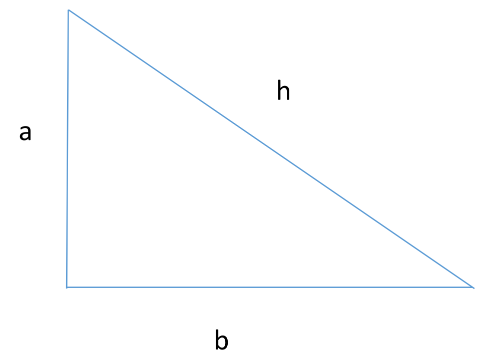
Discussion Board Questions:

<u>Week 1</u>

Imagine a right triangle with area $25\mathrm{m}^2$ How would you express the hypotenuse h as a function of its perimeter P?



Area of a triangle: $\frac{1}{2}(base)(height) = 25$

$$\frac{1}{2}ba = 25 \rightarrow ba = 50$$

Perimeter of a triangle is $P = a + b + h$

$$P - h = a + b$$

Squaring both sides:

$$(P - h)^2 = (a + b)^2$$

$$p^2 + 2Ph + h^2 = a^2 + 2ab + b^2$$

$$p^2 + 2Ph + h^2 = 2(50) + h^2$$

Note that $h^2 = a^2 + b^2$ from Pythagorean theorem

$$p^2 + 2Ph + h^2 = 100 + h^2$$

$$p^2 + 2Ph = 100$$

$$h(P) = \frac{100 - P^2}{2P}$$

<u>Week 2 –</u>

Solve the inequality for x, assuming that a, b, and c are positive constants: $a(bx - c) \geq bc$

$$abx - ac - bc \geq 0$$

$$abx - c(a + b) \geq 0$$

$$abx \geq c(a + b)$$

$$x \geq \frac{c(a + b)}{ab}$$

$$[\frac{c(a + b)}{ab}, \infty)$$

<u>Week 3 –</u>

Show that $f(x) = \ln(x + \sqrt{x^2 + 1})$ is an odd function.
What would the inverse of this function be?

$$f(-x) = \ln(-x + \sqrt{(-x)^2 + 1})$$

$$\ln(-x+\sqrt{(x)^2+1})$$

We could rationalize the numerator to get

$$\ln[-x+\sqrt{(x)^2+1}\times\frac{x+\sqrt{(x)^2+1}}{x+\sqrt{(x)^2+1}}]$$

$$\ln[\frac{-x^2+x^2+1}{x+\sqrt{x^2+1}}]$$

$$\ln\left(\frac{1}{x+\sqrt{x^2+1}}\right)$$

$$\ln\left(x+\sqrt{x^2+1}\right)^{-1}=-\ln\left(x+\sqrt{x^2+1}\right)=-f(x)$$

This is an odd function

In finding the inverse, let $y=\ln(x+\sqrt{x^2+1})$

$x=\ln(y+\sqrt{y^2+1})$

$$e^x=y+\sqrt{y^2+1}$$

$$e^x-y=\sqrt{y^2+1}$$

$$(e^x-y)^2=\left(\sqrt{y^2+1}\right)^2$$

$$e^{2x}-2ye^x+y^2=y^2+1$$

$$e^{2x}-2ye^x=1$$

$$\frac{e^{2x}-1}{2e^x}=y$$

<u>Week 4 –</u>

Show that $\tan\left(\frac{u}{2}\right)=\frac{1-\cos u}{\sin u}$

Method 1: We know that $\sin^2 a=\frac{1-\cos 2a}{2}$, therefore $\sin^2\left(\frac{u}{2}\right)=\frac{1-\cos u}{2}\rightarrow 2\sin^2\left(\frac{u}{2}\right)=1-\cos u$

$$\sin u=2\sin\frac{u}{2}\cos\frac{u}{2}$$

Putting it all together, we get:

$$\frac{2\sin^2\frac{u}{2}}{2\sin\frac{u}{2}\cos\frac{u}{2}}=\frac{\sin\frac{u}{2}}{\cos\frac{u}{2}}=\tan(\frac{u}{2})$$

Method 2: We know that $\sin^2 a = \frac{1-\cos 2a}{2}$, therefore $\sin^2\left(\frac{u}{2}\right) = \frac{1-\cos u}{2} \to sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$

Likewise, $cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}}$

$$\tan(\frac{u}{2}) = \pm\sqrt{\frac{1-\cos u}{1+\cos u}}$$

$$\tan(\frac{u}{2}) = \pm\sqrt{\frac{1-\cos u}{1+\cos u} \times \frac{1-\cos u}{1-\cos u}}$$

$$\tan(\frac{u}{2}) = \pm\sqrt{\frac{(1-\cos u)^2}{1-\cos^2 u}}$$

$$\tan(\frac{u}{2}) = \pm\frac{|1-\cos u|}{|\sin u|}$$

Now, $1-\cos u$ is nonnegative for all values of u. it is also true that $\sin u$ and $\tan\frac{u}{2}$ always have the same sign. It follows that

$$\tan(\frac{u}{2}) = \frac{1-\cos u}{\sin u}$$