the Cobb-Douglas PF:

$$Y = A \; X_1^{\alpha_1} \; X_2^{\alpha_2} \; ... \; X_J^{\alpha_J}$$

α_1, α_2, ..., α_J are technological parameters (all positive).

- For the Cobb-Douglas PF the marginal productivity of input j is:

$$MP_j \; = \; \frac{dY}{dX_j} \; = \; \alpha_j \; X_1^{\alpha_1} \; ... X_j^{\alpha_j - 1} ... \; X_J^{\alpha_J} \; = \; \alpha_j \; \frac{Y}{X_j}$$

- In this PF, MP_j increases with the amount of any other input $k \neq j$. This means that **all the inputs are complements** in production.

$$\frac{dMP_j}{dX_k} \; = \; \frac{\alpha_j}{X_j} \; \frac{dY}{dX_k} \; = \; = \frac{\alpha_j \; \alpha_k \; Y}{X_j \; X_k} > 0$$

PRODUCTION FUNCTION AND COST FUNCTION

- Given a firm's PF and input prices, its **cost function** $C(Y)$ is defined as the minimum cost of producing the amount of output Y:

$$C(Y) = \left[\begin{array}{c} \min_{\{X_1,X_2,...,X_J\}} W_1X_1 + W_2X_2 + ... + W_JX_J \\ \\ \text{subject to: } Y = A\ f(X_1, X_2, ..., X_J) \end{array} \right]$$

where W_1, W_2, ..., W_J are inpit prices.

- Or using a Lagrange representation:

$$C(Y) = \min_{\{\lambda,X_1,...,X_J\}} W_1X_1 + ... + W_JX_J + \lambda \left[Y - A\ f(X_1, ..., X_J) \right]$$

where λ is the Lagrange multiplier of the output restriction.

- The **marginal conditions of optimality** imply that for every input j:

$$W_j - \lambda\ MP_j = 0$$

COST FUNCTION FOR THE COBB-DOUGLAS PF

- Remember that for the Cobb-Douglas PF $MP_j = \alpha_j \ \dfrac{Y}{X_j}$.

- Therefore, the marginal condition of optimality for input j implies:

$$W_j \ X_j = \lambda \ \alpha_j \ Y$$

- And the total cost is equal to:

$$\sum_{j=1}^{J} W_j X_j = \lambda \ \alpha \ Y, \ \ where \ \alpha \equiv \sum_{j=1}^{J} \alpha_j$$

- Parameter α measures the **Returns to Scale** in the PF.

- Returns to Scale: constant if $\alpha = 1$, decreasing if $\alpha < 1$, and increasing if $\alpha > 1$.

DERIVATION OF COST FUNCTION FOR COBB-DOUGLAS [2]

- We need to obtain the value of the Lagrange multiplier λ. For this, we solve the marginal conditions $X_j = \lambda \alpha_j \; Y / W_j$ into the PF:

$$Y = A \, \left(\frac{\lambda \alpha_1 Y}{W_1} \right)^{\alpha_1} \left(\frac{\lambda \alpha_2 Y}{W_2} \right)^{\alpha_2} \ldots \left(\frac{\lambda \alpha_J Y}{W_J} \right)^{\alpha_J}$$

- Solving in this expression for the Lagrange multiplier:

$$\lambda = \left(\frac{W_1}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha}} \left(\frac{W_2}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha}} \ldots \left(\frac{W_J}{\alpha_J} \right)^{\frac{\alpha_J}{\alpha}} \left(\frac{Y}{A} \right)^{\frac{1}{\alpha}} \frac{1}{Y}$$

- Plugging this expression of the multiplier into the equation $C(Y) = \lambda \alpha \; Y$ for the cost, we obtain the cost function:

$$C(Y) = \alpha \, \left(\frac{Y}{A} \right)^{\frac{1}{\alpha}} \, \left(\frac{W_1}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha}} \left(\frac{W_2}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha}} \ldots \left(\frac{W_J}{\alpha_J} \right)^{\frac{\alpha_J}{\alpha}}$$

COST FUNCTION FOR THE COBB-DOUGLAS PF (3)

$$C(Y) = \alpha \left(\frac{Y}{A}\right)^{\frac{1}{\alpha}} \left(\frac{W_1}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha}} \left(\frac{W_2}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha}} \cdots \left(\frac{W_J}{\alpha_J}\right)^{\frac{\alpha_J}{\alpha}}$$

- The marginal cost is:

$$C'(Y) = Y^{\frac{1}{\alpha}-1} \left(\frac{1}{A}\right)^{\frac{1}{\alpha}} \left(\frac{W_1}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha}} \left(\frac{W_2}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha}} \cdots \left(\frac{W_J}{\alpha_J}\right)^{\frac{\alpha_J}{\alpha}}$$

- The sign of $C''(Y)$ is equal to the sign of $\frac{1}{\alpha} - 1$.
  $\alpha = 1$ (*constant returns*): $C''(Y) = 0$ (Constant MC)
  $\alpha < 1$ (*decreasing returns*): $C''(Y) > 0$ (Increasing MC)
  $\alpha > 1$ (*increasing returns*): $C''(Y) < 0$ (Decreasing MC).

MORE ON THE COBB-DOUGLAS

- A nice property of the Cobb-Douglas (for the purpose of estimation) is that its **logarithm transformation is linear in parameters**:

$$\ln(Y) = \ln(A) + \alpha_1 \ \ln(X_1) + \alpha_2 \ \ln(X_2) + \ ... + \alpha_J \ \ln(X_J)$$

- We will represent $\log(Y)$ and $\log(X)$ using the lower letters y and x, resp., and the log-TFP using ω, such that:

$$y = \omega + \alpha_1 \ x_1 + \alpha_2 \ x_2 + \ ... \ + \alpha_J \ x_J$$

- Differences in log-TFP (in ω) are in percentage terms:

  Example: Consider two firms, 1 and 2, using the same amount of inputs X but with $\omega_1 = 1.1$ and $\omega_2 = 1.5$ such that $\omega_2 - \omega_1 = 0.4$. Therefore, firm 2 is 40% more productive than firm 1.

MORE ON THE COBB-DOUGLAS (2)

- Most empirical applications that we will see in this course consider two inputs: **Labor** (L) and **Capital** (K):

$$y = \alpha_L \; \ell + \alpha_K \; k + \omega$$

with $\ell \equiv \ln(L)$ and $k \equiv \ln(K)$.

- Sometimes the specification also includes **Materials** (M):

$$y = \alpha_L \; \ell + \alpha_K \; k + \alpha_M \; m + \omega$$

with $m \equiv \ln(M)$.

2. DATA

DATA

- **Panel Data** of N firms over T time periods with information on output, labor, and capital (in logs):

$$\{\ y_{it},\ \ell_{it},\ k_{it} : i = 1, 2, ..., N\ ;\ t = 1, 2, ... T\ \}$$

- We are interested in the estimation of the parameters α_L and α_K in the Cobb-Douglas PF (in logs):

$$y_{it} = \alpha_L\ \ell_{it} + \alpha_K\ k_{it} + \omega_{it} + e_{it}$$

- ω_{it} = log-TFP. Unobserved inputs (for the researcher) which are known to the firm when it decides K and L: e.g., managerial ability, quality of land, different technologies.

OBSERVING REVENUE INSTEAD OF PHYSICAL OUTPUT

- $Revenue = R_{it} = P_{it}\ Y_{it}$ such that:

$$\ln(R_{it}) = \ln(P_{it}) + \ln(Y_{it}) = p_{it} + y_{it}$$

  but the researcher only observes $\ln(R_{it})$.

- **"Solution" 1**
  - Proxy $\ln(P_{it})$ using industry-level price index, $\ln(P_{Industry,t})$.
  - $\ln(P_{it}) = \ln(P_{Industry,t}) + u_{it}$, where u_{it} is measurement error.
  - This measurement error can interpreted as part of the TFP ω_{it}.

OBSERVING REVENUE INSTEAD OF PHYSICAL OUTPUT [2]

- **"Solution" 2**
- Assume isoelastic demand & monopolisitc competition:

$$y_{it} = b_{it} - \beta \; p_{it}$$

b_{it} is unobservable to the researcher and β is the demand elasticity.

- Then, $p_{it} = \dfrac{b_{it} - y_{it}}{\beta}$ and:

$$\ln(R_{it}) = p_{it} + y_{it} = \frac{b_{it}}{\beta} + \left(1 - \frac{1}{\beta}\right) y_{it}$$

such that:

$$\ln(R_{it}) = \alpha_L^* \; \ell_{it} + \alpha_K^* \; k_{it} + \omega_{it}^* + e_{it}$$

with $\alpha_L^* = \left(1 - \frac{1}{\beta}\right) \alpha_L$, $\alpha_K^* = \left(1 - \frac{1}{\beta}\right) \alpha_K$, and $\omega_{it}^* = \omega_{it} + \frac{b_{it}}{\beta}$.

- This is very relevant for interpretation of results and of "log-TFP".
- For instance, $\alpha_L^* + \alpha_K^* = \left(1 - \frac{1}{\beta}\right) (\alpha_L + \alpha_K) < \alpha_L + \alpha_K$.

3. What determines Total Factor Productivity?

TOTAL FACTOR PRODUCTIVITY (TFP)

- Production function:

$$Y_{it} = A_{it} \; F(K_{it}, L_{it}, M_{it})$$

- A_{it} is denoted Total Factor Productivity (TFP).

- It is a factor-neutral shifter that captures variations in output not explained by observable inputs.

- TFP is a residual.

LARGE & PERSISTENT DIFFERENCES IN TFP ACROSS FIRMS

- **Ubiquitous**: even within narrowly defined industries and products.

- **Large**: 90th to 10th percentile TFP ratios: $\frac{A_{90th}}{A_{10th}}$

  - U.S. manufacturing, **average** within 4-digit SIC industries = **1.92**

  - Denmark: average = **3.75**

  - China or India, average > 5.

- **Persistent**: AR(1) of log-TFP with annual frequency: autoregressive coefficients between 0.6 to 0.8.

- **It matters**: Higher TFP producers are more likely to survive, innovate, invest, ...

WHY DO FIRMS DIFFER IN THEIR PRODUCTIVITY LEVELS?

- What supports such large productivity differences in equilibrium?
- Can producers control the factors that influence productivity or are they purely external effects of the environment?
- If firms can partly control their TFP, what type of choices increase it?

WHY TFP DISPERSION IS POSSIBLE IN EQUILIBRIUM?

- Because the profit function is concave in output and the optimal amount of profit for a monopolist (or duopolist, ...) is smaller than total demand.

- Let the profit of a firm be:

$$\pi_i = P_i(Y_i)\ Y_i - C(Y_i, A_i)$$

  $P_i(Y_i) =$ Inverse demand function; $C(Y_i, A_i) =$ Cost function.

- **Key condition:** either $P_i(Y_i)\ Y_i$ is strictly concave in Y_i, or $C(.)$ is strictly convex in Y_i. [The profit function is strictly concave].

- **Example: DRS even with perfect competition**. $P\ Y_i$ is linear in Y_i but $C(.)$ is strictly convex because DRS.

- **Example: Oligopoly competition even with CRS**. $C(.)$ is linear but $P_i(Y_i)\ Y_i$ is strictly concave in Y_i if demand is downward sloping.

WHY TFP DISPERSION IS POSSIBLE IN EQUILIBRIUM? [2]

- Equilibrium implies the marginal condition for optimal output:

$$MR_i \equiv \frac{\partial \left[P(Y_i) \ \ Y_i \right]}{\partial Y_i} = \frac{\partial C(Y_i, A_i)}{\partial Y_i} \equiv MC_i$$

- If variable profit is strictly concave, this equilibrium can support firms with different TFPs, A_i.

- It is not optimal for the firm with highest TFP to provide all the output in the industry.

- Firms with different TFPs (above a certain threshold value) operate in the same market.

How can a firm affect its TFP?

- Human resources and Managerial Practices.
- Learning-by-Doing
- Organizational structure: vertical integration vs outsourcing.
- Higher-Quality of Labor and Capital inputs.
- Adoption of new technologies.
- Investment in R&D.
- Innovation: process and product innovation.

4. Estimation: Simultaneity Problem

THE SIMULTANEITY PROBLEM

- Consider the PF:

$$y_{it} = \alpha_L \ \ell_{it} + \alpha_K \ k_{it} + \omega_{it}$$

- We are interested in the estimation of α_L and α_K.
- These parameters represent "ceteris paribus" causal effects of labor and capital on output, respectively.
- When the manager decides the optimal (k_{it}, ℓ_{it}) she has some information about log-TFP ω_{it} (that we do not observe).
- This means that there is a correlation between the observable inputs (k_{it}, ℓ_{it}) and the unobservable ω_{it}.
- This correlation implies that the OLS estimators of α_L and α_K are biased and inconsistent.