# ECO 310: Empirical Industrial Organization

## Lecture 9: Models of Competition in Prices or Quantities: Conjectural Variations

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# Outline on today's lecture

1. **Introduction**
2. **Estimating the form of competition when MCs are observed**
3. **Estimating the form of competition without data on MCs**

# 1. Introduction

# Introduction

- In the previous lecture we saw how given a (estimated) demand system and an assumption about competition, **we can obtain (estimate) firms' marginal costs**.
- In today's lecture we will see how given a demand system and firms' marginal costs, **we can identify the form of competition in a market.**
- More specifically, we can identify firms' beliefs about how the other firms in the market respond strategically.
- This approach is called the **conjectural variation approach** or **conjectural variation model**.

# 2. Conjectural variation model: Homogeneous product markets

# Conjectural Variation Model: Homogeneous product markets

- Consider an industry where, at period $t$, the inverse demand curve is $p_t = P\left(Q_t, X_t^D\right)$, and firms, indexed by $i$, have cost functions $C_i(q_{it})$.
- Every firm $i$, chooses its amount of output, $q_{it}$, to maximize its profit, $\Pi_{it} = p_t \ q_{it} - C_i(q_{it})$.
- Without further assumptions, the marginal condition for the profit maximization of a firm is **marginal revenue = marginal cost**, where the marginal revenue of firm $i$ is:

$$MR_{it} = p_t + P_Q'\left(Q_t \ X_t^D\right) \ \left[1 + \frac{dQ_{(-i)t}}{dq_{it}}\right] \ q_{it}$$

- $\dfrac{dQ_{(-i)t}}{dq_{it}}$ represents the **belief** that firm $i$ has about how the other firms will respond if she changes its own amount. We denote this **belief** as the **conjectural variation of firm** $i$, $CV_i$.

# Conjectural Variations and Beliefs

- As researchers, we can consider different assumptions about firms' beliefs or conjectural variations, $CV_{it}$.
- An assumption on CVs implies a particular model of competition.
- Different assumptions imply different equilibrium outcomes, $q_{it}$, $Q_t$, and $p_t$.
- However, not all the assumptions are consistent with an **equilibrium**.
- In fact, most assumptions about CVs imply an equilibrium where firms are not rational in the sense that they have beliefs that do not hold in equilibrium.

# Conjectural Variations: Nash-Cournot equilibrium

- In our model of firm competition, Nash conjecture implies that:

$$CV_{it} \equiv \frac{\partial Q_{(-i)t}}{\partial q_{it}} = 0$$

- This conjecture implies the Cournot equilibrium (or Nash-Cournot equilibrium).

- For every firm $i$, the "perceived" marginal revenue is:

$$MR_{it} = p_t + P'_Q \left( Q_t \;\; X_t^D \right) \; q_{it}$$

and the condition $p_t + P'_Q \left( Q_t \;\; X_t^D \right) \; q_{it} = MC_i(q_{it})$ implies the Cournot equilibrium.

# Conjectural Variations: Perfect Competition

- Are other assumptions on firms' CVs that are consistent with a rational equilibrium?
- Yes, there are CVs that generate **perfect competition equilibrium** and the **collusive or monopoly equilibrium** which are consistent (rational) with the equilibrium outcome that they generate.
- **Perfect competition**. For every firm $i$, $CV_{it} = -1$.
- Note that this conjecture implies that:

$$MR_{it} = p_t + P'_Q\left(Q_t \; X_t^D\right) \; [1 - 1] \; q_{it} = p_t$$

and the conditions $p_t = MC_i(q_{it})$ imply the perfect competition equilibrium.

# Conjectural Variations: Collusion

- There are also beliefs that can generate the collusive outcome (monopoly outcome) as a rational equilibrium.
- **Collusion (Monopoly)**. For every firm $i$, $CV_{it} = N_t - 1$. This conjecture implies:

$$MR_{it} = p_t + P'_Q \left( Q_t \ X_t^D \right) \ N_t \ q_{it}$$

- This conjecture implies the equilibrium conditions:

$$p_t + P'_Q \left( Q_t \ X_t^D \right) \ N_t \ q_{it} = MC_i(q_{it})$$

- When firms have constant and homogeneous MCs, these conditions imply:

$$p_t + P'_Q \left( Q_t \ X_t^D \right) \ Q_t = MC$$

which is the equilibrium condition for the Monopoly (collusive or cartel) outcome.

# Conjectural Variations: Nature of Competition

- The value of the beliefs CV are related to the "nature of competition", i.e., Cournot, Perfect Competition, Cartel (Monopoly).

$$\text{Perfect competition:} \quad CV_{it} = -1; \quad MR_{it} = p_t$$

$$\text{Nash-Cournot:} \quad CV_{it} = 0; \quad MR_{it} = p_t + P'_Q(Q_t) \;\; q_{it}$$

$$\text{Cartel all firms:} \quad CV_{it} = N_t - 1; \quad MR_{it} = p_t + P'_Q(Q_t) \;\; Q_t$$

- Given this result, one can argue that $CV$ is closely related to the **nature of competition**, and therefore with equilibrium price and quantities.
- If $CV$ is negative, the degree of competition is stronger than Cournot. The closer to $-1$, the more competitive.
- If $CV$ is positive, the degree of competition is weaker than Cournot. The closer to $N_t - 1$, the less competitive.

# Conjectural Variations: Nature of Competition [2]

- Interpreting the beliefs CV as an **index of competition** is correct.
- However, it is important to take into account that for values of CV different to $-1$, or 0, or $N_t - 1$, the "Conjectural Variation" equilibrium that we obtain is not a rational equilibrium.
- We can think in the CV as firms' beliefs that are determined over time as the result of firms interactions and learning (a dynamic game).

# Conjectural Variation: Estimation

- Consider an homogeneous product industry and a researcher with data on firms' quantities and marginal costs, and market prices over $T$ periods of time:

$$\text{Data} = \{p_t,\ MC_{it},\ q_{it}\} \text{ for } i = 1, 2, ..., N_t \ \&\ t = 1, 2, ..., T$$

- Under the assumption that every firm chooses the amount of output that maximizes its profit given its belief $CV_{it}$, we have that the following condition holds:

$$p_t + P'_Q\left(Q_t\ X_t^D\right)\ [1 + CV_{it}]\ q_{it} = MC_{it}$$

- And solving for the conjectural variation,

$$CV_{it} = \frac{p_t - MC_{it}}{-P'_Q\left(Q_t\ X_t^D\right)\ q_{it}} - 1 = \left[\frac{(p_t - MC_{it})\,/\,p_t}{q_{it}\,/\,Q_t}\right] |\eta_t| - 1$$

where $\eta_t$ is the demand elasticity.

# Conjectural Variation: Estimation [2]

$$CV_{it} = \left[ \frac{(p_t - MC_{it}) \, / \, p_t}{q_{it} \, / \, Q_t} \right] |\eta_t| - 1$$

- This equation shows that, given data on quantities, prices, demand and marginal costs, we can identify the firms' beliefs that are consistent with these data and with profit maximization.
- Let us denote $\left[ \frac{(p_t - MC_{it}) / p_t}{q_{it} / Q_t} \right]$ as the **Lerner-index-to-market-share ratio** of a firm.
- If the Lerner-index-to-market-share ratios are close zero, then the estimated values of $CV$ will be close to $-1$, unless the absolute demand elasticity is large.
- If the Lerner-index-to-market-share ratios are large (i.e., larger than the inverse demand elasticity), then estimated CV values will be greater than zero, and can reject the hypothesis of Cournot competition.

# Conjectural Variation: Estimation [3]

$$CV_{it} = \left[ \frac{(p_t - MC_{it}) \, / \, p_t}{q_{it} \, / \, Q_t} \right] |\eta_t| - 1$$

- Part of the sample variation of $CV_{it}$ can be due to estimation error in demand and marginal costs.
- To implement a formal statistical test of the value of $CV_{it}$ we need to take into account this error.
- For instance, let $\overline{CV}$ be the sample mean of the values $CV_{it}$. Under the null hypothesis of Cournot competition, $CV_{it} = 0$ for every $(i, t)$ and $\overline{CV}$ has a Normal distribution $(0, , s^2)$. We can estimate $s$ and implement a t-test based on the statistic $\overline{CV} / \widehat{s}$.

# 3. Estimating CV parameters without data on MCs

# Estimating CV parameters without data on MCs

- So far, we have considered the estimation of CV parameters when the researcher knows both demand and firms' marginal costs.
- We now consider the case where the **researcher knows the demand, but it does not know firms' marginal costs**.
- Identification of CVs requires also de identification of MCs.
- Under some conditions, we can **jointly identify CVs and MCs** using the marginal conditions of optimality and the demand.

# Data

- Researcher observes data:

$$\text{Data} = \left\{ P_t,\ q_{it},\, X_t^D,\, X_t^{MC} :\ i = 1, ...N_t;\ t = 1, ..., T \right\}$$

- $X_t^D$ are variables affecting consumer demand, e.g., average income, population.
- $X_t^{MC}$ are variables affecting marginal costs, e.g., some input prices.

# Model: Demand and MCs

- Consider the linear (inverse) demand equation:

$$P_t = \alpha_0 + \alpha_1 \ X_t^D - \alpha_2 \ Q_t + \varepsilon_t^D$$

with $\alpha_2 \geq 0$, and $\varepsilon_t^D$ is unobservable to the researcher.

- Consider the marginal cost function:

$$MC_{it} = \beta_0 + \beta_1 \ X_t^{MC} + \beta_2 \ q_{it} + \varepsilon_{it}^{MC}$$

with $\beta_2 \geq 0$, and $\varepsilon_{it}^{MC}$ is unobservable to the researcher.

# Model: Profit maximization

- Profit maximization implies $MR_{it} = MC_{it}$, or equivalently:

$$P_t + \frac{dP_t}{dQ_t} \left[ 1 + CV_{it} \right] \; q_{it} = MC_{it}$$

- In the model above, $\dfrac{dP_t}{dQ_t} = -\alpha_2$. Therefore,

$$P_t - \alpha_2 \left[ 1 + CV_{it} \right] \; q_{it} = \beta_0 + \beta_1 \; X_t^{MC} + \beta_2 \; q_{it} + \varepsilon_{it}^{MC}$$

- Or equivalently,

$$P_t = \beta_0 + \beta_1 \; X_t^{MC} + \left[ \beta_2 + \alpha_2 (1 + CV_{it}) \right] \; q_{it} + \varepsilon_{it}^{MC}$$

- This equation describes the marginal condition for profit maximization. We assume now that $CV_{it} = CV$ for every observation $i, t$ in the data.

# Complete structural model

- The structural equations of the model are:

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t + \varepsilon_t^D$$

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \; X_t^{MC} + [\beta_2 + \alpha_2 (1 + CV)] \; q_{it} + \varepsilon_{it}^{MC}$$

- Using this model and data, **can we identify (estimate consistently, without asymptotic bias) the CV parameter?**

- First, we will see that NO. In this model we cannot separately identify CV and MC.

- Second, we will see that a simple modification of this model implies separate identification of CV and MC.

# Identification of demand parameters

$$\text{Demand:} \quad P_t = \ \alpha_0 + \alpha_1 \ X_t^D - \alpha_2 \ Q_t + \varepsilon_t^D$$

- Endogeneity problem: in equilibrium, $cov(Q_t, \varepsilon_t^D) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, $Q_t$ depends on $X_t^{MC}$. Note that $X_t^{MC}$ does not enter in demand. If $X_t^{MC}$ is not correlated with $\varepsilon_t^D$, then $X_t^{MC}$ satisfies all the conditions for being a valid instrument.
- Parameters $\alpha_0$, $\alpha_1$, and $\alpha_2$ are identified using this IV estimator.

# Identification of CV and MCs

$$\text{F.O.C.:} \quad P_t = \quad \beta_0 + \beta_1 \; X_t^{MC} + [\beta_2 + \alpha_2(1 + CV)] \;\; q_{it} + \varepsilon_{it}^{MC}$$

- Endogeneity problem: in equilibrium, $cov(q_{it}, \varepsilon_{it}^{MC}) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, $q_{it}$ depends on $X_t^D$. Note that $X_t^D$ does not enter enter in the F.O.C. If $X_t^D$ is not correlated with $\varepsilon_{it}^{MC}$, then $X_t^D$ satisfies all the conditions for being a valid instrument.
- Parameters $\beta_0$, $\beta_1$, and $\gamma \equiv \beta_2 + \alpha_2(1 + CV)$ are identified using this IV estimator.

# The identification problem

$$\text{F.O.C.:} \quad P_t = \quad \beta_0 + \beta_1 \; X_t^{MC} + \gamma \; q_{it} + \varepsilon_{it}^{MC}$$

- Note that we can identify the parameter $\gamma$, where $\gamma \equiv \beta_2 + \alpha_2(1 + CV)$, and the slope of inverse demand function, $\alpha_2$.
- However, knowledge of $\gamma$ and $\alpha_2$ is not sufficient to identify separately $CV$ and the slope of the MC, $\beta_2$.
- Suppose that $\gamma = 1$ and $\alpha_2 = 0.4$, such that we have the constraint:

$$1 = \beta_2 + 0.4 \; (1 + CV)$$

- This equation is satisfied by any of the following:
  [Perfect competition] $CV = -1$ and $\beta_2 = 1.0$
  [Cournot] $CV = 0$ and $\beta_2 = 0.6$
  [Cartel, with $N = 3$] $CV = N - 1 = 2$ and $\beta_2 = 0.2$

# The identification problem [2]

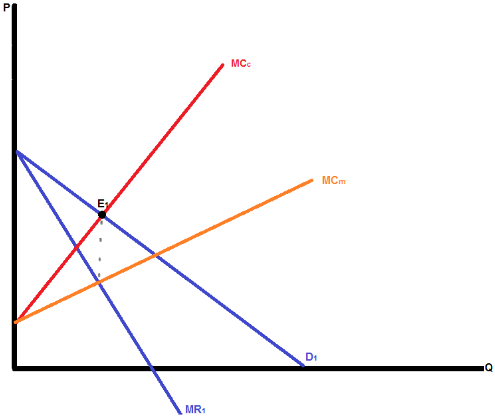
- The IV estimator identifies the MC by using the instrument $X_t^D$ that shifts the demand.

- When we make an assumption about the form of competition, shifts in the demand curve are able to trace out the marginal cost curve, i.e., to identify the MC parameters.

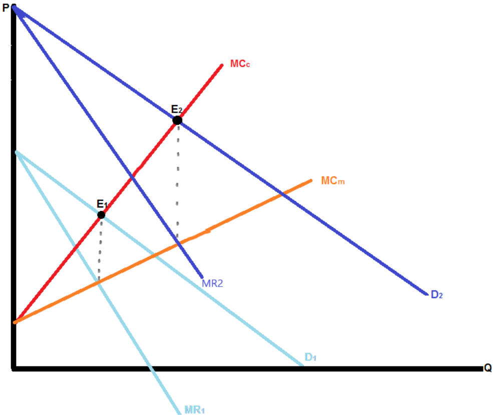
- However, without specifying the form of competition, shifts in the demand alone are not sufficient to separately identify MC and CV.

- Let $\widehat{q}_{it}(X_t^D)$ be the part of $q_{it}$ explained $X_t^D$. When $X_t^D$ varies, we see a positive correlation between $P_t$ and $\widehat{q}_{it}(X_t^D)$. But the magnitude of this correlation can be explained by the combination of:
  - either zero/negative $CV$ and positive and large $\beta_2$;
  - or positive $CV$ and small or zero $\beta_2$.

# The identification problem [3]



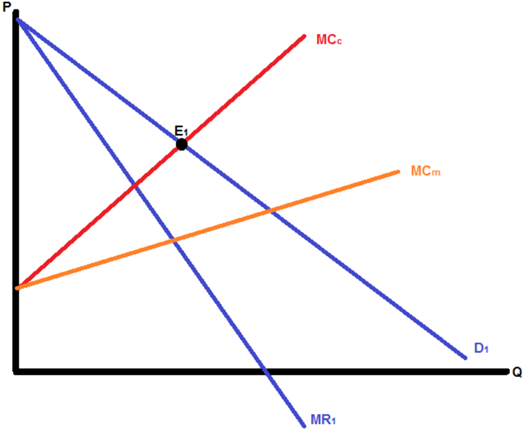
# The identification problem [4]



# Solving the identification problem

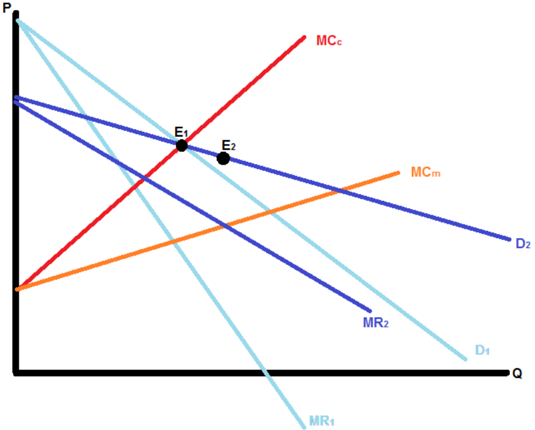
- Solving the identification problem involves generalizing demand so that changes in exogenous variables do **more than just parallel shift** the demand curve and MR.
- In particular, we need to allow for additional exogenous variables that are capable of **rotating** the demand curve as well.
- "Demand Rotators" are exogenous variables affecting the slope of the demand curve:

# Solving the identification problem [2]




- Note that $E_1$ could be an equilibrium either for a perfectly competitive industry with cost $MC_c$ or for a monopolist with cost $MC_m$.
- There is no observable distinction between the hypotheses of competition and

# Solving the identification problem [3]




- Now, rotate the demand curve to $D_2$, with $MR_2$
- Competitive equilibrium stays at $E_1$. But monopoly equilibrium moves to $E_2$

# Solving the identification problem [4]

- Consider now the following demand equation:

$$P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t - \alpha_3 \; [R_t \; Q_t] + \varepsilon_t^D$$

- $R_t$ is an observable variable that affects the slope of the demand, i.e., the price of a substitute or complement product.
- Key condition: $\alpha_3 \neq 0$.
- That is, when $R_t$ varies, there should be rotation (i.e., change in the slope of the demand curve).

# Solving the identification problem [5]

- Given this demand model, we have that:

$$\frac{dP_t}{dQ_t} = -\alpha_2 - \alpha_3 \ R_t$$

- And the F.O.C. for profit maximization

$$P_t + \frac{dP_t}{dQ_t} [1 + CV] \ q_{it} = MC_{it}$$

become:

$$P_t + (-\alpha_2 - \alpha_3 \ R_t) \ \ [1 + CV] \ q_{it} = MC_{it}$$

or equivalently:

$$P_t = MC_{it} + (\alpha_2 + \alpha_3 \ R_t) \ \ [1 + CV] \ q_{it}$$

# Solving the identification problem [6]

- Combining this F.O.C. with the MC function, $MC_{it} = \beta_0 + \beta_1 X_t^{MC} + \beta_2\ q_{it} + \varepsilon_{it}^{MC}$, we have:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \beta_2\ q_{it} + (\alpha_2 + \alpha_3\ R_t)\ \ [1 + CV]\ \ q_{it} + \varepsilon_{it}^{MC}$$

- That we can represent using the following regression model:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \gamma_1\ q_{it} + \gamma_2\ \ (R_t\ q_{it}) + \varepsilon_{it}^{MC}$$

with $\gamma_1 \equiv \beta_2 + \alpha_2\ [1 + CV]$ and $\gamma_2 \equiv \alpha_3\ [1 + CV]$.

# Solving the identification problem [7]

- The structural equations of the model are:

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \ X_t^D - \alpha_2 \ Q_t - \alpha_3 \ [R_t \ Q_t] + \varepsilon_t^D$$

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \ X_t^{MC} + \gamma_1 \ q_{it} + \gamma_2 \ (R_t \ q_{it}) + \varepsilon_{it}^{MC}$$

- Using this model and data, **we can identify separately CV and MC parameters.**

# Identification of demand parameters

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t - \alpha_3 \; [R_t \; Q_t] + \varepsilon_t^D$$

- Endogeneity problem: in equilibrium, $cov(Q_t, \varepsilon_t^D) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, $Q_t$ depends on $X_t^{MC}$. Note that $X_t^{MC}$ does not enter in demand. If $X_t^{MC}$ is not correlated with $\varepsilon_t^D$, then $X_t^{MC}$ satisfies all the conditions for being a valid instrument.
- Parameters $\alpha_0$, $\alpha_1$, $\alpha_2$, and $\alpha_3$ are identified using this IV estimator.

# Identification of CV and MCs

$$\text{F.O.C.:} \quad P_t = \quad \beta_0 + \beta_1 \ X_t^{MC} + \gamma_1 \ q_{it} + \gamma_2 \ \left( R_t \ q_{it} \right) + \varepsilon_{it}^{MC}$$

- Endogeneity problem: in equilibrium, $cov(q_{it}, \varepsilon_{it}^{MC}) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, $q_{it}$ depends on $X_t^D$. Note that $X_t^D$ does not enter enter in the F.O.C. If $X_t^D$ is not correlated with $\varepsilon_{it}^{MC}$, then $X_t^D$ satisfies all the conditions for being a valid instrument.
- Parameters $\beta_0$, $\beta_1$, $\gamma_1$, and $\gamma_2$ are identified.

# Identification of CV and MCs [2]

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \, X_t^{MC} + \gamma_1 \, q_{it} + \gamma_2 \, \left( R_t \, q_{it} \right) + \varepsilon_{it}^{MC}$$

- Note that:

$$\begin{aligned} \gamma_1 &= \beta_2 + \alpha_2 \left[ 1 + CV \right] \\ \gamma_2 &= \alpha_3 \left[ 1 + CV \right] \end{aligned}$$

- It is clear that given $\gamma_2$ and $\alpha_3$, we identify $CV$.
- And given $\gamma_1$, $\alpha_2$, and $CV$ we identify $\beta_2$.

# Identification of CV and MCs [3]

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \; X_t^{MC} + \gamma_1 \; q_{it} + \gamma_2 \; (R_t \; q_{it}) + \varepsilon_{it}^{MC}$$

- with

$$\gamma_2 = \alpha_3 \left[1 + CV\right]$$

- The identification of $CV$ is very intuitive: $1 + CV = \gamma_2 / \alpha_3$ . It measures the ratio between the sensitivity of $P_t$ with respect to $(R_t \; q_{it})$ in the F.O.C. and the sensitivity of $P_t$ with respect to $(R_t \; Q_t)$ in the demand.

- Example: $\alpha_3 = 0.5$ and $N = 3$.
  [Perfect competition] $CV = -1$ such that $\gamma_2 / \alpha_3 = 0$
  [Cournot] $CV = 0$ such that $\gamma_2 / \alpha_3 = 1/0.5 = 2$
  [Cartel, with $N = 3$] $CV = N - 1 = 2$ such that $\gamma_2 / \alpha_3 = 2/0.5 = 4$