ECO310 : Problem Set 2019

$$Y_{it} = A_{it} \left(L_{it}^{P} + \lambda_L L_{it}^{T} \right)^{\alpha_L} \left(K_{it} + \lambda_K I_{it} \right)^{\alpha_K}$$

Taking logs,

$$y_{it} = \alpha_L \log \left(L_{it}^{P} + \lambda_L L_{it}^{T} \right) + \alpha_K \log \left(K_{it} + \lambda_K I_{it} \right) + \omega_{it}$$

$$= \alpha_L \log \left(L_{it}^{P} \left(1 + \lambda_L \frac{L_{it}^{T}}{L_{it}^{P}} \right) \right) + \alpha_K \log \left(K_{it} \left(1 + \lambda_K \frac{I_{it}}{K_{it}} \right) \right) + \omega_{it}$$

(*) Note: $\log(1+x) \approx x$

Therefore, we have

$$y_{it} = \alpha_L \log \left(L_{it}^{P} \right) + \alpha_L \lambda_L \left(\frac{L_{it}^{T}}{L_{it}^{P}} \right)$$

$$+ \alpha_K \log(K_{it}) + \alpha_K \lambda_K \left(\frac{I_{it}}{K_{it}}\right) + \omega_{it}$$

Question 1

Our regression parameters are the following:

$$\beta_{ep} = \alpha_L \qquad \beta_{rt} = \alpha_L \lambda_L$$

$$\beta_K = \alpha_K \qquad \beta_{ik} = \alpha_K \lambda_K$$

Therefore, given our parameter estimates $\hat{\beta}$, we can identify the (λ_L, λ_K) parameters:

$$\hat{\lambda}_L = \frac{\hat{\beta}_{rt}}{\hat{\beta}_{ep}} \qquad \hat{\lambda}_K = \frac{\hat{\beta}_{ik}}{\hat{\beta}_k}$$

Question 3

First, we will assume that the transitory shock is serially correlated according to an AR(1) process:

$$u_{it} = \rho u_{it-1} + a_{it}$$

We quasi-first difference our model to get rid of the serial correlation:

$$y_{it} - \rho y_{it-1}$$

$=$

$$\alpha_L l_{it}^p - \rho\alpha_L l_{it-1}^p + \alpha_L\lambda_L r_{it} - \rho\alpha_L\lambda_L r_{it-1}$$

$$+\alpha_K k_{it} - \rho\alpha_K k_{it-1} + \alpha_K\lambda_K ik_{it} - \rho\alpha_K\lambda_K ik_{it-1}$$

$$+ \eta_i - \rho\eta_i + \gamma_t - \rho\gamma_{t-1}$$

$$+ \rho u_{it-1} + a_{it} - \rho u_{it-1}$$

We obtain the following quasi-first differenced regression model :

$$y_{it} = \beta_{y\_} y_{it-1} + \beta_{lp} lp_{it} + \beta_{lp\_} lp_{it-1}$$

$$+ \beta_{rt} rt_{it} + \beta_{rt\_} rt_{it-1} + \beta_k k_{it}$$

$$+ \beta_{k\_} k_{it-1} + \beta_{ik} ik_{it} + \beta_{ik\_} ik_{it-1}$$

$$+ \eta_i^* + \gamma_t^* + a_{it}$$

where * variables are the variables in quasi-first differences, and

$$\beta_{y-} = \rho \qquad \beta_{lp} = \alpha_L \qquad \beta_{lp-} = -\rho\alpha_L$$

$$\beta_{rt} = \alpha_L \lambda_L \qquad \beta_{rt-} = -\rho\alpha_L\lambda_L \qquad \beta_{k} = \alpha_K$$

$$\beta_{k-} = -\rho\alpha_K \qquad \beta_{ik} = \alpha_K\lambda_K \qquad \beta_{ik-} = -\rho\alpha_K\lambda_K$$

We therefore have the following restrictions on the parameters:

$$\beta_{lp} = \frac{-\beta_{lp-}}{\beta_{y-}} \qquad \beta_{k} = \frac{-\beta_{k-}}{\beta_{y-}}$$

$$\beta_{rt} = \frac{-\beta_{rt-}}{\beta_{y-}} \qquad \beta_{ik} = \frac{-\beta_{ik-}}{\beta_{y-}}$$

We can identify (λ_L, λ_K) in two ways:

① $\hat{\lambda}_L = \dfrac{\hat{\beta}_{rt}}{\hat{\beta}_{lp}}$ $\hat{\lambda}_K = \dfrac{\hat{\beta}_{ik}}{\hat{\beta}_{k}}$

② $\hat{\lambda}_L = \dfrac{\hat{\beta}_{rt\_}}{\hat{\beta}_{lp\_}}$ $\hat{\lambda}_K = \dfrac{\hat{\beta}_{ik\_}}{\hat{\beta}_{k\_}}$

Question 4

We first difference our model:

$(y_{it} - y_{it-1})$

$$= \alpha_L (lp_{it} - lp_{it-1}) + \alpha_L \lambda_L (rt_{it} - rt_{it-1})$$

$$+ \alpha_K (k_{it} - k_{it-1}) + \alpha_K \lambda_K (ik_{it} - ik_{it-1})$$

$$+ \omega_{it} - \omega_{it-1}$$

This yields the following regression parameters:

$$\beta_{lp} = \alpha_L \qquad \beta_{rt} = \alpha_L \lambda_L$$

$$\beta_k = \alpha_K \qquad \beta_{ik} = \alpha_K \lambda_K .$$

Question 5

Here, we assume an AR(1) process for the transitory shock. We therefore consider the quasi-first differenced model of Q3 :

$$y_{it} = \beta_{y\_}\, y_{it-1} + \beta_{lp}\, lp_{it} + \beta_{lp\_}\, lp_{it-1}$$

$$+ \beta_{rt}\, rt_{it} + \beta_{rt\_}\, rt_{it-1} + \beta_{k}\, k_{it}$$

$$+ \beta_{k\_}\, k_{it-1} + \beta_{ik}\, ik_{it} + \beta_{ik\_}\, ik_{it-1}$$

$$+ \eta_i^{*} + \gamma_t^{*} + a_{it}$$

We then first difference the model in order to apply the Arellano - Bond estimator :

$$\Delta y_{it} = \beta_{y\_}\, \Delta y_{it-1} + \beta_{lp}\, \Delta lp_{it} + \beta_{lp\_}\, \Delta lp_{it-1}$$

$$+ \beta_{rt}\, \Delta rt_{it} + \beta_{rt\_}\, \Delta rt_{it-1} + \beta_{k}\, \Delta k_{it}$$

$$+ \beta_{k\_}\, \Delta k_{it-1} + \beta_{ik}\, \Delta ik_{it} + \beta_{ik\_}\, \Delta ik_{it-1}$$

$$+ \Delta \gamma_t^* + \Delta a_{it}$$

Given the regression parameters from Q3, we have the following restrictions:

$$\beta_{lp} = \frac{-\beta_{lp\_}}{\beta_{y\_}} \qquad \beta_{k} = \frac{\beta_{k\_}}{\beta_{y\_}}$$

$$\beta_{rt} = \frac{-\beta_{rt\_}}{\beta_{y\_}} \qquad \beta_{ik} = \frac{-\beta_{ik\_}}{\beta_{y\_}}$$