Empirical Industrial Organization: Models, Methods, and Applications

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5. Market Entry

5.1 Introduction

In a model of market entry the endogenous variables are firms' decisions to be active in the market and, in some cases, the characteristics of the products that firms provide. In the previous chapters, we have taken the number of firms and products in a market as exogenously given or, more precisely, as predetermined in the first stage of a two-stage game of competition. In this chapter, we study the first stage of the competition game.

Empirical games of market entry in retail markets share as common features that the payoff of being active in the market depends on market size, entry cost, and the number and characteristics of other active firms. The set of structural parameters of the model varies considerably across models and applications, but it typically includes parameters that represent the entry cost and the strategic interactions between firms (competition effects). These parameters play a key role in the determination of the number of firms in the market, their characteristics, and their spatial configuration. These costs cannot be identified from the estimation of demand equations, production functions, or marginal conditions of optimality for prices or quantities. Instead, in a structural entry model, entry costs are identified using the principle of revealed preference: if we observe a firm operating in a market it is because its value in that market is greater than the value of shutting down and putting its assets to alternative uses. Under this principle, firms' entry decisions reveal information about the underlying or latent profit function. Empirical games of market entry can be also useful to identify strategic interactions between firms that occur through variable profits. In empirical applications where sample variation in prices is very small but there is substantial variation in entry decisions, an entry model can provide more information about demand substitution between stores and products than the standard approach of using prices and quantities to estimate demand. Furthermore, data on prices and quantities at the store level are sometimes difficult to obtain, while data on firms entry/exit decisions are more commonly available.

In empirical applications of games of market entry, structural parameters are estimated using data on firms' entry decisions in a sample of markets. The estimated model is used to answer empirical questions on the nature of competition and the structure of costs in an industry, and to make predictions about the effects of changes in structural parameters or of counterfactual public policies affecting firms' profits, for example, subsidies, taxes, or zoning laws.

An important application of models of entry is the study of firms' decision about the spatial location of their products, their production plants, or their stores. Competition in differentiated product markets is often characterized by the importance of product location in the space of product characteristics, and therefore, the geographic location of stores is important in retail markets. As shown in previous chapters, the characteristics of firms' products relative to those of competing products can have substantial effects on demand and costs, and consequently on prices, quantities, profits, and consumer welfare. Firms need to choose product location carefully so that they are accessible to many potential customers. For instance, opening a store in attractive locations is typically more expensive (for example, higher land prices) and it can be associated with stronger competition. Firms should consider this trade-off when choosing the best store location. The study of the determinants of spatial location of products is necessary to inform public policy and business debates such as the value of a merger between multiproduct firms, spatial pre-emption, cannibalization between products of the same firm, or the magnitude of economies of scope. Therefore, it is not surprising that models of market entry, store location, and spatial competition have played a fundamental role in the theory of industrial organization at least since the work of Hotelling (1929). However, empirical work on structural estimation of these models has been much more recent and it has followed the seminal work by Bresnahan and Reiss (1990, 1991).

5.2 General ideas

5.2.1 What is a model of market entry?

Models of market entry in IO can be characterized in terms of three main features. First, the key endogenous variable is a firm decision to operate or not in a market. Entry in a market should be understood in a broad sense. The standard example is the decision of a firm to enter in an industry for the first time. However, applications of entry models include also decisions of opening a new store, introducing a new product, adopting a new technology, the release of a new movie, or the decision to bid in an auction, among others. A second important feature is that there is an entry cost associated with being active in the market. Finally, the payoff of being active in the market depends on the number (and the characteristics) of other firms active in the market, that is, the model is a game.

Consider a market with N firms that decide whether to be active. We index firms with $i \in \{1, 2, ..., N\}$. Let $a_i \in \{0, 1\}$ be a binary variable that represents the decision of firm i of being active in a market ($a_i = 1$) or not ($a_i = 0$). The profit of not being active is zero. The profit of an active firm is $V_i(n) - F_i$ where V_i is the variable profit of firm i when there are n firms active in the market, and F_i is the entry cost for firm i. The number of active firms, n, is endogenous and is equal to $n = \sum_{i=1}^{N} a_i$. Under the Nash assumption, every firm takes as given the actions of the other firms and makes a decision that maximizes its own profit. Therefore, the best response of firm i under the

Nash equilibrium is:

$$a_i = \begin{cases} 1 & \text{if} \quad V_i\left(1+\sum_{j\neq i} a_j\right) - F_i \geq 0 \\ 0 & \text{if} \quad V_i\left(1+\sum_{j\neq i} a_j\right) - F_i < 0 \end{cases} \tag{5.1}$$

For instance, consider a market with two potential entrants with $V_1(n) = V_2(n) = 100 - 20\,n$ and $F_1 = F_2 = F$, such that $V_i\left(1+a_j\right) - F_i = 80 - F - 20\,a_j$. The best responses are:

	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	$(0\ ,0)$	$(0\ ,80-F)$
$a_1 = 1$	$(80-F\ ,0)$	$(60-F\ ,60-F)$

(5.2)

We can see that the model has different predictions about market structure depending on the value of the fixed cost. If $F \leq 60$, duopoly, $(a_1,a_2) = (1,1)$, is the unique Nash equilibrium. If $60 < F \leq 80$, then either the monopoly of firm 1 $(a_1,a_2) = (1,0)$ or the monopoly of firm 2 $(a_1,a_2) = (0,1)$ are Nash equilibria. If $F > 80$, then no firm in the market $(a_1,a_2) = (0,0)$ is the unique Nash equilibrium. The observed actions of the potential entrants reveal information about profits, and about fixed costs.

Principle of Revealed Preference. The estimation of structural models of market entry is based on the principle of Revealed Preference. In the context of these models, this principle establishes that if we observe a firm operating in a market it is because its value in that market is greater than the value of shutting down and putting its assets in alternative uses. Under this principle, firms' entry decisions reveal information about the underlying latent firm's profit (or value).

Static models. In this chapter, we study static games of market entry. We study dynamic models of market entry in chapters 7 and 8. There are several differences between static and dynamic models of market entry. But there is a simple difference that should be already pointed out. For static models of entry, we should understand entry as "being active in the market" and not as a transition from being "out" of the market to being "in" the market. That is, in these static models we ignore the fact that, when choosing whether to be active or not in the market, some firms are already active (incumbents) and other firms not (potential entrants). In other words, we ignore that the choice of not being active in the market means "exit" for some firms and "stay out" for others.

5.2.2 Why estimating entry models?

The specification and estimation of models of market entry is motivated by the need to endogenize the number of firms in the market, as well as some characteristics that operate at the extensive margin. Endogenizing the number of firms in the market is a key aspect in any model of IO where market structure is treated as endogenous. Once we endogenize the number of firms in the market, we need to identify entry cost parameters, and these parameters cannot be identified from demand equations, production functions, and marginal conditions of optimality for prices and quantities. We identify entry costs from the own entry model. More generally, we can distinguish the following motives for the estimation of models of market entry.

(a) Identification of entry cost parameters. Parameters such as fixed production costs, entry costs, or investment costs do not appear in demand or production equations, or in the marginal conditions of optimality in firms' decisions of prices or quantities. However, fixed costs contribute to the market entry decision. These parameters can be important in the determination of market structure and market power in an industry.

(b) Data on prices and quantities may not be available at the level of individual firm, product, and market. Many countries have excellent surveys of manufacturers or retailers with information at the level of the specific industry (5 or 6 digits NAICS, SIC) and local markets (census tracts) on the number of establishments and some measure of firm size such as aggregate revenue. Though we observe aggregate revenue at the industry-market level, we do not observe P and Q at that level. Under some assumptions, it is possible to identify structural parameters using these data and the structure of an entry model.

(c) Econometric efficiency. The equilibrium entry conditions contain useful information for the identification of structural parameters. Using this information can increase significantly the precision of our estimates. In fact, when the sample variability in prices and quantities is small, the equilibrium entry conditions may have a more significant contribution to the identification of demand and cost parameters than demand equations or production functions.

(d) Dealing with endogenous selection problem in the estimation of demand or production functions. In some applications, the estimation of a demand system or a production function requires dealing with the endogeneity of firms' and products' entry. For instance, Olley and Pakes (1996) show that ignoring the endogeneity of a firm's decision to exit from the market can generate significant biases in the estimation of production functions. Similarly, in the estimation of demand of differentiated products, not all the products are available in every market and time period. We observe a product only in markets where demand for this product is high enough to make it profitable to introduce that product. Ignoring the endogeneity of the presence of products can introduce important biases in the estimation of demand (Ciliberto, Murry, and Tamer, 2020; Gandhi and Houde, 2019; and Li et al., 2018). Dealing with the endogeneity of product presence may require the specification and estimation of a model of market product entry.

The type of data used, the information structure of the entry game, and the assumptions about unobserved heterogeneity, are important characteristics of an entry game that have implications on the identification, estimation, and predictions of the model.

5.3 Data

The datasets that have been used in empirical applications of structural models of entry in retail markets consist of a sample of geographic markets with information on firms' entry decisions and consumer socio-economic characteristics over one or several periods of time. In these applications, the number of firms and time periods is typically small such that statistical inference (that is, the construction of sample moments and the application of law of large numbers and central limit theorems) is based on a 'large' number of markets. In most applications, the number of geographic markets is between a few hundred and a few thousand. Within these common features, there is substantial heterogeneity in the type of data that have been used in empirical applications.

In this section, we concentrate on four features of the data that are particularly important, as they have substantial implications on the type of model that can be estimated, the empirical questions that we can answer, and the econometric methods to be used. These features are: (1) selection of geographic markets; (2) presence or not of within-market spatial differentiation; (3) information on prices, quantities, or sales at the store level; and (4) information on potential entrants.

5.3.1 Geographic markets

In a seminal paper, Bresnahan and Reiss (1990) use cross-sectional data from 149 small US towns to estimate a model of entry of automobile dealerships. For each town, the dataset contains information on the number of stores in the market, demographic characteristics such as population and income, and input prices such as land prices. The selection of the 149 small towns is based on the following criteria: the town belongs to a county with fewer than 10 000 people; there is no other town with a population of over 1000 people within 25 miles of the central town; and there is no large city within 125 miles. These conditions for the selection of a sample of markets are typically described as the 'isolated small towns' market selection. This approach has been very influential and has been followed in many empirical applications of entry in retail markets.

The main motivation for using this sample selection is in the assumptions of spatial competition in the Bresnahan–Reiss model. The model assumes that the location of a store within a market does not have any implication on its profits or in the degree of competition with other stores. This assumption is plausible only in small towns where the possibilities for spatial differentiation are very limited. If this model were estimated using a sample of large cities, we would spuriously find very small competition effects simply because there is negligible or no competition at all between stores located far away from each other within the city. The model also assumes that there is no competition between stores located in different markets. This assumption is plausible only if the market under study is not geographically close to other markets; otherwise the model would ignore relevant competition from stores outside the market.

Although the 'isolated small towns' approach has generated a good number of important applications, it has some limitations. The extrapolation to urban markets of the empirical findings obtained in these samples of rural markets is in general not plausible. Focusing on rural areas makes the approach impractical for many interesting retail industries that are predominantly urban. Furthermore, when looking at national retail chains, these rural markets account for a very small fraction of these firms' total profits.

5.3.2 Spatial competition

The limitations of the 'isolated small towns' approach have motivated the development of empirical models of entry in retail markets that take into account the spatial locations and differentiation of stores within a city market. The work by Seim (2006) was seminal in this evolution of the literature. In Seim's model, a city is partitioned into many small locations or blocks, for example, census tracts, or a uniform grid of square blocks. A city can be partitioned into dozens, hundreds, or even thousands of these contiguous blocks or locations. In contrast to the 'isolated small towns' approach, these locations are not isolated, and the model allows for competition effects between stores at different locations.

The datasets in these applications contain information on the number of stores, consumer demographics, and input prices at the block level. This typically means that the information on store locations should be geocoded, that is, should include the exact latitude and longitude of each store location. Information on consumer demographics is usually available at a more aggregate geographic level.

The researcher's choice for the size of a block depends on multiple considerations, including the retail industry under study, data availability, specification of the unobservables, and computational cost. In principle, a finer grid entails a more flexible model in measuring spatial substitution between stores. The computational cost of estimating the model can increase rapidly with the number of locations. The assumption on the distribution of the unobservables across locations is also important.

A common approach is to define a block/location where demographic information is available. For example, the set of locations can be equal to the set of census tracts within the city. While convenient, a drawback of this approach is that some blocks, especially those in the periphery of a city, tend to be very large. These large blocks are often problematic because (1) within-block spatial differentiation seems plausible, and (2) the distance to other blocks becomes highly sensitive to choices of block centroids. In particular, a mere use of geometric centroids in these large blocks can be quite misleading as the spatial distribution of population is often quite skewed.

To avoid this problem, Seim (2006) uses population weighted centroids rather than (unweighted) geometric centroids. An alternative approach to avoid this problem is to draw a square grid on the entire city and use each square as a possible location, as in Datta and Sudhir (2013) and Nishida (2015). The value of consumer demographics in a square block is equal to the weighted average of the demographics at the census tracts that overlap with the square. The advantage of this approach is that each submarket has a uniform shape. In practice, implementation of this approach requires the removal of certain squares where entry cost is prohibitive. These areas include those with some particular natural features (for example, lakes, mountains, and wetlands) or where commercial space is prohibited by zoning. For example, Nishida (2015) excludes areas with zero population, and Datta and Sudhir (2013) remove areas that do not have any 'big box' stores, as these areas are very likely to be zoned for either residential use or small stores.

So far, all the papers that have estimated this type of model have considered a sample of cities (but not locations within a city) that is still in the spirit of the Bresnahan–Reiss isolated small markets approach. For instance, Seim selects US cities with population between 40 000 and 150 000, and without other cities with more than 25 000 people within 20 miles. The main reason for this is to avoid the possibility of outside competition at the boundaries of a city.

It is interesting that in the current generation of these applications, statistical inference is based on the number of cities and not on the number of locations. A relevant question is whether this model can be estimated consistently using data from a single city with many locations, that is, the estimator is consistent when the number of locations goes to infinity. This type of application can be motivated by the fact that city characteristics that are relevant for these models, such as the appropriate measure of geographic distance, transportation costs, or land use regulations and zoning, can be city specific. Xu (2018) studies an empirical game of market entry for a single city (network) and presents conditions for consistency and asymptotic normality of estimators as the number of locations increases. As far as we know, there are not yet empirical applications following that approach.

5.3.3 Store level data

Most applications of models of entry in retail markets do not use data on prices and quantities due to the lack of such data. The most popular alternative is to estimate the structural (or semi-structural) parameters of the model using market entry data only, for example,Bresnahan and Reiss (1990), Mazzeo (2002), Seim (2006), or Jia (2008), among many others. Typically, these studies either do not try to separately identify variable profits from fixed costs, or they do it by assuming that the variable profit is proportional to an observable measure of market size. Data on prices and quantities at the store level can substantially help the identification of these models. In particular, it is possible to consider a richer specification of the model that distinguishes between demand, variable cost, and fixed cost parameters, and includes unobservable variables into each of these components of the model.

A sequential estimation approach is quite convenient for the estimation of this type of model. In a first step, data on prices and quantities at the store level can be used to estimate a spatial demand system as in Davis (2006) for movie theatres or Houde (2012) for gas stations. Note that, in contrast to standard applications of demand estimation of differentiated products, the estimation of demand models of this class should deal with the endogeneity of store locations. In other words, in these demand models, not only are prices endogenous, but also the set of products or stores available at each location, as they are potentially correlated with unobserved errors in the demand system. In a second step, variable costs can be estimated using firms' best response functions in a Bertrand or Cournot model. Finally, in a third step, we estimate fixed cost parameters using the entry game and information of firms' entry and store location decisions. It is important to emphasize that the estimation of a demand system of spatial differentiation in the first step provides the structure of spatial competition effects between stores at different locations, such that the researcher does not need to consider other types of semi-reduced form specifications of strategic interactions, as in Seim (2006) among others.

In some applications, price and quantity are not available, but there is information on revenue at the store level. This information can be used to estimate a (semi reduced form) variable profit function in a first step, and then in a second step the structure of fixed costs is estimated. This is the case in the applications in Ellickson and Misra 2012, Suzuki 2013), and Aguirregabiria, Clark, and Wang 2016.

5.3.4 Potential entrants

An important modelling decision in empirical entry games is to define the set of potential entrants. In most cases, researchers have limited information on the number of potential entrants, let alone their identity. This problem is particularly severe when entrants are mostly independent small stores (for example, mom-and-pop stores). A practical approach is to estimate the model under different numbers of potential entrants and examine how estimates are sensitive to these choices, for example, Seim (2006) and Jia (2008). The problem is less severe when most entrants belong to national chains (for example, big box stores) because the names of these chains are often obvious and the number is typically small.

It is important to distinguish three types of data sets. The specification and the identification of the model is different for each of these three types of data.

(1) Only global potential entrants. The same N firms are the potential entrants in every market. We know the identity of these "global" potential entrants. Therefore, we observe the decision of each of these firms in every independent market. We observe market characteristics, and sometimes firm characteristics which may vary or not across markets. The data set is $\{s_m, x_{im}, a_{im} : m = 1,2,...,M; i = 1,2,...,N\}$ where m is the market index; i is the firm index; s_m is a vector of characteristics of market m such as market size, average consumer income, or other demographic variables; x_{im} is a vector of characteristics of firm i; and a_{im} is the indicator of the event "firm i is active in market m".

Examples. Berry (1992) considers entry in airline markets. A market is a city pair (for instance, Boston-Chicago). The set of markets consists of all the pairs of US cities with airports. Every airline company operating in the US is a potential entrant in each of these markets. a_{im} is the indicator of the event "airline i operates in city pair m". Toivanen and Waterson (2005) consider entry in local markets by fast-food restaurants in UK. Potential entrants are Burger King, McDonalds, KFC, Wendys, etc.

(2) Only local potential entrants. We do not know the identity of the potential entrants. In fact, most potential entrants may be local, that is, they consider entry in only one local market. For this type of data we only observe market characteristics and the number of active firms in the market. The data set is: $\{s_m, n_m : m = 1,2,...,M\}$ where n_m is the number of firms operating in market m. Notice also that we do not know the number of potential entrants N, and this may vary over markets.

Examples. Bresnahan and Reiss (1990). Car dealers in small towns. Bresnahan and Reiss (1991). Restaurants, dentists and other retailers and professional services in small towns. Seim (2006). Video rental stores.

(3) Both global and local potential entrants. This case combines and encompasses the previous two cases. There are N_G firms which are potential entrants in all the markets, and we now the identity of these firms. But there are also other potential entrants that are just local. We observe $\{s_m, n_m, z_{im}, a_{im} : m = 1,2,...,M; i = 1,2,...,N_G\}$.With this data we can nonparametrically identify $\Pr(n_m, a_m | x_m)$. We can allow for heterogeneity between global players in a very general way. Heterogeneity between local players should be much more restrictive.

5.4 Models

Road map.

(a) Bresnahan and Reiss. We start with a simple and pioneer model in this literature: the models in Bresnahan and Reiss (1991). This paper together with Bresnahan and Reiss (1990) were significant contributions to the structural estimation of models of market entry that opened a new literature that has grown significantly during the last 20 years. In their paper, Bresnahan and Reiss show that given a cross-section of "isolated" local markets where we observe the number of firms active, and some exogenous market characteristics, including market size, it is possible to identify fixed costs and the "degree of competition" or the "nature of competition" in the industry. By "nature of competition", these authors (and after them, this literature) mean a measure of how a firm's variable profit declines with the number of competitors. What is most remarkable about Bresnahan and Reiss's result is how with quite limited information (for instance, no information about prices of quantities) the researcher can identify the degree of competition using an entry model.

(b) Relaxing the assumption of homogeneous firms. Bresnahan and Reiss's model is based on some important assumptions. In particular, firms are homogeneous and they have complete information. The assumption of firm homogeneity (both in demand and costs) is strong and can be clearly rejected in many industries. Perhaps more importantly, ignoring firm heterogeneity when it is in fact present can lead to biased and misleading results about the degree of competition in a industry. Therefore, the first assumption that we relax is the one of homogeneous firms.

As shown originally in the own work of Bresnahan and Reiss (1991), relaxing the assumption of firm homogeneity implies two significant econometric challenges. First, the entry model becomes a system of simultaneous equations with endogenous binary choice variables. Dealing with endogeneity in a binary choice system of equations is not a simple econometric problem. In general, IV estimators are not available. Furthermore, the model now has multiple equilibria. Dealing with both endogeneity and multiple equilibria in this class of nonlinear models is an interesting but challenging problem in econometrics.

(c) Dealing with endogeneity and multiple equilibria in games of complete information. We will go through different approaches that have been used in this literature to deal with the problems of endogeneity and multiple equilibria. It is worthwhile to distinguish two groups of approaches or methods.

The first group of methods is characterized by imposing restrictions that imply equilibrium uniqueness for any value of the exogenous variables. Of course, firm homogeneity is a type of assumption that implies equilibrium uniqueness. But there are other assumptions that imply uniqueness even when firms are heterogeneous. For instance, a triangular structure in the strategic interactions between firms (Heckman, 1978), or sequential entry decisions (Berry, 1992) imply equilibrium uniqueness. Given these assumptions, these papers deal with the endogeneity problem by using a maximum likelihood approach.

The second group of methods do not impose equilibrium uniqueness. The pioneering work by Jovanovic (1989) and Tamer (2003) were important contributions to this approach. These authors showed (Jovanovic for a general but stylized econometric model, and Tamer for a two-player binary choice game) that identification and multiple equilibria are very different issues in econometric models.

Models with multiple equilibria can be (point or set) identified, and we do not need to impose equilibrium uniqueness as a form to get identification. Multiple equilibria can be a computational nuisance in the estimation of these models, but it is not an identification problem. This simple idea has generated a significant and growing literature that deals with computationally simple methods to estimate models with multiple equilibria, and more specifically with the estimation of discrete games.

(d) Games of incomplete information. Our next step will be to relax the assumption of complete information by introducing some variables that are private information to each firm. We will see that the identification and estimation of these models can be significantly simpler than in the case of models of complete information.

5.4.1 Single- and Multi-store firms

Single- and Multi-store firms

We start with the description of a static entry game between single-store firms. Later, we extend this framework to incorporate dynamics and multi-store firms. There are N retail firms that are potential entrants in a market. We index firms by $i \in \{1,2,\ldots,N\}$. From a geographic point of view, the market is a compact set $\mathbb{C}$ in the Euclidean space $\mathbb{R}^2$, and it contains L locations where firms can operate stores. These locations are exogenously given and they are indexed by $\ell \in \{1,2,\ldots,L\}$.

Firms play a two-stage game. In the first stage, firms make their entry and store location decisions. Each firm decides whether to be active or not in the market, and if active, chooses the location of its store. We can represent a firm's decision using an L-dimensional vector of binary variables, $a_i \equiv \{a_{i\ell} : \ell = 1,2,\ldots,L\}$, where $a_{i\ell} \in \{0,1\}$ is the indicator of the event 'firm i has a store in location ℓ'. For single-store firms, there is at most one component in the vector a_i that is equal to one while the rest of the binary variables must be zero. In the second stage they compete in prices (or quantities) taking entry decisions as given. The equilibrium in the second stage determines equilibrium prices and quantities at each active store.

The market is populated by consumers. Each consumer is characterized by her preference for the products that firms sell and by her geographical location or home address h that belongs to the set of consumer home addresses $\{1,2,\ldots,H\}$. The set of consumer home addresses and the set of feasible business locations may be different. Following Smith (2004), Davis (2006), or Houde (2012), aggregate consumer demand comes from a discrete choice model of differentiated products where both product characteristics and transportation costs affect demand. For instance, in a spatial logit model, the demand for firm i with a store in location ℓ is:

$$q_{i\ell} = \sum_{h=1}^{H} M(h) \frac{a_{i\ell}\ \exp\{x_i\,\beta - \alpha\ p_{i\ell} - \tau(d_{h\ell})\}}{1 + \sum_{j=1}^{N} \sum_{\ell'=1}^{L} a_{j\ell'}\ \exp\{x_j\,\beta - \alpha\ p_{j\ell'} - \tau\ d_{h\ell'}\}} \tag{5.3}$$

where $q_{i\ell}$ and $p_{i\ell}$ are the quantity sold and the price, respectively, at store (i,ℓ); $M(h)$ represents the mass of consumers living in address h; the term within the square brackets is the market share of store (i,ℓ) among consumers living in address h; x_i is a vector of observable characteristics (other than price) of the product of firm i; and β is the vector of marginal utilities of these characteristics; α is the marginal utility of income; $d_{h\ell}$ represents the geographic distance between home address h and business location ℓ; and $\tau(d_{h\ell})$ is an increasing real-valued function that represents consumer transportation costs.

Given this demand system, active stores compete in prices à la Nash–Bertrand to maximize their respective variable profits, $(p_{i\ell} - c_{i\ell})\ q_{i\ell}$, where $c_{i\ell}$ is the marginal cost of store (i,ℓ), that is exogenously given. The solution of the system of best response functions can be described as a vector of equilibrium prices for each active firm/store.

Let $p_i^*(\ell, a_{-i}, x)$ and $q_i^*(\ell, a_{-i}, x)$ represent the equilibrium price and quantity for firm i given that this firm has a store at location ℓ. The rest of the firms' entry/location decisions are represented by the vector $a_{-i} \equiv \{a_j : j \neq i\}$, and the firms' characteristics are denoted by $x \equiv (x_1, x_2, \ldots, x_N)$. Similarly, we can define the equilibrium (indirect) variable profit,

$$VP_i^*(\ell, a_{-i}, x) = [p(\ell, a_{-i}, x) - c_{i\ell}]\ q_i^*(\ell, a_{-i}, x) \tag{5.4}$$

Consider now the entry stage of the game. The profit of firm i if it has a store in location ℓ is:

$$\pi_i(\ell, a_{-i}, x) = VP_i^*(\ell, a_{-i}, x) - EC_{i\ell} \tag{5.5}$$

where $EC_{i\ell}$ represents the entry cost of firm i at location ℓ, that for the moment is exogenously given. The profit of a firm that is not active in the market is normalized to zero, that is, $\pi_i(0, a_{-i}, x) = 0$, where with some abuse of notation, we use $\ell = 0$ to represent the choice alternative of no entry in any of the L locations.

The description of an equilibrium in this model depends on whether firms have complete or incomplete information about other firms' costs. The empirical literature on entry games has considered both cases.

Complete information game. In the complete information model, a Nash equilibrium is an N-tuple $\{a_i^* : i = 1, 2, \ldots, N\}$ such that for every firm i the following best response condition is satisfied:

$$a_{i\ell}^* = 1\{\pi_i(\ell, a_{-i}^*, x) \geq \pi_i(\ell', a_{-i}^*, x) \text{ for any } \ell' \neq \ell\} \tag{5.6}$$

where $1\{.\}$ is the indicator function. In equilibrium, each firm is maximizing its own profit given the entry and location decisions of the other firms.

Incomplete information game. In a game of incomplete information, there is a component of a firm's profit that is private information to the firm. For instance, suppose that the entry cost of firm i is $EC_{i\ell} = ec_{i\ell} + \varepsilon_{i\ell}$, where $ec_{i\ell}$ is public information for all the firms, and $\varepsilon_{i\ell}$ is private information to firm i. These private cost shocks can be correlated across locations for a given firm, but they are independently distributed across firms, that is, $\varepsilon_i \equiv \{\varepsilon_{i\ell} : \ell = 1, 2, \ldots, L\}$ is independently distributed across firms with a distribution function F_i that is continuously differentiable over $\mathbb{R}^L$ and common knowledge to all the firms.

A firm's strategy is an L-dimensional mapping $\alpha_i(\varepsilon_i; x) \equiv \{\alpha_{i\ell}(\varepsilon_i; x) : \ell = 1, 2, \ldots, L\}$, where $\alpha_{i\ell}(\varepsilon_i; x)$ is a binary-valued function from the set of possible private information values $\mathbb{R}^L$ and the support of x into $\{0, 1\}$, such that $\alpha_{i\ell}(\varepsilon_i; x) = 1$ means that firm i enters location ℓ when the value of private information is ε_i. A firm has uncertainty about the actual entry decisions of other firms because it does not know the realization of other firms' private information. Therefore, firms maximize expected profits. Let $\pi_i^e(\ell, \alpha_{-i}, x)$ be the expected profit of firm i if it has a store at location ℓ and the other firms follow their respective strategies in α_{-i}^*. By definition, $\pi_i^e(\ell, \alpha_{-i}, x) \equiv \mathbb{E}_{\varepsilon_{-i}}[\pi_i(\ell, \alpha_{-i}(\varepsilon_{-i}; x), x)]$, where $\mathbb{E}_{\varepsilon_{-i}}$ represents the expectation over the distribution of the private information of firms other than i. A Bayesian Nash equilibrium in this game of incomplete information is an N-tuple of strategy functions $\{\alpha_{-i}^* : i = 1, 2, \ldots, N\}$ such that every firm maximizes its expected profit: for any ε_i,

$$\alpha_{i\ell}^*(\varepsilon_i; x) = 1\{\pi_i^e(\ell, \alpha_{-i}^*, x) \geq \pi_i^e(\ell', \alpha_{-i}^*, x) \text{ for any } \ell' \neq \ell\} \tag{5.7}$$

In an entry game of incomplete information, firms' strategies (and therefore, a Bayesian Nash equilibrium) can also be described using firms' probabilities of market entry, instead of the strategy functions $\alpha_i(\varepsilon_i; x)$. In sections 2.2.1 and 2.2.4, we present examples of this representation in the context of more specific models.

Multi-store firms

Multi-store firms, or retail chains, have become prominent in many retail industries such as supermarkets, department stores, apparel, electronics, fast food restaurants, or coffee shops, among others. Cannibalization and economies of scope between stores of the same chain are two important factors in the entry and location decisions of a multi-store firm. The term cannibalization refers to the business stealing effects between stores of the same chain. Economies of scope may appear if some operating costs are shared between stores of the same retail chain such that these costs are not duplicated when the number of stores in the chain increases. For instance, some advertising, inventory, personnel, or distribution costs can be shared among the stores of the same firm. These economies of scope may become quantitatively more important when store locations are geographically closer to each other. This type of economies of scope is called economies of density.

The recent empirical literature on retail chains has emphasized the importance of these economies of density, that is, Holmes (2011), Jia (2008), Ellickson, Houghton, and Timmins (2013), and Nishida (2015). For instance, the transportation cost associated with the distribution of products from wholesalers to retail stores can be smaller if stores are close to each other. Also, geographic proximity can facilitate sharing inventories and even personnel across stores of the same chain. We now present an extension of the basic framework that accounts for multi-store firms.

A multi-store firm decides its number of stores and their locations. We can represent a firm's entry decision using the L-dimensional vector $a_i \equiv \{a_{i\ell} : \ell = 1, 2, \ldots, L\}$, where $a_{i\ell} \in \{0, 1\}$ is still the indicator of the event 'firm i has a store in location ℓ'. In contrast to the case with single-store firms, now the vector a_i can take any value within the choice set $\{0, 1\}^L$. The demand system still can be described using equation (5.4.1). The variable profit of a firm is the sum of variable profits over every location where the firm has stores, $\sum_{\ell=1}^{L} a_{i\ell}\,(p_{i\ell} - c_{i\ell}) q_{i\ell}$.

Firms compete in prices taking their store locations as given. A retail chain may choose to have a uniform price across all its stores, or to charge a different price at each store. In the Bertrand pricing game with spatial price discrimination (that is, different prices at each store), the best response of firm i can be characterized by the first-order conditions:

$$q_{i\ell} + (p_{i\ell} - c_{i\ell})\,\frac{\partial q_{i\ell}}{\partial p_{i\ell}} + \sum_{\ell' \neq \ell} (p_{i\ell'} - c_{i\ell'})\,\frac{\partial q_{i\ell'}}{\partial p_{i\ell}} = 0 \tag{5.8}$$

The first two terms represent the standard marginal profit of a single-store firm. The last term represents the effect on the variable profits of all other stores within the firm, and it captures how the pricing decision of the firm internalizes the cannibalization effect among its own stores.

A Nash-Bertrand equilibrium is a solution in prices to the system of best response equations in (5.4.1). The equilibrium (indirect) variable profit of firm i is:

$$VP_i^*(a_i, a_{-i}; x) = \sum_{\ell=1}^{L} (p_i^*(\ell, a_{-i}; x) - c_{i\ell})\, q_i^*(\ell, a_{-i}; x) \tag{5.9}$$

where $p_{i\ell}^*(\ell, a_{-i}; x)$ and $q_i^*(\ell, a_{-i}; x)$ represent Bertrand equilibrium prices and quantities, respectively.

The total profit of the retail chain is equal to total variable profit minus total entry cost: $\pi_i(a_i, a_{-i}; x) = VP_i^*(a_i, a_{-i}; x) - EC_i(a_i)$. The entry costs of a retail chain may depend on the number of stores (that is, (dis)economies of scale) and on the distance between the stores (for example, economies of density). In section 2.2.5, we provide examples of specifications of entry costs for multi-store retailers.

The description of an equilibrium in this game of entry between retail chains is similar to the game between single-store firms. With complete information, a Nash equilibrium is an N-tuple $\{a_i^* : i = 1, 2, \ldots, N\}$ that satisfies the following best response conditions:

$$\pi_i(a_i^*, a_{-i}^*; x) \geq \pi_i(a_i, a_{-i}^*; x) \ \text{ for any } a_i \neq a_i^* \tag{5.10}$$

With incomplete information, a Bayesian Nash equilibrium is an N-tuple of strategy functions $\{\alpha_i^*(\varepsilon_i; x) : i = 1, 2, \ldots, N\}$ such that every firm maximizes its expected profit: for any ε_i:

$$\pi_i^e(\alpha_i^*(\varepsilon_i; x), \alpha_{-i}^*, x) \geq \pi_i^e(a_i, \alpha_{-i}^*, x) \quad \text{for any } a_i \neq \alpha_i^*(\varepsilon_i; x) \tag{5.11}$$

Specification assumptions

The games of entry in retail markets that have been estimated in empirical applications have imposed different types of restrictions on the framework that we have presented above. For example, restrictions on firm and market heterogeneity, firms' information, spatial competition, multi-store firms, dynamics, or the form of the structural functions.

The motivations for these restrictions are diverse. Some restrictions are imposed to achieve identification or precise enough estimates of the parameters of interest, given the researcher's limited information on the characteristics of markets and firms. For instance, as we describe in section 5.3.3, prices and quantities at the store level are typically not observable to the researcher, and most sample information comes from firms' entry decisions. These limitations in the available data have motivated researchers to use simple specifications for the indirect variable profit function.

Other restrictions are imposed for computational convenience in the solution and estimation of the model, for example, to obtain closed form solutions, to guarantee equilibrium uniqueness as it facilitates the estimation of the model, or to reduce the dimensionality of the space of firms' actions or states. In this subsection, we review some important models in this literature and discuss their main identification assumptions. We have organized these models in an approximate chronological order.

5.4.2 Homogeneous firms

Work in this field was pioneered by Bresnahan and Reiss. In Bresnahan and Reiss (1991), they study several retail and professional industries in the US, specifically pharmacies, tire dealers, doctors, and dentists. The main purpose of the paper is to estimate the 'nature' or 'degree' of competition for each of the industries: how fast variable profits decline when the number of firms in the market increases. More specifically, the authors are interested in estimating how many entrants are needed to achieve an oligopoly equilibrium equivalent to the competitive equilibrium, that is, the hypothesis of contestable markets (Baumol 1982; Baumol, Panzar, and Willig 1982; Martin 2000).

For each industry, their dataset consists of a cross-section of M small 'isolated markets'. In section 5.3, we discussed the empirical motivation and implementation of the 'isolated markets' restriction. For the purpose of the model, a key aspect of this restriction is that the M local markets are independent in terms of demand and competition such that the equilibrium in one market is independent of the one in the other markets. The model also assumes that each market consists of a single location, that is, $L = 1$, such that spatial competition is not explicitly incorporated in the model. For each local market, the researcher observes the number of active firms (n), a measure of market size (s), and some exogenous market characteristics that may affect demand and/or costs (x).

Given this limited information, the researcher needs to restrict firm heterogeneity. Bresnahan and Reiss propose a static game between single-store firms where all the potential entrants in a market are identical and have complete information on demand and costs. The profit of a store is:

$$\pi(n) = s\,V(x,n) \;-\; EC(x) \;-\varepsilon, \tag{5.12}$$

where $V(x,n)$ represents variable profit per capita (per consumer) that depends on the number of active firms n, and $EC(x)+\varepsilon$ is the entry cost, where ε is unobservable to the researcher. The form of competition between active firms is not explicitly modelled. Instead, the authors consider a flexible specification of the variable profit per-capita that is strictly decreasing but nonparametric in the number of active stores. Therefore, the specification is consistent with a general model of competition between homogeneous firms, or even between symmetrically differentiated firms.

Given these assumptions, the equilibrium in a local market can be described as a number of firms n^* that satisfies two conditions: (1) every active firm is maximizing profits by being active in the market, that is, $\pi(n^*) \geq 0$; and (2) every inactive firm is maximizing profits by being out of the market, that is, $\pi(n^*+1) < 0$. In other words, every firm is making its best response given the actions of the others. Since the profit function is strictly decreasing in the number of active firms, the equilibrium is unique and it can be represented using the following expression: for any value $n \in \{0,1,2,\ldots\}$,

$$\begin{aligned} \{n^* = n\} \quad &\Leftrightarrow \quad \{\pi(n) \geq 0 \;\text{ and } \pi(n+1) < 0\} \\ &\Leftrightarrow \quad \{s\,V(x,n+1) - EC(x) < \varepsilon \leq \; s\,V(x,n) - EC(x)\} \end{aligned} \tag{5.13}$$

Also, this condition implies that the distribution of the equilibrium number of firms given exogenous market characteristics is:

$$\Pr(n^* = n \mid s,x) = F(s\,V(x,n) - EC(x)) - F(s\,V(x,n+1) - EC(x)) \tag{5.14}$$

where $F$is the CDF of ε. This representation of the equilibrium as an ordered discrete choice model is convenient for estimation.

In the absence of price and quantity data, the separate identification of the variable profit function and the entry cost function is based on the exclusion restrictions that variable profit depends on market size and on the number of active firms while the entry cost does not depend on these variables.

Private information. The previous model can be slightly modified to allow for firms' private information. This variant of the original model maintains the property of equilibrium uniqueness and most of the simplicity of the previous model. Suppose that now the entry cost of a firm is $EC(x) + \varepsilon_i$, where ε_i is private information of firm i and it is independently and identically distributed across firms with a CDF F. There are N potential entrants in the local market. The presence of private information implies that, when potential entrants make entry decisions, they do not know ex ante the actual number of firms that will be active in the market. Instead, each firm has beliefs about the probability distribution of the number of other firms that are active. We represent these beliefs, for say firm i, using the function $G_i(n) \equiv \Pr(n^*_{-i} = n|s,x)$, where n^*_{-i} represents the number of firms other than i that are active in the market. Then, the expected profit of a firm if active in the market is:

$$\pi_i^e = \left[\sum_{n=0}^{N-1} G_i(n)\ s\ V(x,n+1)\right] - EC(x) - \varepsilon_i \tag{5.15}$$

The best response of a firm is to be active in the market if and only if its expected profit is positive or zero, that is, $a_i = 1\{\pi_i^e \geq 0\}$. Integrating this best response function over the distribution of the private information ε_i we obtain the best response probability of being active for firm i, that is:

$$P_i \equiv F\left(\left[\sum_{n=0}^{N-1} G_i(n)\ s\ V(x,n+1)\right] - EC(x)\right) \tag{5.16}$$

Since all firms are identical, up to their independent private information, it seems reasonable to impose the restriction that in equilibrium they all have the same beliefs and, therefore, the same best response probability of entry. Therefore, in equilibrium, firms' entry decisions $\{a_i\}$ are independent Bernoulli random variables with probability P, and the number of active firms other than i in the market has a Binomial distribution with argument $(N-1,P)$ such that $\Pr(n^*_{-i} = n) = B(n|N-1,P)$.

In equilibrium, the belief function $G(n)$ should be consistent with firms' best response probability P. Therefore, a Bayesian Nash Equilibrium in this model can be described as a probability of market entry P^*, which is the best response probability when firms' beliefs about the distribution of other firms active in the market are $G(n) = B(n|N-1,P^*)$. We can represent this equilibrium condition using the following equation:

$$P^* = F\left(\left[\sum_{n=0}^{N-1} B(n|N-1,P^*)\ s\ V(x,n+1)\right] - EC(x)\right) \tag{5.17}$$

When the variable profit $V(x,n)$ is a decreasing function in the number of active stores, the right-hand side in equation (5.17) is also a decreasing function in the probability of entry P, and this implies equilibrium uniqueness. In contrast to the complete information model in Bresnahan and Reiss (1991), this incomplete information model does not have a closed form solution for the equilibrium distribution of the number of active firms in the market. However, the numerical solution to the fixed point problem in equation (5.17) is computationally very simple, and so are the estimation and comparative statistics using this model.

Given that the only difference between the two models described in this section is in their assumptions about firms' information, it seems reasonable to consider whether these models are observationally different or not. In other words, does the assumption on complete versus incomplete information have implications on the model predictions about competition? Grieco (2014) investigates this question in the context of an empirical application to local grocery markets. In Grieco's model, firms are heterogeneous in terms of (common knowledge) observable variables, and this observable heterogeneity plays a key role in his approach to empirically distinguish between firms' public and private information. Note that the comparison of equilibrium conditions in equations (5.14) and (5.17) shows other testable difference between the two models. In the game of incomplete information, the number of potential entrants N has an effect on the whole probability distribution of the number of active firms: a larger number of potential entrants implies a shift to the right in the whole distribution of the number of active firms. In contrast, in the game of complete information, the value of N affects only the probability $Pr(n^* = N|s,x)$ but not the distribution of the number of active firms at values smaller than N. This empirical prediction has relevant economic implications: with incomplete information, the number of potential entrants has a positive effect on competition even in markets where this number is not binding.

Bresnahan and Reiss (1991)

The authors study several retail and professional industries in the US: Doctors; Dentists; Pharmacies; Plumbers; car dealers; etc. For each industry, the dataset consists of a cross-section of M small, "isolated" markets. We index markets by m. For each market m, we observe the number of active firms (n_m), a measure of market size (s_m), and some exogenous market characteristics that may affect demand and/or costs (x_m).

$$\text{Data} = \{\, n_m,\; s_m, x_m : m = 1, 2, ..., M\} \tag{5.18}$$

There are several empirical questions that they wish to answer. First, they want to estimate the "nature" or "degree" of competition for each of the industries: that is, how fast variable profits decline when the number of firms in the market increase. Second, but related to the estimation of the degree of competition, BR are also interested in estimating how many entrants are needed to achieve an equilibrium equivalent to the competitive equilibrium, that is, hypothesis of contestable markets.

Model. Consider a market m. There is a number N of potential entrants in the market. Each firm decides whether to be active or not in the market. Let $\Pi_m(n)$ be the profit of an active firm in market m when there are n active firms. The function $\Pi_m(n)$ is strictly decreasing in n. If n_m is the equilibrium number of firms in market m, then it should satisfy the following conditions:

$$\Pi_m(n_m) \geq 0 \qquad \text{and} \qquad \Pi_m(n_m + 1) < 0 \tag{5.19}$$

That is, every firm is making her best response given the actions of the others. For active firms, their best response is to be active, and for inactive firms their best response is to not enter in the market.

To complete the model we have to specify the structure of the profit function $\Pi_m(n)$. Total profit is equal to variable profit, $V_m(n)$, minus fixed costs, $F_m(n)$:

$$\Pi_m(n) = V_m(n) - F_m(n) \tag{5.20}$$

In this model, where we do not observe prices or quantities, the key difference in the specification of variable profit and fixed cost is that variables profits increase with market size (in fact, they are proportional to market size) and fixed costs do not.

The variable profit of a firm in market m when there are n active firms is:

$$V_m(n) = s_m \; v_m(n) = s_m \; \left(x_m^D \beta - \alpha(n)\right) \tag{5.21}$$

where s_m represents market size; $v_m(n)$ is the variable profit per-capita; x_m^D is a vector of market characteristics that may affect the demand of the product, for instance, per capita income, age distribution; β is a vector of parameters; and $\alpha(1)$, $\alpha(2)$, ...$\alpha(N)$ are parameters that capture the degree of competition, such that we expect that $\alpha(1) \leq \alpha(2) \leq \alpha(3)$ ... $\leq \alpha(N)$. Given that there is no firm-heterogeneity in the variable profit function, there is an implicit assumption of homogeneous product or symmetrically differentiated product as in, for instance, Salop circle city (Salop, 1979).

The specification for the fixed cost is:

$$F_m(n) = x_m^C \; \gamma + \delta(n) + \varepsilon_m \tag{5.22}$$

where x_m^C is a vector of observable market characteristics that may affect the fixed cost, for instance, rental price; ε_m is a market characteristic that is unobservable to the researchers but observable to the firms; $\delta(1)$, $\delta(2)$, ...$\delta(N)$ are parameters. The dependence of the fixed cost with respect to the number of firms is very unconventional or non-standard in IO. Bresnahan and Reiss allow for this possibility and provide several interpretations. However, the interpretation of the parameters $\delta(n)$ is not completely clear. In some sense, BR allow the fixed cost to depend on the number firms in the market for robustness reasons. There are several possible interpretations for why fixed costs may depend on the number of firms in the market: (a) entry deterrence: incumbents create barriers to entry; (b) a shortcut to allow for firm heterogeneity in fixed costs, in the sense that late entrants are less efficient in fixed costs; and (c) actual endogenous fixed costs, for instance rental prices or other components of the fixed costs, not included in x_m^C, may increase with the number of incumbents (for instance, demand effect on rental prices). For any of these interpretations we expect $\delta(n)$ to be an increasing function of n.

Since both $\alpha(n)$ and $\delta(n)$ increase with n, it is clear that the profit function $\Pi_m(n)$ declines with n. Therefore, as we anticipated above, the equilibrium condition for the number of firms in the market can be represented as follows. For $n \in \{0, 1, ..., N\}$

$$\{n_m = n\} \Leftrightarrow \{ \; \Pi_m(n) \geq 0 \quad \text{AND} \quad \Pi_m(n+1) < 0 \; \} \tag{5.23}$$

It is simple to show that the model has a unique equilibrium for any value of the exogenous variables and structural parameters. This is just a direct implication of the strict monotonicity of the profit function $\Pi_m(n)$.

We have a random sample $\{n_m,\ s_m,\ x_m^D,\ x_m^C : m = 1,2,...,M\}$ and we want to use this sample to estimate the vector of parameters:

$$\theta = \{\beta,\ \gamma,\ \sigma,\ \alpha(1),...,\alpha(N),\ \delta(1),...,\delta(N)\} \tag{5.24}$$

The unobserved component of the entry cost, ε_m, is assumed independent of (s_m, x_m^D, x_m^C) and it is i.i.d. over markets with distribution $N(0,\sigma)$. As usual in discrete choice models, σ is not identified. We normalize $\sigma = 1$, which means that we are really identifying the rest of the parameters up to scale. We should keep this in mind for the interpretation of the estimation results.

Given this model and sample, BR estimate θ by maximum likelihood:

$$\hat{\theta} = \arg\max_{\theta} \sum_{m=1}^{M} \log \Pr(n_m \mid \theta, s_m, x_m^D, x_m^C) \tag{5.25}$$

What is the form of the probabilities $\Pr(n_m|\theta, s_m, x_m)$ in the BR model? This entry model has the structure of an *ordered Probit model*. We can represent the equilibrium condition $\{\Pi_m(n) \geq 0 \ \text{ AND } \ \Pi_m(n+1) < 0\}$ in terms of thresholds for the unobservable variable ε_m:

$$\{n_m = n\} \Leftrightarrow \{T_m(n+1) < \varepsilon_m \leq T_m(n)\}, \tag{5.26}$$

where, for any $n \in \{1,2,...,N\}$,

$$T_m(N) \equiv s_m x_m^D \beta - x_m^C \gamma - \alpha(n) s_m - \delta(n) \tag{5.27}$$

and $T_m(0) = +\infty$, $T_m(N^* + 1) = -\infty$. This is the structure of an ordered probit model. Therefore, the distribution of the number of firms conditional on the observed exogenous market characteristics is:

$$\begin{aligned}
\Pr(n_m = n | s_m, x_m) &= \Phi(T_m(n)) - \Phi(T_m(n+1)) \\
&= \Phi\left(\ s_m x_m^D \beta - x_m^C \gamma - \alpha(n) s_m - \delta(n)\ \right) \\
&- \Phi\left(\ s_m x_m^D \beta - x_m^C \gamma - \alpha(n+1) s_m - \delta(n+1)\ \right)
\end{aligned} \tag{5.28}$$

This model is simple to estimate and most econometric software packages include a command for the estimation of the ordered probit.

Data. The dataset consists of a cross-section of 202 "isolated" local markets. Why isolated local markets? It is very important to include in our definition of market all the firms that are actually competing in the market and not more or less. Otherwise, we can introduce significant biases in the estimated parameters. If our definition of market is too narrow, such that we do not include all the firms that are actually in a market, we will conclude that there is little entry either because fixed costs are too large or the degree of competition is strong: that is, we will overestimate the $\alpha's$ or the $\delta's$ or both. If our definition of market is too broad, such that we include firms that are not actually competing in the same market, we will conclude that there is significant entry and to rationalize this we will need fixed costs to be small or to have a low degree of competition between firms. Therefore, we will underestimate the $\alpha's$ or the $\delta's$ or both.

Under a broad definition of a market, the most common mistake is having a large city as a single market. Conversely, under a narrow definition of a market, the most common mistake is having small towns that are close to each other, or close to a large town, as single markets. To avoid these type of errors, BR construct "isolated local markets". The criteria to select isolated markets in the US are: (a) at least 20 miles from the nearest town of 1000 people or more; (b) at least 100 miles from cities with 100,000 people or more.

Empirical results. Let $S(n)$ be the minimum market size to sustain n firms in the market. $S(n)$ are called *market dize entry thresholds* and they can be obtained using the estimated parameters of the model. They do not depend on the normalization $\sigma = 1$. Brenahan and Reiss find that, for most industries, both $\alpha(n)$ and $\delta(n)$ increase with n. There are very significant cross-industry differences in entry thresholds $S(n)$. For most of the industries, entry thresholds $S(n)/N$ become constant for values of n greater than 4 or 5. This result supports the hypothesis of contestable markets (Baumol, 1982).

5.4.3 Endogenous product choice

Mazzeo (2002) studies market entry in the motel industry using local markets along US interstate highways. A local market is defined as a narrow region around a highway exit. Mazzeo's model maintains most of the assumptions in Bresnahan and Reiss (1991), such as no spatial competition (that is, L=1), ex ante homogeneous firms, complete information, no multi-store firms, and no dynamics. However, he extends the Bresnahan–Reiss model in an interesting dimension: he introduces endogenous product differentiation.

More specifically, firms not only decide whether to enter in a market but they also choose the type of product to offer: low-quality product E (that is, economy hotel), or high-quality product H (that is, upscale hotel). Product differentiation makes competition less intense, and it can increase firms' profits. However, firms also have an incentive to offer the type of product for which demand is stronger.

The profit of an active hotel of type $T \in \{E, H\}$ is:

$$\pi_T(n_E, n_H) = s\, V_T(x, n_E, n_H) - EC_T(x) - \varepsilon_T \tag{5.29}$$

where n_E and n_H represent the number of active hotels with low and high quality, respectively, in the local market. Similarly to the Bresnahan–Reiss model, V_T is the variable profit per capita and $EC_T(x) + \varepsilon_T$ is the entry cost for type T hotels, where ε_T is unobservable to the researcher.

Mazzeo solves and estimates his model under two different equilibrium concepts: Stackelberg and what he terms a 'two-stage game'. A computational advantage of the two-stage game is that under the assumptions of the model the equilibrium is unique. In the first stage, the total number of active hotels, $n \equiv n_E + n_H$, is determined in a similar way as in the Bresnahan–Reiss model. Hotels enter the market as long as there is some configuration (n_E, n_H) where both low-quality and high-quality hotels make positive profits. Define the first-stage profit function as:

$$\Pi(n) \equiv \max_{n_E, n_H : n_E + n_H = n} \min[\pi_E(n_E, n_H)\ ,\ \pi_H(n_E, n_H)] \tag{5.30}$$

Then, the equilibrium number of hotels in the first stage is the value n^* that satisfies two conditions: (1) every active firm wants to be in the market, that is, $\Pi(n^*) \geq 0$; and

(2) every inactive firm prefers to be out of the market, that is, $\Pi(n^* + 1) < 0$. If the profit functions π_E and π_H are strictly decreasing functions in the number of active firms (n_E, n_H), then $\Pi(n)$ is also a strictly decreasing function, and the equilibrium number of stores in the first stage, n^*, is unique.

In the second stage, active hotels choose simultaneously their type or quality level. In this second stage, an equilibrium is a pair (n_E^*, n_H^*) such that every firm chooses the type that maximizes its profit given the choices of the other firms. That is, low quality firms are not better off by switching to high quality, and vice versa:

$$\begin{aligned} \pi_E(n_E^*, n_H^*) &\geq \pi_H(n_E^* - 1, n_H^* + 1) \\ \pi_H(n_E^*, n_E^*) &\geq \pi_E(n_E^* + 1, n_E^* - 1) \end{aligned} \tag{5.31}$$

Mazzeo shows that the equilibrium pair (n_E^*, n_H^*) in this second stage is also unique.

Using these equilibrium conditions, it is possible to obtain a closed form expression for the (quadrangle) region in the space of the unobservables $(\varepsilon_E, \varepsilon_H)$ that generate a particular value of the equilibrium pair (n_E^*, n_H^*). Let $R_\varepsilon(n_E, n_H; s, x)$ be the quadrangle region in $\mathbb{R}^2$ associated with the pair (n_E, n_H) given exogenous market characteristics (s, x) , and let $F(\varepsilon_E, \varepsilon_H)$ be the CDF of the unobservable variables. Then, we have that:

$$\Pr(n_E^* = n_E, n_H^* = n_H | s, x) = \int 1\{(\varepsilon_E, \varepsilon_H) \in R_\varepsilon(n_E, n_H; s, x)\; dF(\varepsilon_E, \varepsilon_H) \tag{5.32}$$

In the empirical application, Mazzeo finds that hotels have strong incentives to differentiate from their rivals to avoid nose-to-nose competition.

Ellickson and Misra (2008) estimate a game of incomplete information for the US supermarket industry where supermarkets choose the type of 'pricing strategy': 'everyday low price' (EDLP) versus 'high-low' pricing. The choice of pricing strategy can be seen as a form of horizontal product differentiation. The authors find evidence of strategic complementarity between supermarkets' pricing strategies: firms competing in the same market tend to adopt the same pricing strategy not only because they face the same type of consumers but also because there are positive synergies in the adoption of the same strategy.

From an empirical point of view, Ellickson and Misra's result is more controversial than Mazzeo's finding of firms' incentives to differentiate from each other. In particular, the existence of unobservables that are positively correlated across firms but are not fully accounted for in the econometric model, may generate a spurious estimate of positive spillovers in the adoption of the same strategy.

Vitorino (2012) estimates a game of store entry in shopping centers that allows for incomplete information, positive spillover effects among stores, and also unobserved market heterogeneity for the researcher that is common knowledge to firms. Her empirical results show that, after controlling for unobserved market heterogeneity, firms face business stealing effects but also significant incentives to collocate, and that the relative magnitude of these two effects varies substantially across store types.

5.4.4 Firm heterogeneity

The assumption that all potential entrants and incumbents are homogeneous in their variable profits and entry costs is very convenient and facilitates the estimation, but it is also very unrealistic in many applications. A potentially very important factor in the determination of market structure is that firms, potential entrants, are ex-ante heterogeneous. In many applications we want to take into account this heterogeneity. Allowing for firm heterogeneity introduces two important issues in these models: endogenous explanatory variables, and multiple equilibria. We will comment on different approaches that have been used to deal with these issues.

Consider an industry with N potential entrants. For instance, the airline industry. These potential entrants decide whether to be active or not in a market. We observe M different realizations of this entry game. These realizations can be different geographic markets (different routes or city pairs, for instance, Toronto-New York, Montreal-Washington, etc) or different periods of time. We index firms with $i \in \{1,2,...,N\}$ and markets with $m \in \{1,2,...,M\}$.

Let $a_{im} \in \{0,1\}$ be the binary indicator of the event "firm i is active in market m". For a given market m, the N firms choose simultaneously whether to be active or not in the market. When making its decision, a firm wants to maximize its profit.

Once firms have decided whether to be active or not in the market, active firms compete in prices or in quantities and firms' profits are realized. For the moment, we do not make explicit the specific form of competition in this second part of the game, or the structure of demand and variable costs. We take as given an "indirect profit function" that depends on exogenous market and firm characteristics and on the number and the identity of the active firms in the market. This indirect profit function comes from a model of price or quantity competition, but at this point we do not make that model explicit here. Also, we consider that the researcher does not have access to data on firms' prices and quantities such that demand and variable cost parameters in the profit function cannot be estimated from demand, and/or Bertrand/Cournot best response functions.

The (indirect) profit function of an incumbent firm depends on market and firm characteristics affecting demand and costs, and on the entry decisions of the other potential entrants:

$$\Pi_{im} = \begin{cases} \Pi_i\left(x_{im},\ \varepsilon_{im}, a_{-im}\right) & \text{if} \quad a_{im} = 1 \\ \\ 0 & \text{if} \quad a_{im} = 0 \end{cases} \tag{5.33}$$

where x_{im} and ε_{im} are vectors of exogenous market and firm characteristics, and a_{-im} $\equiv \{a_{jm} : j \neq i\}$. The vector x_{im} is observable to the researcher while ε_{im} is unobserved to the researcher. For the moment we assume that $x_m \equiv \{x_{1m}, x_{2m}, ..., x_{Nm}\}$ and $\varepsilon_m \equiv \{\varepsilon_{1m}, \varepsilon_{2m}, ..., \varepsilon_{Nm}\}$ are common knowledge for all players.

For instance, in the example of the airline industry, the vector x_{im} may include market characteristics such as population and socioeconomic characteristics in the two cities that affect demand, characteristics of the airports such as measures of congestion (that affect costs), and firm characteristics such as the number of other connections that the airline has in the two airports (that affect operating costs due to economies of scale and scope).

The N firms choose simultaneously $\{a_{1m}, a_{2m}, ..., a_{Nm}\}$ and the assumptions of Nash equilibrium hold. A Nash equilibrium in this entry game is an N-tuple $a^*_m = (a^*_{1m}, a^*_{2m}, ..., a^*_{Nm})$ such that for any player i:

$$a^*_{im} = 1\left\{\ \Pi_i\left(x_{im},\ \varepsilon_{im}, a^*_{-im}\right) \geq 0\ \right\} \tag{5.34}$$

where $1\{.\}$ is the indicator function.

Given a dataset with information on $\{a_{im}, x_{im}\}$ for every firm in the M markets, we want to use this model to learn about the structure of the profit function Π_i. In these applications, we are particularly interested in the effect of other firms' entry decisions on a firm's profit. For instance, how Southwest's entry in the Chicago-Boston market affectsthe profit of American Airlines.

For the sake of concreteness, consider the following specification of the profit function:

$$\Pi_{im} = x_{im}\ \beta_i - \sum\nolimits_{j \neq i} a_{jm}\ \delta_{ij} + \varepsilon_{im} \tag{5.35}$$

where x_{im} is a $1 \times K$ vector of observable market and firm characteristics; β_i is a $K \times 1$ vector of parameters; $\delta_i = \{\delta_{ij} : j \neq i\}$ is a $(N-1) \times 1$ vector of parameters, with δ_{ij} being the effect of firm $j's$ entry on firm $i's$ profit; ε_{im} is a zero mean random variable that is observable to the players but unobservable to the econometrician.

We assume that ε_{im} is independent of x_m, and it is $i.i.d.$ over m, and independent across i. If x_{im} includes a constant term, then without loss of generality $\mathbb{E}(\varepsilon_{im}) = 0$. Define $\sigma_i^2 \equiv Var(\varepsilon_{im})$. Then, we also assume that the probability distribution of $\varepsilon_{im}/\sigma_i$ is known to the researcher. For instance, $\varepsilon_{im}/\sigma_i$ has a standard normal distribution.

The econometric model can be described as a system of N simultaneous equations where the endogenous variables are the entry dummy variables:

$$a_{im} = 1\left\{\ x_{im}\ \beta_i - \sum\nolimits_{j \neq i} a_{jm}\ \delta_{ij} + \varepsilon_{im} \geq 0\ \right\} \tag{5.36}$$

We want to estimate the vector of parameters $\theta = \{\beta_i/\sigma_i, \delta_i/\sigma_i : i = 1, 2, ..., N\}$.

There are two main econometric issues in the estimation of this model: (1) endogenous explanatory variables, a_{jm}; and (2) multiple equilibria.

Endogeneity of other players' actions

In the system of structural equations in (5.36), the actions of the other players, $\{a_{jm} : j \neq i\}$ are endogenous in an econometric sense. That is, a_{jm} is correlated with the unobserved term ε_{im}, and ignoring this correlation can lead to serious biases in our estimates of the parameters β_i and δ_i.

There two sources of endogeneity or correlation between a_{jm} and ε_{im}: (i) simultaneity; and (ii) correlation between ε_{im} and ε_{jm}. It is interesting to distinguish between these two sources of endogeneity because they bias the parameter δ_{ij} in opposite directions.

(i) Simultaneity. An equilibrium of the model is a reduced form equation where we represent the action of each player as a function of only exogenous variables in x_m and ε_m. In this reduced form, a_{jm} depends on ε_{im}. It is possible to show that this dependence is negative: keeping all the other exogenous variables constant, if ε_{im} is small enough then $a_{jm} = 0$, and if ε_{im} is large enough then $a_{jm} = 1$. Suppose that our estimator of δ_{ij} ignores this dependence. Then, the negative dependence between a_{jm} and ε_{im} contributes to generate a upward bias in the estimator of δ_{ij}.

That is, we will spuriously over-estimate the negative effect of Southwest on the profit of American Airlines because Southwest tends to enter in markets where AA has low values of ε_{im}.

(ii) Positively correlated unobservables. It is reasonable to expect that ε_{im} and ε_{jm} are positively correlated. This is because both ε_{im} and ε_{jm} contain unobserved market characteristics that affect in a similar way, or at least in the same direction, all the firms in the same market. Some markets are more profitable than others for every firm, and part of this market heterogeneity is observable to firms but unobservable to us as researchers. The positive correlation between ε_{im} and ε_{jm} generates also a positive dependence between a_{jm} and ε_{im}.

For instance, suppose that $\varepsilon_{im} = \omega_m + u_{im}$, where ω_m represents the common market effect, and u_{im} is independent across firms. Then, keeping x_m and the unobserved u variables constant, if ω_m is small enough then ε_{im} is small and $a_{jm} = 0$, and if ω_m is large enough then ε_{im} is large and $a_{jm} = 1$. Suppose that our estimator of δ_{ij} ignores this dependence. Then, the positive dependence between a_{jm} and ε_{im} contributes to generate a downward bias in the estimator of δ_{ij}. In fact, the estimate of δ_{ij} could have the wrong sign, that is, could be negative instead of positive.

Therefore, we could spuriously find that American Airlines benefits from the operation of Continental in the same market because we tend to observe that these firms are always active in the same markets. This positive correlation between a_{im} and a_{jm} can be completely driven by the positive correlation between ε_{im} and ε_{jm}.

These two sources of endogeneity generate biases of opposite sign in δ_{ij}. There is evidence from different empirical applications that the bias due to unobserved market effects is much more important than the simultaneity bias. For instance, among others, Orhun (2013) in the US supermarket industry, Collard-Wexler (2013) in the US cement industry, Aguirregabiria and Mira (2007) in several retail industries in Chile, Igami and Yang (2016) in the Canadian fast-food restaurant industry, and Aguirregabiria and Ho (2012) in the US airline industry.

How do we deal with this endogeneity problem? The intuition for the identification in this model is similar to the identification using standard instrumental variables (IV) and control function (CF) estimation methods.

IV approach. There are exogenous firm characteristics in x_{jm} that affect the action of firm j but do not have a direct effect on the action of firm i: that is, observable characteristics with $\beta_j \neq 0$ but $\beta_i = 0$.

CF approach. There is an observable variable C_{it} that "proxies" or "controls for" the endogenous part of ε_{im} such that if we include C_{it} in the equation for firm i then the new error term in that equation and a_{jm} become independent (conditional on C_{it}).

The method of instrumental variables is the most common approach to deal with endogeneity in linear models. However, IV or GMM cannot be applied to estimate discrete choice models with endogenous variables. Control function approaches: Rivers and Vuong (1988), Vytlacil and Yildiz (2007). These approaches have not been extended yet to deal with models with multiple equilibria.

An alternative approach is Maximum likelihood. If we derive the probability distribution of the dummy endogenous variables conditional on the exogenous variables (that is, the reduced form of the model), we can use these probabilities to estimate the model by maximum likelihood.

$$\ell(\theta) = \sum_{m=1}^{M} \ln \Pr(a_{1m}, a_{2m}, ..., a_{Nm} \mid x_m,\ \theta) \tag{5.37}$$

This is the approach that has been most commonly used in this literature. However, we will have to deal with the problem of multiple equilibria.

Multiple equilibria

Consider the model with two players and assume that $\delta_1 \geq 0$ and $\delta_2 \geq 0$.

$$\begin{aligned} a_1 &= 1\{\ x_1\beta_1 - \delta_1\ a_2 + \varepsilon_1 \geq 0\ \} \\ a_2 &= 1\{\ x_2\beta_2 - \delta_2\ a_1 + \varepsilon_2 \geq 0\ \} \end{aligned} \tag{5.38}$$

The reduced form of the model is a representation of the endogenous variables (a_1, a_2) only in terms of exogenous variables and parameters. This is the reduced form of this model:

$$\begin{aligned} \{x_1\beta_1 + \varepsilon_1 < 0\}\ \&\ \{x_2\beta_2 + \varepsilon_2 < 0\} &\Rightarrow (a_1, a_2) = (0,0) \\ \{x_1\beta_1 - \delta_1 + \varepsilon_1 \geq 0\}\,\&\,\{x_2\beta_2 - \delta_2 + \varepsilon_2 \geq 0\} &\Rightarrow (a_1, a_2) = (1,1) \\ \{x_1\beta_1 - \delta_1 + \varepsilon_1 < 0\}\ \&\ \{x_2\beta_2 + \varepsilon_2 \geq 0\} &\Rightarrow (a_1, a_2) = (0,1) \\ \{x_1\beta_1 + \varepsilon_1 \geq 0\}\ \&\ \{x_2\beta_2 - \delta_2 + \varepsilon_2 < 0\} &\Rightarrow (a_1, a_2) = (1,0) \end{aligned} \tag{5.39}$$

The graphical representation in the space $(\varepsilon_1, \varepsilon_2)$ is in Figure 5.1.

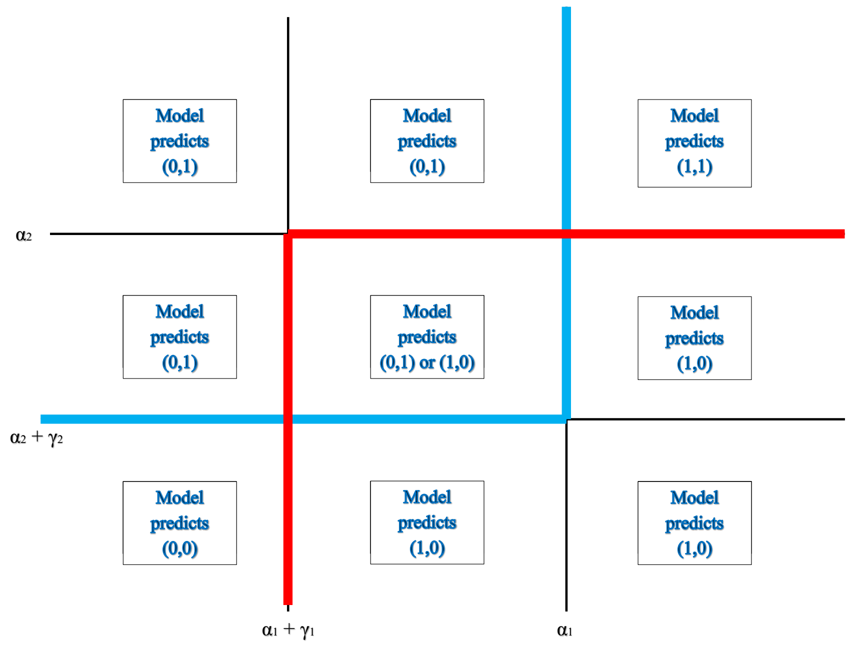

Figure 5.1: Outcomes in the Region of the Unobservables

Note that when:

$$\{0 \leq x_1\beta_1 + \varepsilon_1 < \delta_1\} \text{ and } \{0 \leq x_2\beta_2 + \varepsilon_2 < \delta_2\} \tag{5.40}$$

we have two Nash equilibria: $(a_1, a_2) = (0,1)$ and $(a_1, a_2) = (1,0)$. For this range of values of $(\varepsilon_1, \varepsilon_2)$, the reduced form (that is, the equilibrium) is not uniquely determined.

Therefore, we can not uniquely determine the probability $\Pr(a_{1m}, a_{2m}|x_m;\theta)$ that we need to estimate the model by maximum likelihood. We know $\Pr(1,1|\theta)$, and $\Pr(0,0|\theta)$, but we only have lower and upper bounds for $\Pr(0,1|\theta)$ and $\Pr(1,0|\theta)$.

The problem of indeterminacy of the probabilities of different outcomes becomes even more serious in empirical games with more than 2 players or/and more than two choice alternatives.

There have been different approaches to deal with the problem of multiple equilibria. Some authors have imposed additional structure in the model to guarantee equilibrium uniqueness or at least uniqueness of some observable outcome, for instance, number of entrants). A second group of studies do not impose additional structure and use methods such as moment inequalities or pseudo maximum likelihood to estimate structural parameters. The main motivation of this second group of studies is that identification and multiple equilibria are different problems and we do not need equilibrium uniqueness to identify parameters. We discuss these methods below.

5.4.5 Incomplete information

Model and basic assumptions

Consider a market with N potential entrants. If firm i does not operate in market m ($a_{im} = 0$), its profit is zero. If the firm is active in the market ($a_{im} = 1$), the profit is:

$$\Pi_{im} = \Pi_i(x_m, a_{-im}) - \varepsilon_{im} \tag{5.41}$$

For instance,

$$\Pi_{im} = x_{im}\ \beta_i - \varepsilon_{im} - \sum\nolimits_{j \neq i} \delta_{ij}\ a_{jm} \tag{5.42}$$

where β_i and δ_i are parameters. These parameters and the vector $s_m = (s_{1m}, s_{2m}, ..., s_{Nm})$ contain the variables which are common knowledge for all players. Now ε_{im} is private information of firm i. For the moment, we assume that private information variables are independent of s_m, and independently distributed over firms with distribution functions $G_i(\varepsilon_{im})$. The distribution function G_i is strictly increasing in $\mathbb{R}$. The information of player i is (s_m, ε_{im}).

A player's strategy depends on the variables in her information set. Let $\alpha \equiv \{\alpha_i(s_m, \varepsilon_{im}) : i = 1, 2, ..., N\}$ be a set of strategy functions, one for each player, such that $\alpha_i : S \times \mathbb{R} \to \{0, 1\}$. The actual profit Π_{im} is unknown to player i because the private information of the other players is unknown to player i. Players maximize expected profits:

$$\pi_i(s_m, \varepsilon_{im}, \alpha_{-i}) = s_{im}\ \beta_i - \varepsilon_{im} - \sum\nolimits_{j \neq i} \delta_{ij} \left[\int 1\left\{ \alpha_j(s_m, \varepsilon_{jm}) = 1 \right\} dG_j(\varepsilon_{jm}) \right] \tag{5.43}$$

or:

$$\begin{aligned} \pi_i(s_m, \varepsilon_{im}, \alpha_{-i}) &= s_{im}\ \beta_i - \varepsilon_{im} - \sum\nolimits_{j \neq i} \delta_{ij}\ P_j^{\alpha}(s_m) \\ &= s_{im}\ \beta_i - \varepsilon_{im} - P_{-i}^{\alpha}(s_m)'\delta_i \end{aligned} \tag{5.44}$$

where $P_j^{\alpha}(s_m) \equiv \int 1\left\{ \alpha_j(s_m, \varepsilon_{jm}) = 1 \right\} dG_j(\varepsilon_{jm})$ is player j's probability of entry if she behaves according to her strategy in α.

Suppose that players other than i play their respective strategies in α. What is player i's best response? Let $b_i(s_m, \varepsilon_{im}, \alpha_{-i})$ be player i's best response function. This function is:

$$\begin{aligned} b_i(s_m, \varepsilon_{im}, \alpha_{-i}) &= 1\{\ \pi_i(s_m, \varepsilon_{im}, \alpha_{-i}) \geq 0\ \} \\ &= 1\left\{\ \varepsilon_{im} \leq s_{im}\ \beta_i - P^{\alpha}_{-i}(s_m)'\delta_i\ \right\} \end{aligned} \tag{5.45}$$

Associated with the best response function b_i (in the space of strategies), we can define a *best response probability function* in the space of probabilities as:

$$\begin{aligned} \Psi_i(s_m, P^{\alpha}_{-i}) &= \int 1\left\{\ b_i(s_m, \varepsilon_{im}, \alpha_{-i}) = 1\ \right\} dG_i(\varepsilon_{im}) \\ &= \int 1\left\{\ \varepsilon_{im} \leq s_{im}\ \beta_i - P^{\alpha}_{-i}(s_m)'\delta_i\ \right\} dG_i(\varepsilon_{im}) \\ & G_i\left(s_{im}\ \beta_i - P^{\alpha}_{-i}(s_m)'\delta_i\right) \end{aligned} \tag{5.46}$$

A *Bayesian Nash equilibrium* (BNE) in this model is a set of strategy functions α^* such that, for any player i and any value of (s_m, ε_{im}), we have that:

$$\alpha^*_i(s_m, \varepsilon_{im}) = b_i(s_m, \varepsilon_{im}, \alpha^*_{-i}) \tag{5.47}$$

Associated with the set of strategies α^* we can define a set of choice probability functions $P^* = \{P^*_i(s_m) : i = 1,2,...,N\}$ such that $P^*_i(s_m) \equiv \int 1\{\alpha^*_i(s_m, \varepsilon_{im}) = 1\}\, dG_i(\varepsilon_{im})$. Note that these equilibrium choice probabilities are such that, for any player i and any value of s_m:

$$\begin{aligned} P^*_i(s_m) &= \Psi_i(s_m, P^*_{-i}) \\ &= G_i\left(s_{im}\ \beta_i - P^*_{-i}(s_m)'\delta_i\right) \end{aligned} \tag{5.48}$$

Therefore, we can define a BNE in terms of strategy functions α^* or in terms of choice probabilties P^*. There is a one-to-one relationship between α^* and P^*. Given α^*, it is clear that there is only one set of choice probabilities P^* defined as $P^*_i(s_m) \equiv \int I\{\alpha^*_i(s_m, \varepsilon_{im}) = 1\}\, dG_i(\varepsilon_{im})$. And given P^*, there is only one set of strategies α^* that is a BNE and is consistent with P^*. These strategy functions are:

$$\alpha^*_i(s_m, \varepsilon_{im}) = 1\left\{\ \varepsilon_{im} \leq s_{im}\ \beta_i - P^*_{-i}(s_m)'\delta_i\ \right\} \tag{5.49}$$

Suppose that the distribution of ε_{im} is known up to some scale parameter σ_i. For instance, suppose that $\varepsilon_{im} \sim iid\ N(0,1)$. Then, we have that equilibrium choice probabilities in market m solve the fixed point mapping in probability space:

$$P^*_i(s_m) = \Phi\left(s_{im}\ \frac{\beta_i}{\sigma_i} - P^{\alpha}_{-i}(s_m)'\frac{\delta_i}{\sigma_i}\right) \tag{5.50}$$

For notational simplicity we will use β_i and δ_i to represent β_i/σ_i and δ_i/σ_i, respectively.

We use θ to represent the vector of structural parameters $\{\beta_i, \delta_i : i = 1,2,...,N\}$. To emphasize that equilibrium probabilities depend on θ we use $P(s_m, \theta) = \{P_i(s_m, \theta) : i = 1,2,...,N\}$ to represent a vector of equilibrium probabilities associated with the exogenous conditions (s_m, θ). In general, there are values of $(s_m; \theta)$ for which the model has multiple equilibria. This is very common in models where players are heterogeneous, but we can find also multiple symmetric equilibria in models with homogeneous players, especially if there is strategic complementarity (that is, $\delta_i < 0$) as in coordination games.

Data and identification

Suppose that we observe this game played in M independent markets. We observe players' actions and a subset of the common knowledge state variables, $x_{im} \subseteq s_{im}$. That is,

$$Data = \{x_{im}, a_{im} : m = 1, 2, ..., M;\ i = 1, 2, ..., N\} \tag{5.51}$$

The researcher does not observe private information variables. It is important to distinguish two cases: (Case I) No common knowledge unobservables, that is, $x_{im} = s_{im}$; (Case II) Common knowledge unobservables, that is, $s_{im} = (x_{im}, \omega_{im})$, where ω_{im} is unobservable.

Case I: No common knowledge unobservables. Suppose that we have a random sample of markets and we observe:

$$\{x_{im}, a_{im} : m = 1, 2, ..., M;\ i = 1, 2, ..., N\} \tag{5.52}$$

We can describe this type of dataset as *data with global players* as all the firms play the entry game in the M markets. Let $P^0 = \{P_i^0(x) : i = 1, 2, .., N; x \in X\}$ be players' entry probabilities in the population under study. The population is an equilibrium of the model. That is, for any i and any $x \in X$:

$$P_i^0(x) = \Phi\left(x_i\ \beta_i - P_{-i}^0(x)'\delta_i\right) \tag{5.53}$$

From our sample, we can nonparametrically identify the population P^0, that is, $P_i^0(x) = \mathbb{E}(a_{im}|x_m = x)$. Given P^0 and the equilibrium conditions in (5.53), can we uniquely identify θ? Notice that we can write these equations as:

$$\Phi^{-1}\left(P_i^0(x_m)\right) = x_{im}\ \beta_i - P_{-i}^0(x_m)'\delta_i = Z_{im}\ \theta_i \tag{5.54}$$

Define $Y_{im} \equiv \Phi^{-1}\left(P_i^0(x_m)\right)$; $Z_{im} \equiv (x_{im}, P_{-i}^0(x_m))$; and $\theta_i^0 \equiv (\beta_i^0, \delta_i^0)$. Then,

$$Y_{im} = Z_{im}\ \theta_i \tag{5.55}$$

And we can also write this system as:

$$\mathbb{E}(Z'_{im} Y_{im}) = \mathbb{E}(Z'_{im} Z_{im})\ \theta_i \tag{5.56}$$

It is clear that θ_i is uniquely identified if $\mathbb{E}(Z'_{im} Z_{im})$ is a nonsigular matrix. Note that if x_{im} contains variables that vary both over markets and over players then we have exclusion restrictions that imply that $\mathbb{E}(Z'_{im} Z_{im})$ is a nonsigular matrix.

I some empirical applications, the dataset includes only local players. That is, firms that are potential entrants in only one local market. In this case, we have a random sample of markets and we observe:

$$\{x_m, n_m : m = 1, 2, ..., M\} \tag{5.57}$$

Let $P^0 = \{P^0(x) : x \in X\}$ be the entry probabilities in the population under study. The population is an equilibrium of the model, and therefore there is a θ such that for any $x \in X$:

$$P^0(x) = \Phi\left(x\ \beta - \delta\ H(P^0[x])\right) \tag{5.58}$$

From our sample, we can nonparametrically identify the population P^0. To see this, notice that: (1) we can identify the distribution for the number of firms: $\Pr(n_m = n|x_m = x)$; (2) the model implies that conditional on $x_m = x$ the number of firms follows a Binomial distribution with arguments N and $P^0(x)$, and therefore:

$$\Pr(n_m = n|x_m = x) = \binom{n}{N} P^0(x)^n \left(1 - P^0(x)\right)^{N-n}; \tag{5.59}$$

and (3) given the previous expression, we can obtain the $P^0(x)$ associated with $\Pr(n_m = n|x_m = x)$. Given P^0 and the equilibrium condition $P^0(x) = \Phi\left(x\,\beta - \delta\, H(P^0[x])\right)$, can we uniquely identify θ? Notice that we can write these equations as:

$$Y_m = x_m\,\beta - \delta\, H(P^0[x_m]) = Z_m\,\theta \tag{5.60}$$

where $Y_m \equiv \Phi^{-1}\left(P^0(x_m)\right)$; $\theta \equiv (\beta, \delta)$; and $Z_m \equiv (x_m, H(P^0[x_m]))$. And we can also write this system as:

$$\mathbb{E}(Z_m' Y_m) = \mathbb{E}(Z_m' Z_m)\,\theta \tag{5.61}$$

It is clear θ is uniquely identified if $\mathbb{E}(Z_m' Z_m)$ is a nonsingular matrix.

Case II: Common knowledge unobservables. Conditional on x_m, players' actions are still correlated across markets. This is evidence that ....

In applications where we do not observe the identity of the potential entrants, we consider a model without firm heterogeneity:

$$\Pi_{im} = x_m\,\beta - \delta\, h\left(1 + \sum\nolimits_{j\neq i} a_{jm}\right) + \varepsilon_{im} \tag{5.62}$$

A symmetric Bayesian Nash equilibrium in this model is a probability of entry $P^*(x_m;\theta)$ that solves the fixed point problem:

$$P^*(x_m;\theta) = \Phi\left(x_m\,\beta - \delta\, H(P[x_t,\theta])\right) \tag{5.63}$$

where $H(P)$ is the expected value of $h\left(1 + \sum_{j\neq i} a_j\right)$ conditional on the information of firm i, and under the condition that the other firms behave according to their entry probabilities in P. That is,

$$H(P) = \sum\nolimits_{a_{-i}} \left(\prod\nolimits_{j\neq i} P_j^{a_j}\ \left[1 - P_j\right]^{1-a_j}\right) h\left(1 + \sum\nolimits_{j\neq i} a_j\right) \tag{5.64}$$

and $\sum_{a_{-i}}$ represents the sum over all the possible actions of firms other than i.

Pseudo ML estimation

The goal is to estimate the vector of structural parameters θ^0 given a random sample $\{x_{im}, a_{im}\}$. Equilibrium probabilities are not uniquely determined for some values of the primitives. However, for any vector of probabilities P, the best response probability functions $\Phi\left(x_{im}\,\beta_i - \sum_{j\neq i} \delta_{ij}\, P_j(x_m)\right)$ are always well-defined. We define a pseudo likelihood function based on best responses to the population probabilities:

$$\begin{aligned} Q_M(\theta, P^0) &= \sum\nolimits_{m=1}^{M} \sum\nolimits_{i=1}^{N} a_{im}\ \ln\Phi\left(x_{im}\,\beta_i - \sum\nolimits_{j\neq i} \delta_{ij}\, P_j^0(x_m)\right) \\ &+ \ (1 - a_{im})\ \ln\Phi\left(-x_{im}\,\beta_i + \sum\nolimits_{j\neq i} \delta_{ij}\, P_j^0(x_m)\right) \end{aligned} \tag{5.65}$$

It is possible to show that θ uniquely maximizes $Q_\infty(\theta, P^0)$. The PML estimator of θ^0 maximizes $Q_M(\theta, \hat{P}^0)$, where $\hat{P}^0$ is a consistent nonparametric estimator of P^0. This estimator is consistent and asymptotically normal. Iterating in this procedure can provide efficiency gains both in finite samples and asymptotically (Aguirregabiria, 2004).

5.4.6 Entry and spatial competition

How do market power and profits of a retail firm depend on the location of its store(s) relative to the location of competitors? How important is spatial differentiation in explaining market power? These are important questions in the study of competition in retail markets. Seim (2006) studies these questions in the context of the video rental industry. Seim's work is the first study that endogenizes store locations and introduces spatial competition in a game of market entry.

Seim's model has important similarities with the static game with single-store firms and incomplete information that we have presented above. The main difference is that Seim's model does not include an explicit model of spatial consumer demand and price competition. Instead, she considers a 'semi-structural' specification of a store's profit that captures the idea that the profit of a store declines when competing stores get closer in geographic space. The specification seems consistent with the idea that consumers face transportation costs, and therefore spatial differentiation between stores can increase profits.

From a geographical point of view, a market in this model is a compact set in the two-dimension Euclidean space. There are L locations in the market where firms can operate stores. These locations are a set grid points where the grid can be as fine as we want. We index locations by ℓ that belongs to the set $\{1,2,...,L\}$.

There are N potential entrants in the market. Each firm makes two decisions: (1) whether to be active or not in the market; and (2) if it decides to be active, where to open its store. Note that Seim does not model multi-store firms. Aguirregabiria and Vicentini (2016) present an extension of Seim's model with multi-store firms, endogenous consumer behavior, and dynamics.

Let a_i represent the decisions of firm i, such that $a_i \in \{0,1,...,L\}$ and $a_i = 0$ represents "no entry", and $a_i = \ell > 0$ represents entry in location ℓ.

The profit of not being active in the market is normalized to zero. Let $\Pi_{i\ell}$ be the profit of firm i if it has a store in location ℓ. These profits depend on the store location decisions of the other firms. In particular, $\Pi_{i\ell}$ declines with the number of other stores "close to" location ℓ.

Of course, the specific meaning of being close to location ℓ is key for the implications of this model. This should depend on how consumers perceive as close substitutes stores in different locations. In principle, if we have data on quantities and prices for the different stores that are active in this city, we could estimate a demand system that would provide a measures of consumers' transportation costs and of the degree of substitution in demand between stores at different locations. Houde (2012) applies this approach to gasoline markets. However, for this industry we may not have information on prices and quantities at the store level. Fortunately, store location decisions may contain useful (and even better) information for identifying the degree of competition between stores at different locations.

Seim's specification of the profit function is "semi-structural": it does not model explicitly consumer behavior,but it is consistent with the idea that consumers face transportation costs, and therefore spatial differentiation (ceteris paribus) can increase profits.

For every location ℓ in the city, Seim defines B rings around the location: a first ring of radius d_1 (say half a mile); a second ring of radius $d_2 > d_1$ (say one mile), and so on.

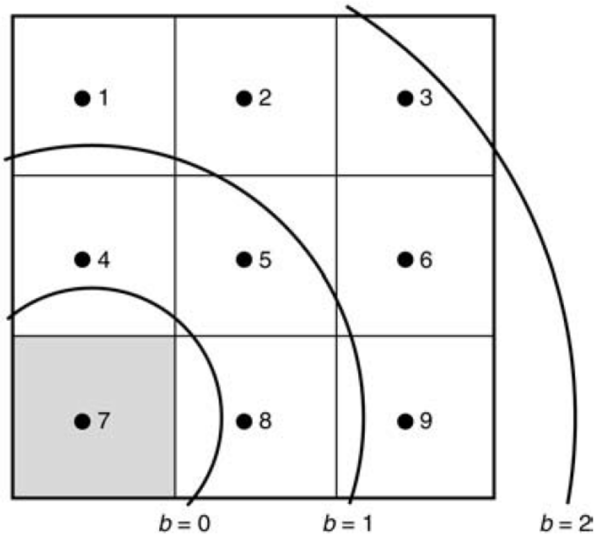

Figure 5.2: Seim, 2006): Defition of Local Markets

The profit of a store depends on the number of other stores located within each of the B rings. We expect that closer stores should have stronger negative effects on the store's profits. The profit function of an active store at location ℓ is:

$$\Pi_{i\ell} = x_\ell \, \beta + \sum_{b=1}^{B} \gamma_b \, N_{b\ell} + \xi_\ell + \varepsilon_{i\ell} \tag{5.66}$$

where β, γ_1, γ_2, ..., and γ_B are parameters; x_ℓ is a vector of observable exogenous characteristics that affect profits in location ℓ; $N_{b\ell}$ is the number of stores in ring b around location ℓ excluding i; ξ_ℓ represents exogenous characteristics of location ℓ that are unobserved to the researcher but common and observable to firms; and $\varepsilon_{i\ell}$ is a component of the profit of firm i in location ℓ that is private information to this firm. For the "no entry" choice, $\Pi_{i0} = \varepsilon_{i0}$.

Assumption. Let $\varepsilon_i = \{\varepsilon_{i\ell} : \ell = 0, 1, ..., L\}$ be the vector with the private information variables of firm i at every possible location. ε_i is i.i.d. over firms and locations with a extreme value type 1 distribution.

The information of firm i is (x, ξ, ε_i), where x and ξ represent the vectors with x_ℓ and ξ_ℓ, respectively, at every location ℓ in the city. Firm i does not know the $\varepsilon's$ of other firms. Therefore, $N_{b\ell}$ is unknown to the firm. Firms only know the probability distribution of $N_{b\ell}$. Therefore, firms maximize expected profits. The expected profit of firm i is:

$$\Pi^e_{i\ell} = x_\ell \, \beta + \sum_{b=1}^{B} \gamma_b \, N^e_{b\ell} + \xi_\ell + \varepsilon_{i\ell} \tag{5.67}$$

where $N^e_{b\ell}$ represents $\mathbb{E}(N_{b\ell}|x, \xi)$.

A firm's strategy depends on the variables in its information set. Let $\alpha_i(x, \xi, \varepsilon_i)$ be a strategy function for firm i such that $\alpha_i : X \times \mathbb{R}^2 \to \{0, 1, ..., L\}$. Given expectations $N^e_{b\ell}$, the best response strategy of firm i is:

$$\alpha_i(x, \xi, \varepsilon_i) = \arg \max_{\ell \in \{0,1,...,L\}} \left\{ x_\ell \, \beta + \sum_{b=1}^{B} \gamma_b \, N^e_{b\ell} + \xi_\ell + \varepsilon_{i\ell} \right\} \tag{5.68}$$

Or similarly, $\alpha_i(x, \xi, \varepsilon_i) = \ell$ if and only if $x_\ell \, \beta + \sum_{b=1}^{B} \gamma_b \, N^e_{b\ell} + \xi_\ell + \varepsilon_{i\ell}$ is greater that $x_{\ell'}$ $\beta + \sum_{b=1}^{B} \gamma_b \, N^e_{b\ell'} + \xi_{\ell'} + \varepsilon_{i\ell'}$ for any other location ℓ'.

From the point of view of other firms that do not know the private information of firm i but know the strategy function $\alpha_i(x, \xi, \varepsilon_i)$, the strategy of firm i can be described as a probability distribution: $P_i \equiv \{P_{i\ell} : \ell = 0, 1, ..., L\}$ where $P_{i\ell}$ is the probability that firm i chooses location ℓ when following her strategy $\alpha_i(x, \xi, \varepsilon_i)$. That is,

$$P_{i\ell} \equiv \int 1\{\alpha_i(x, \xi, \varepsilon_i) = \ell\} \, dF(\varepsilon_i) \tag{5.69}$$

where $F(\varepsilon_i)$ is the CDF of ε_i. By construction, $\sum_{\ell=0}^{L} P_{i\ell} = 1$.

Given expectations $N^e_{b\ell}$, we can also represent the best response strategy of firm i as a choice probability. A best response probability $P_{i\ell}$ is:

$$P_{i\ell} = \int 1 \left[\ell = \arg\max_{\ell'} \left\{ x_{\ell'} \, \beta + \sum_{b=1}^{B} \gamma_b \, N^e_{b\ell'} + \xi_{\ell'} + \varepsilon_{i\ell'} \right\} \right] \, dF(\varepsilon_i) \tag{5.70}$$

And given the extreme value assumption on ε_i:

$$P_{i\ell} = \frac{\exp\left\{ x_\ell \beta + \sum_{b=1}^{B} \gamma_b \, N^e_{b\ell} + \xi_\ell \right\}}{1 + \exp\left\{ x_{\ell'} \beta + \sum_{b=1}^{B} \gamma_b \, N^e_{b\ell'} + \xi_{\ell'} \right\}} \tag{5.71}$$

In this application, there is no information on firms' exogenous characteristics, and Seim assumes that the equilibrium is symmetric: $\alpha_i(x, \xi, \varepsilon_i) = \alpha(x, \xi, \varepsilon_i)$ and $P_{i\ell} = P_\ell$ for every firm i.

The expected number of firms in ring b around location ℓ, $N^e_{b\ell}$, is determined by the vector of entry probabilities $P \equiv \{P_{\ell'} : \ell' = 1, 2, ..., L\}$. That is:

$$N^e_{b\ell} = \sum_{\ell'=1}^{L} 1\{\ell' \text{belongs to ring } b \text{ around } \ell\} \, P_{\ell'} \, N \tag{5.72}$$

To emphasize this dependence we use the notation $N^e_{b\ell}(P)$.

Therefore, we can define a (symmetric) equilibrium in this game as a vector of probabilities $P \equiv \{P_\ell : \ell = 1,2,...,L\}$ that solve the following system of equilibrium conditions: for every $\ell = 1,2,...,L$:

$$P_\ell = \frac{\exp\left\{x_\ell\beta + \sum_{b=1}^{B} \gamma_b\, N^e_{b\ell}(P) + \xi_\ell\right\}}{1 + \exp\left\{x_{\ell'}\beta + \sum_{b=1}^{B} \gamma_b\, N^e_{b\ell'}(P) + \xi_{\ell'}\right\}} \tag{5.73}$$

By Brower's Theorem an equilibrium exists. The equilibrium may not be unique. Seim shows that if the γ parameters are not large and they decline fast enough with b, then the equilibrium is unique.

Let $\theta = \{N, \beta, \gamma_1, \gamma_2, ..., \gamma_B\}$ be the vector of parameters of the model. These parameters can be estimated even if we have data only from one city. Suppose that the data set is $\{x_\ell, n_\ell : \ell = 1,2,...,L\}$ for L different locations in a city, where L is large, and n_ℓ represents the number of stores in location ℓ. We want to use these data to estimate θ. We describe below the estimation with data from only one city. Later, we will see that the extension to data from more than one city is trivial.

Let x be the vector $\{x_\ell : \ell = 1,2,...,L\}$. All the analysis is conditional on x, which is a description of the "landscape" of observable socioeconomic characteristics in the city. Given x, we can think of $\{n_\ell : \ell = 1,2,...,L\}$ as *one realization of a spatial stochastic process*. In terms of the econometric analysis, this has similarities with time series econometrics in the sense that a time series is a single realization from a stochastic process. Despite having just one realization of a stochastic process, we can estimate consistently the parameters of that process as long as we make some stationarity assumptions.

This is the model considered by Seim (2006): there is city unobserved heterogeneity (her dataset includes multiple cities) but within a city there is no unobserved location heterogeneity.

Conditional on x, spatial correlation/dependence in the unobservable variables ξ_ℓ can generate dependence between the number of firms at different locations $\{n_\ell\}$. We start with the simpler case where there is no unobserved location heterogeneity: that is, $\xi_\ell = 0$ for every location ℓ.

Without unobserved location heterogeneity, and conditional on x, the variables n_ℓ are independently distributed, and n_ℓ is a random draw from a Binomial random variable with arguments $(N, P_\ell(x,\theta))$, where $P_\ell(x,\theta)$ are the equilibrium probabilities defined above where now we explicitly include (x,θ) as arguments.

$$n_\ell \sim i.i.d. \text{ over } \ell \text{ Binomial}(N, P_\ell(x,\theta)) \tag{5.74}$$

Therefore,

$$\begin{aligned} \Pr(n_1, n_2, ..., n_L \mid x, \theta) &= \textstyle\prod_{\ell=1}^{L} \Pr(n_\ell \mid x, \theta) \\ &= \textstyle\prod_{\ell=1}^{L} \dfrac{N!}{n_\ell(N-n_\ell)!} P_\ell(x,\theta)^{n_\ell}(1 - P_\ell(x,\theta))^{N-n_\ell} \end{aligned} \tag{5.75}$$

The log-likelihood function is:

$$\ell(\theta) = \sum_{\ell=1}^{L} \ln\left(\frac{N!}{(N-n_\ell)!}\right) + n_\ell \ln P_\ell(x,\theta) + (N-n_\ell)\ln(1 - P_\ell(x,\theta)) \tag{5.76}$$

The maximum likelihood estimator, $\hat{\theta}$, is the value of θ that maximizes this likelihood. The parameters of the model, including the number of potential entrants N, are identified. Partly, the identification comes from functional form assumptions. However, there are also exclusion restrictions that can provide identification even if some of these assumptions are relaxed. In particular, for the identification of β and γ_b, the model implies that $N^e_{b\ell}$ depends on socioeconomic characteristics at locations other than ℓ (that is, $x_{\ell'}$ for $\ell' \neq \ell$). Therefore, $N^e_{b\ell}$ has sample variability that is independent of x_ℓ and this implies that the effects of x_ℓ and $N^e_{b\ell}$ on a firm's profit can be identified even if we relax the linearity assumption.[1]

Now, let's consider the model where $\xi_\ell \neq 0$. A simple (but restrictive approach) is to assume that there is a number R of "regions" or districts in the city, where the number of regions R is small relative to the number of locations L, such that all the unobserved heterogeneity is between regions but there is no unobserved heterogeneity within regions. Under this assumption, we can control for unobserved heterogeneity by including region dummies. In fact, this case is equivalent to the previous case without unobserved location heterogeneity with the only difference being that the vector of observables x_ℓ now includes region dummies.

A more interesting case is when the unobserved heterogeneity is at the location level. We assume that $\xi = \{\xi_\ell : \ell = 1,2,...,L\}$ is independent of x and it is a random draw from a spatial stochastic process. The simplest process is when ξ_ℓ is $i.i.d.$ with a known distribution, say $N(0,\sigma^2_\xi)$ where the zero mean is without loss of generality. However, we can allow for spatial dependence in this unobservable. For instance, we may consider a Spatial autoregressive process (SAR):

$$\xi_\ell = \rho \ \bar{\xi}^C_\ell + u_\ell \tag{5.77}$$

where u_ℓ is $i.i.d.$ $N(0,\sigma^2_u)$, ρ is a parameter, and $\bar{\xi}^C_\ell$ is the mean value of ξ at the C locations closest to location ℓ, excluding location ℓ itself. To obtain a random draw of the vector ξ from this stochastic process it is convenient to write the process in vector form:

$$\xi = \rho \ \mathbf{W}^C \ \xi + u \tag{5.78}$$

where ξ and u are $L \times 1$ vectors, and $\mathbf{W}^C$ is a $L \times L$ weighting matrix such that every row, say row ℓ, has values $1/C$ at positions that correspond to locations close to location ℓ, and zeroes otherwise. Then, we can write $\xi = (I - \rho \ \mathbf{W}^C)^{-1} u$. First, we take independent draws from $N(0,\sigma^2_u)$ to generate the vector u, and then we pre-multiply that vector by $(I - \rho \ \mathbf{W}^C)^{-1}$ to obtain ξ.

Note that now the vector of structural parameters includes the parameters in the stochastic process of ξ, that is, σ_u and ρ.

Now, conditional on both x and ξ, the variables n_ℓ are independently distributed, and n_ℓ is a random draw from Binomial random variable with arguments $(N, P_\ell(x,\xi,\theta))$, where $P_\ell(x,\xi,\theta)$ are the equilibrium probabilities. Importantly, for different values of ξ

[1] Xu (2018) studies the asymptotics of this type of estimator. His model is a bit different to Seim's model because players and locations are interchangeable.

we have different equilibrium probabilities. Then,

$$\begin{aligned}
\Pr(n_1, n_2, ..., n_L \mid x, \theta) &= \int \Pr(n_1, n_2, ..., n_L \mid x, \xi, \theta)\ dG(\xi) \\
&= \int \left[\prod_{\ell=1}^{L} \Pr(n_\ell \mid x, \xi, \theta)\right]\ dG(\xi) \\
&= \prod_{\ell=1}^{L} \frac{N!}{n_\ell (N - n_\ell)!} \\
&\quad \int \left[\prod_{\ell=1}^{L} P_\ell(x, \xi, \theta)^{n_\ell} (1 - P_\ell(x, \xi, \theta))^{N-n_\ell}\right] dG(\xi)
\end{aligned} \tag{5.79}$$

And the log-likelihood function is:

$$\ell(\theta) = \sum_{\ell=1}^{L} \ln\left(\frac{N!}{(N-n_\ell)!}\right) \tag{5.80}$$

$$\tag{5.81}$$

$$+\ln\left(\int \left[\prod_{\ell=1}^{L} P_\ell(x, \xi, \theta)^{n_\ell} (1 - P_\ell(x, \xi, \theta))^{N-n_\ell}\right] dG(\xi)\right) \tag{5.82}$$

The maximum likelihood estimator is defined as usual.

In their empirical study on competition between big-box discount stores in the US (that is, Kmart, Target and Walmart), Zhu and Singh (2009) extend Seim's entry model by introducing firm heterogeneity. The model allows competition effects to be asymmetric across three different chains. For example, the model can incorporate a situation where the impact on the profits of Target from a Walmart store 10 miles away is stronger than the impact from a Kmart store located 5 miles away. The specification of the profit function of a store of chain i at location ℓ is:

$$\pi_{i\ell} = x_\ell\ \beta_i + \sum_{j \neq i} \sum_{b=1}^{B} \gamma_{bij}\ n_{b\ell j} + \xi_\ell + \varepsilon_{i\ell} \tag{5.83}$$

where $n_{b\ell j}$ represents the number of stores that chain j has within the $b - ring$ around location ℓ. Despite the paper studying competition between retail chains, it still makes similar simplifying assumptions as in Seim's model that ignores important aspects of competition between retail chains. In particular, the model ignores economies of density, and firms' concerns about cannibalization between stores of the same chain. It assumes that the entry decisions of a retail chain are made independently at each location. Under these assumptions, the equilibrium of the model can be described as a vector of $N * L$ entry probabilities, one for each firm and location, that solves the following fixed point problem:

$$P_{i\ell} = \frac{\exp\left\{x_\ell \beta_i + \sum_{j \neq i} \sum_{b=1}^{B} \gamma_{bij}\ N\left[\sum_{\ell'=1}^{L} D_{\ell\ell'}^{b}\ P_{j\ell'}\right] + \xi_\ell\right\}}{1 + \sum_{\ell'=1}^{L} \exp\left\{x_{\ell'} \beta_i + \sum_{j \neq i} \sum_{b=1}^{B} \gamma_{bij}\ N\left[\sum_{\ell''=1}^{L} D_{\ell'\ell''}^{b}\ P_{j\ell''}\right] + \xi_{\ell'}\right\}} \tag{5.84}$$

The authors find substantial heterogeneity in the competition effects between these three big-box discount chains, and in the pattern of how these effects decline with distance. For instance, Walmart's supercenters have a very substantial impact even at a large distance.

Datta and Sudhir (2013) estimate an entry model of grocery stores that endogenizes both location and product type decisions. They are interested in evaluating the effects of zoning restrictions on market structure. Zoning often reduces firms' ability to avoid competition by locating remotely each other. Theory suggests that in such a market firms have a stronger incentive to differentiate their products. Their estimation results support this theoretical prediction. The authors also investigate different impacts of various types of zoning ('centralized zoning', 'neighborhood zoning', and 'outskirt zoning') on equilibrium market structure.

5.4.7 Multi-store firms

As we have mentioned above, economies of density and cannibalization are potentially important factors in store location decisions of retail chains. A realistic model of competition between retail chains should incorporate this type of spillover effects. Taking into account these effects requires a model of competition between multi-store firms similar to the one in section 2.1.2. The model takes into account the joint determination of a firm's entry decisions at different locations. A firm's entry decision is represented by the L-dimension vector $a_i \equiv \{a_{i\ell} : \ell = 1, 2, \ldots, L\}$, with $a_{i\ell} \in \{0, 1\}$, such that the set of possible actions contains 2^L elements. For instance, Jia (2008) studies competition between two chains (Walmart and Kmart) over 2065 locations (US counties). The number of possible decisions of a retail chain is 2^{2065}. Without further restrictions, computing firms' best responses is intractable.

In her paper, Jia therefore imposes restrictions on the specification of firms' profits that imply the supermodularity of the game and facilitate substantially the computation of an equilibrium. Suppose that we index the two firms as i and j. The profit function of a firm, say i, is $\Pi_i = V_i(a_i, a_j) - EC_i(a_i)$, where $V_i(a_i, a_j)$ is the variable profit function such that:

$$V_i(a_i, a_j) = \sum_{\ell=1}^{L} a_{i\ell} \left[x_\ell \, \beta_i + \gamma_{ij} \, a_{j\ell} \right] \tag{5.85}$$

x_ℓ is a vector of market/location characteristics. γ_{ij} is a parameter that represents the effect on the profit of firm i of competition from a store of chain j. $EC_i(a_i)$ is the entry cost function such that:

$$EC_i(a_i) = \sum_{\ell=1}^{L} a_{i\ell} \left[\theta_{i\ell}^{EC} - \frac{\theta^{ED}}{2} \sum_{\ell'=1}^{L} \frac{a_{i\ell'}}{d_{\ell\ell'}} \right] \tag{5.86}$$

$\theta_{i\ell}^{EC}$ is the entry cost that firm i would have in location ℓ in the absence of economies of density (that is, if it were a single-store firm); θ^{ED} is a parameter that represents the magnitude of the economies of density and is assumed to be positive; and $d_{\ell\ell'}$ is the distance between locations ℓ and ℓ'.

Jia further assumes that the entry cost $\theta_{i\ell}^{EC}$ consists of three components:

$$\theta_{i\ell}^{EC} = \theta_i^{EC} + (1 - \rho) \, \xi_\ell + \varepsilon_{i\ell}, \tag{5.87}$$

where θ_i^{EC} is chain-fixed effects, ρ is a scale parameter, ξ_ℓ is a location random effect, and $\varepsilon_{i\ell}$ is a firm-location error term. Both ξ_ℓ and $\varepsilon_{i\ell}$ are i.i.d. draws from the standard normal distribution and known to all the players when making decisions. To capture economies of density, the presence of stores from the same firm at other locations is weighted by the inverse of the distance between locations, $1/d_{\ell\ell'}$. This term is multiplied by one-half to avoid double counting in the total entry cost of the retail chain.

The specification of the profit function in equations (5.85) and () imposes some important restrictions. Under this specification, locations are interdependent only through economies of density. In particular, there are no cannibalization effects between stores of the same chain at different locations. Similarly, there is no spatial competition between stores of different chains at different locations. In particular, this specification ignores the spatial competition effects between Kmart, Target, and Walmart that Zhu and Singh (2009) find in their study. The specification also rules out cost savings that do not depend on store density such as lower wholesale prices owing to strong bargaining power of chain stores. The main motivation for these restrictions is to have a supermodular game that facilitates very substantially the computation of an equilibrium, even when the model has a large number of locations.

In a Nash equilibrium of this model, the entry decisions of a firm, say i, should satisfy the following L optimality conditions:

$$a_{i\ell} = 1\left\{ x_\ell\, \beta_i + \gamma_{ij}\, a_{j\ell} - \theta^{EC}_{i\ell} + \frac{\theta^{ED}}{2} \sum_{\ell'=1}^{L} \frac{a_{i\ell'}}{d_{\ell\ell'}} \geq 0 \right\} \tag{5.88}$$

These conditions can be interpreted as the best response of firm i in location ℓ given the other firm's entry decisions, and given also firm i's entry decisions at locations other than ℓ. We can write this system of conditions in a vector form as $a_i = br_i(a_i, a_j)$. Given a_j, a fixed point of the mapping $br_i(., a_j)$ is a (full) best response of firm i to the choice a_j by firm j. With $\theta^{ED} > 0$ (that is, economies of density), it is clear from equation (9.31) that the mapping br_i is increasing in a_i. By Topkis's theorem, this increasing property implies that: (1) the mapping has at least one fixed point solution; (2) if it has multiple fixed points they are ordered from the lowest to the largest; and (3) the smallest (largest) fixed point can be obtained by successive iterations in the mapping br_i using as starting value $a_i = 0$ ($a_i = 1$). Given these properties, Jia shows that the following algorithm provides the Nash equilibrium that is most profitable for firm i:

Step [firm i]: Given the lowest possible value for $a_j = 0$, that is, $a_i = (0, 0, \ldots 0)$, we apply successive iterations with respect to a_i in the fixed point mapping $br_i(., a_j = 0)$ starting at $a_i = (1, 1, \ldots 1)$. These iterations converge to the largest best response of firm i, that we denote by $a_i^{(1)} = BR_i^{(High)}(0)$.

Step [firm j]: Step [j]: Given $a_i^{(1)}$, we apply successive iterations with respect to a_j in the fixed point mapping $br_j(., a_i^{(1)})$ starting at $a_j = 0$. These iterations converge to the lowest best response of firm j, that we denote by $a_j^{(1)} = BR_j^{(Low)}(a_i^{(1)})$.

We keep iterating in Step [firm i] and Step [firm j] until convergence. At every iteration, say k, given $a_j^{(k-1)}$ we first apply (Step [i]) to obtain $a_i^{(k)} = BR_i^{(High)}(a_j^{(k-1)})$, and then we apply (Step [j]) to obtain $a_j^{(k)} = BR_j^{(Low)}(a_i^{(k)})$. The supermodularity of the game ensures the convergence of this process and the resulting fixed point is the Nash equilibrium that most favors firm i. Jia combines this solution algorithm with a simulation of unobservables to estimate the parameters of the model using the method of simulated moments (MSM).

In his empirical study of convenience stores in Okinawa Island of Japan, Nishida (2015) extends Jia's model in two directions. First, a firm is allowed to open multiple stores (up to four) in the same location. Second, the model explicitly incorporates some form of spatial competition: a store's revenue is affected not only by other stores in the same location but also by those in adjacent locations.

Although the approach used in these two studies is elegant and useful, its use in other applications is somewhat limited. First, supermodularity requires that the own network effect on profits is monotonic, that is, the effect is either always positive ($\theta^{ED} > 0$) or always negative ($\theta^{ED} < 0$). This condition rules out situations where the net effect of cannibalization and economies of density varies across markets. Second, the number of (strategic) players must be equal to two. For a game to be supermodular, players' strategies must be strategic complements. In a model of market entry, players' strategies are strategic substitutes. However, when the number of players is equal to two, any game of strategic substitutes can be transformed into one of strategic complements by changing the order of strategies of one player (for example, use zero for entry and one for no entry). This trick no longer works when we have more than two players.

Ellickson, Houghton, and Timmins (2013, hereafter EHT) propose an alternative estimation strategy and apply it to data of US discount store chains. Their estimation method is based on a set of inequalities that arise from the best response condition of a Nash equilibrium. Taking its opponents' decisions as given, a chain's profit associated with its observed entry decision must be larger than the profit of any alternative entry decision. EHT consider particular deviations that relocate one of the observed stores to another location.

Let a_i^* be the observed vector of entry decisions of firm i, and suppose that in this observed vector the firm has a store in location ℓ but not in location ℓ'. Consider the alternative (hypothetical) choice $a_i^{\ell\to\ell'}$ that is equal to a_i^* except that the store in location ℓ is closed and relocated to location ℓ'. Revealed preference implies that $\pi_i(a_i^*) \geq \pi_i(a_i^{\ell\to\ell'})$. EHT further simplify this inequality by assuming that there are no economies of scope or density (for example, $\theta^{ED} = 0$), and that there are no firm-location-specific factors unobservable to the researcher, that is, $\varepsilon_{i\ell} = 0$. Under these two assumptions, the inequality above can be written as the profit difference between two locations:

$$[x_\ell - x_{\ell'}]\beta_i + \sum_{j\neq i} \gamma_{ij} \left[a_{j\ell}^* - a_{j\ell'}^*\right] + [\xi_\ell - \xi_{\ell'}] \geq 0 \tag{5.89}$$

Now, consider another chain, say k, that has an observed choice a_k^* with a store in location ℓ' but not in location ℓ. For this chain, we consider the opposite (hypothetical) relocation decision from firm i above: the store in location ℓ' is closed and a new store is open in location ℓ. For this chain, revealed preference implies that

$$[x_{\ell'} - x_\ell]\beta_k + \sum_{j\neq k} \gamma_{kj} \left[a_{j\ell}^* - a_{j\ell'}^*\right] + [\xi_{\ell'} - \xi_\ell] \geq 0 \tag{5.90}$$

Summing up the inequalities for firms i and k, we generate an inequality that is free from location fixed effects ξ.

$$[x_{\ell'} - x_\ell]\ [\beta_i - \beta_k] + \sum_{j\neq i} \gamma_{ij} \left[a_{j\ell}^* - a_{j\ell'}^*\right] + \sum_{j\neq k} \gamma_{kj} \left[a_{j\ell}^* - a_{j\ell'}^*\right] \geq 0 \tag{5.91}$$

EHT construct a number of inequalities of this type and obtain estimates of the parameters of the model by using a smooth maximum score estimator (Manski, 1975; Horowitz, 1992; Fox, 2007).

Unlike the lattice theory approach of Jia and Nishida, the approach applied by EHT can accommodate more than two players, allows the researcher to be agnostic about equilibrium selections, and is robust to the presence of unobserved market heterogeneity. Their model, however, rules out any explicit interdependence between stores in different locations, including spatial competition, cannibalization and economies of density. Although incorporating such inter-locational interdependencies does not seem to cause any fundamental estimation issue, doing so can be difficult in practice as it considerably increases the amount of computation. Another possible downside of this approach is the restriction it imposes on unobservables. The only type of structural errors that this model includes are the variables ξ_ℓ that are common for all firms. Therefore, to accommodate observations that are incompatible with the inequalities in EHT model, the model requires non-structural errors, which may be interpreted as firms' optimization errors.

5.5 Estimation

The estimation of games of entry and spatial competition in retail markets should deal with some common issues in the econometrics of games and dynamic structural models. Here we do not try to present a detailed discussion of this econometric literature. Instead, we provide a brief description of the main issues, with an emphasis on aspects that are particularly relevant for empirical applications in retail industries.

5.5.1 Multiple Equilibria

Entry models with heterogeneous firms often generate more than one equilibrium for a given set of parameters. Multiple equilibria pose challenges to the researcher for two main reasons. First, standard maximum likelihood estimation no longer works because the likelihood of certain outcomes is not well defined without knowing the equilibrium selection mechanism. Second, without further assumptions, some predictions or counterfactual experiments using the estimated model are subject to an identification problem. These predictions depend on the type of equilibrium that is selected in a hypothetical scenario not included in the data.

Several approaches have been proposed to estimate an entry game with multiple equilibria. Which method works the best depends on assumptions imposed in the model, especially its information structure. In a game of complete information, there are at least four approaches. The simplest approach is to impose some particular equilibrium selection rule beforehand and estimate the model parameters under this rule. For instance, Jia (2008) estimates the model of competition between big-box chains using the equilibrium that is most preferable to K-mart. She also estimates the same model under alternative equilibrium selection rules to check for the robustness of some of her results. The second approach is to construct a likelihood function for some endogenous outcomes of the game that are common across all the equilibria. Bresnahan and Reiss (1991) estimate their model by exploiting the fact that, in their model, the total number of entrants is unique in all the equilibria.

A third approach is to make use of inequalities that are robust to multiple equilibria. One example is the profit inequality approach of EHT, which we described above. Another example is the method of moment inequality estimators proposed by Ciliberto and Tamer (2009). They characterize the lower and upper bounds of the probability of a certain outcome that are robust to any equilibrium selection rule. Estimation of structural parameters relies on the set of probability inequalities constructed from these bounds. In the first step, the researcher nonparametrically estimates the probabilities of equilibrium outcomes conditional on observables. The second step is to find a set of structural parameters such that the resulting probability inequalities are most consistent with the data. The application of Ciliberto and Tamer's approach to a spatial entry model may not be straightforward. In models of this class, the number of possible outcomes (that is, market structures) is often very large. For example, consider a local market consisting of ten sub-blocks. When two chains decide whether they enter into each of these sub-blocks, the total number of possible market structures is 2^{10}. Such a large number of possible outcomes makes it difficult to implement this approach for two reasons. The first stage estimate is likely to be very imprecise even when a sample size is reasonably large. The second stage estimation can be computationally intensive because one needs to check, for a given set of parameters, whether each possible outcome meets the equilibrium conditions or not.

A fourth approach proposed by Bajari, Hong, and Nekipelov (2010) consists in the specification of a flexible equilibrium selection mechanism and in the joint estimation of the parameters in this mechanism and the structural parameters in firms' profit functions. Together with standard exclusion restrictions for the identification of games, the key specification and identification assumption in this paper is that the equilibrium selection function depends only on firms' profits.

In empirical games of incomplete information, the standard way to deal with multiple equilibria is to use a two-step estimation method (Aguirregabiria and Mira 2007); Bajari, Hong, and Ryan 2010). In the first step, the researcher estimates the probabilities of firms' entry conditional on market observables (called policy functions) in a nonparametric way, for example, a sieves estimator. The second step is to find a set of structural parameters that are most consistent with the observed data and these estimated policy functions. A key assumption for the consistency of this approach is that, in the data, two markets with the same observable characteristics do not select different types of equilibria, that is, same equilibria conditional on observables. Without this assumption, the recovered policy function in the first stage would be a weighted sum of firms' policies under different equilibria, making the second-stage estimates inconsistent. Several authors have recently proposed extensions of this method to allow for multiplicity of equilibria in the data for markets with the same observable characteristics.

Identification and multiple equilibria

Tamer (2003) showed that all the parameters of the previous entry model with $N = 2$ is (point) identified under standard exclusion restrictions, and that multiple equilibria do not play any role in this identification result. Tamer's result can be extended to any number N of players, as long as we have the appropriate exclusion restrictions.

More generally, equilibrium uniqueness is neither a necessary nor a sufficient condition for the identification of a model (Jovanovic, 1989). To see this, consider a model with a vector of structural parameters $\theta \in \Theta$, and define the mapping $C(\theta)$ from the set of parameters Θ to the set of measurable predictions of the model. For instance, $C(\theta)$ may contain the probability distribution of players' actions conditional on exogenous variables $\Pr(a_1, a_2, ..., a_N | x, \theta)$.

Multiple equilibria implies that the mapping C is a correspondence. A model is not point-identified if at the observed data (say $P^0 = \Pr(a_1, a_2, ..., a_N | x, \theta)$ for any vector of actions and $x's$) the inverse mapping C^{-1} is a correspondence. In general, C being a function (that is, equilibrium uniqueness) is neither a necessary nor a sufficient condition for C^{-1} being a function (that is, for point identification).

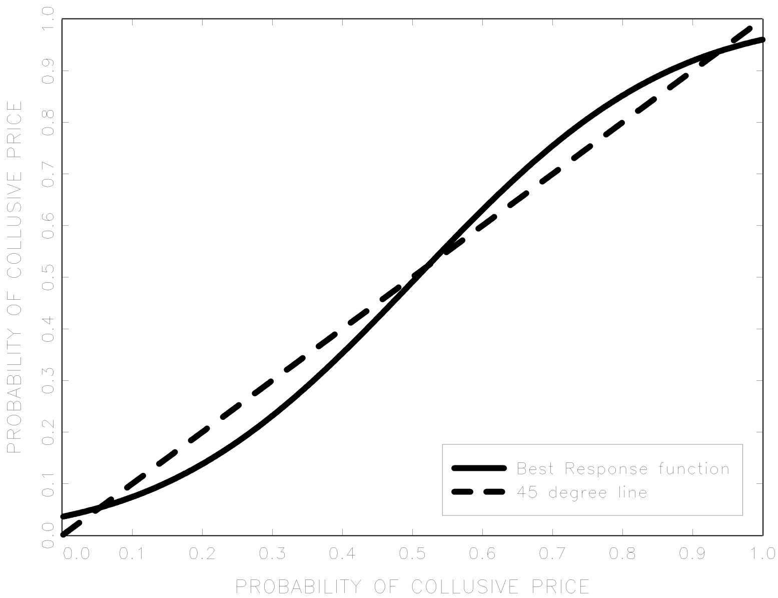

Figure 5.3: Multiple Equilibria

To illustrate the identification of a game with multiple equilibria, we start with a simple binary choice game with identical players and where the equilibrium probability P is implicitly defined as the solution to the condition $P = \Phi(-1.8 + \theta\ P)$, where θ is a structural parameter, and $\Phi(.)$ is the CDF of the standard normal. Suppose that the true value θ_0 is 3.5. It is possible to verify that the set of equilibria associated with θ_0 is $C(\theta_0) = \{\ P^{(A)}(\theta_0) = 0.054$, $P^{(B)}(\theta_0) = 0.551$, and $P^{(C)}(\theta_0) = 0.924\}$. The game has been played M times and we observe players' actions for each realization of the game $\{a_{im} : i, m\}$. Let P_0 be the population probability $\Pr(a_{im} = 1)$. Without further assumptions the probability P_0 can be estimated consistently from the data. For instance, a simple frequency estimator $\hat{P}_0 = (NM)^{-1} \sum_{i,m} a_{im}$ is a consistent estimator of P_0. Without further assumption, we do not know the relationship between population probability P_0 and the equilibrium probabilities in $C(\theta_0)$. If all the sample observations come from the same equilibrium, then P_0 should be one of the points in $C(\theta_0)$. However, if the observations come from different equilibria in $C(\theta_0)$, then P_0 is a mixture of

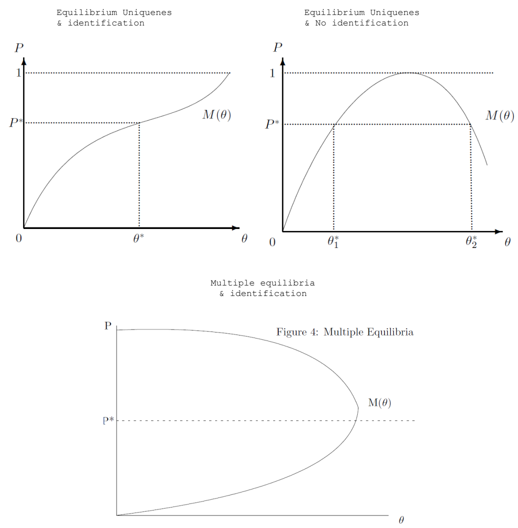

Figure 5.4: Multiple Equilibria versus Identification

the elements in $C(\theta_0)$. To obtain identification, we can assume that every observation in the sample comes from the same equilibrium. Under this condition, since P_0 is an equilibrium associated with θ_0, we know that $P_0 = \Phi(-1.8 + \theta_0\, P_0)$. Given that Φ is an invertible function, we have that $\theta_0 = (\Phi^{-1}(P_0) + 1.8)/P_0$. Provided that P_0 is not zero, it is clear that θ_0 is point identified regardless of the existence of multiple equilibria in the model.

5.5.2 Unobserved market heterogeneity

Some market characteristics affecting firms' profits may not be observable to the researcher. For example, consider local attractions that spur the demand for hotels in a particular geographic location. Observing and controlling for all the relevant attractions are often impossible to the researcher. This demand effect implies that markets with such attractions should have more hotels than those without such attractions but with equivalent observable characteristics. Therefore, without accounting for this type of unobservables, researchers may wrongly conclude that competition boosts profits, or underestimate the negative effect of competition on profits.

Unobserved market heterogeneity usually appears as an additive term (ω_ℓ) in the firm's profit function ($\pi_{i\ell}$) where ω_ℓ is a random effect from a distribution known up to some parameters. The most common assumption (for example, Seim 2006; Zhu and Singh 2009; Datta and Sudhir 2013) is that these unobservables are common across locations in the same local market (that is, $\omega_\ell = \omega$ for all ℓ). Under this assumption, the magnitude of unobserved market heterogeneity matters in terms of whether the firm enters some location in this market, but not the location itself. Orhun (2013) relaxes this assumption by allowing unobserved heterogeneity to vary across locations in the same market.

In a game of complete information, accommodating unobserved market heterogeneity does not require a fundamental change in the estimation process. In a game of incomplete information, however, unobserved market heterogeneity introduces an additional challenge. Consistency of the two-step method requires that the initial nonparametric estimator of firms' entry probabilities in the first step should account for the presence of unobserved market heterogeneity. A possible solution is to use a finite mixture model. In this model, every market's ω_ℓ is drawn from a distribution with finite support. Aguirregabiria and Mira (2007) show how to accommodate such market-specific unobservables into their nested pseudo likelihood (NPL) algorithm. arcidiacono and Miller (2011) propose an expectation-maximization (EM) algorithm in a more general environment. An alternative way to deal with this problem is to use panel data with a reasonably long time horizon. In that way, we can incorporate market fixed effects as parameters to be estimated. This approach is popular when estimating a dynamic game (for example, Ryan 2012; Suzuki 2013). A necessary condition to implement this approach is that every market at least observes some entries during the sample period. Dropping markets with no entries from the sample may generate a selection bias.

5.5.3 Computation

The number of geographic locations, L, introduces two dimensionality problems in the computation of firms' best responses in games of entry with spatial competition. First, in a static game, a multi-store firm's set of possible actions includes all the possible spatial configurations of its store network. The number of alternatives in this set is equal to 2^L, and this number is extremely large even with modest values of L, such as a few hundred geographic locations. Without further assumptions, the computation of best responses becomes impractical. This is an important computational issue that has deterred some authors from accounting for multi-store retailers in their spatial competition models, for example, Seim (2006), or Zhu and Singh (2009), among many others. As we have described in section 2.2.5, two approaches that have been applied to deal with this issue are (1) to impose restrictions that guarantee supermodularity of the game (that is, only two players, no cannibalization effects), and (2) to avoid the exact computation of best responses and use instead inequality restrictions implied by these best responses.

Looking at the firms' decision problem as a sequential or dynamic problem helps also to deal with the dimensionality in the space of possible choices. In a given period of time (for example, year, quarter, or month), we typically observe that a retail chain makes small changes in its network of stores, that is, it opens only a few new stores, or closes only a few existing stores. Imposing these small changes as a restriction on the model implies a very dramatic reduction in the dimension of the action space such that the computation of best responses becomes practical, at least in a 'myopic' version of the sequential decision problem.

However, to fully take into account the sequential or dynamic nature of a firm's decision problem, we also need to acknowledge that firms are forward-looking. In the firm's dynamic programming problem, the set of possible states is equal to all the possible spatial configurations of a store network, and it has 2^L elements. Therefore, by going from a static model to a dynamic forward-looking model, we have just 'moved' the dimensionality problem from the action space into the state space. Recent papers propose different approaches to deal with this dimensionality problem in the state space.

arcidiacono et al. (2013) present a continuous-time dynamic game of spatial competition in a retail industry and propose an estimation method of this model. The continuous-time assumption eliminates the curse of dimensionality associated with integration over the state space. Aguirregabiria and Vicentini (2016) propose a method of spatial interpolation that exploits the information provided by the (indirect) variable profit function.

5.6 Further topics

Spillovers between different retail sectors. Existing applications of games of entry and spatial competition in retail markets concentrate on a single retail industry. However, there are also interesting spillover effects between different retail industries. Some of these spillovers are positive, such as good restaurants making a certain neighborhood more attractive for shopping. There are also negative spillovers effects through land prices. Retail sectors with high value per unit of space (for example, jewelry stores) are willing to pay higher land prices than supermarkets that have low markups and are intensive in the use of land. The consideration and measurement of these spillover effects are interesting in and of themselves, and they can help to explain the turnover and reallocation of industries in different parts of a city. Relatedly, endogenizing land prices would also open the possibility of using these models for the evaluation of specific public policies at the city level.

Richer datasets with store level information on prices, quantities, inventories. The identification and estimation of competition effects based mainly on data of store locations have been the rule more than the exception in this literature. This approach typically requires strong restrictions in the specification of demand and variable costs. The increasing availability of datasets with rich information on prices and quantities at the product and store level should create a new generation of empirical games of entry and spatial competition that relax these restrictions. Also, data on store characteristics such as product assortments or inventories will enable the introduction of these important decisions as endogenous variables in empirical models of competition between retail stores.

Measuring spatial pre-emption. So far, all the empirical approaches to measure the effects of spatial pre-emption are based on the comparison of firms' actual entry with firms' behavior in a counterfactual scenario characterized by a change in either (1) a structural parameter (for example, a store exit value), or (2) firms' beliefs (for example, a firm believes that other firms' entry decisions do not respond to this firm's entry behavior). These approaches suffer from the serious limitation in which they do not only capture the effect of pre-emption, but also other effects. The development of new approaches to measure the pure effect of pre-emption would be a methodological contribution with relevant implications in this literature.

Geography. Every local market is different in its shape and its road network. These differences may have important impacts on the resulting market structure. For example, the center of a local market may be a quite attractive location for retailers when all highways go through there. However, it may not be the case anymore when highways encircle the city center (for example, Beltway in Washington DC). These differences may affect retailers' location choices and the degree of competition in an equilibrium. The development of empirical models of competition in retail markets that incorporate, in a systematic way, these idiosyncratic geographic features will be an important contribution in this literature.