Online Mathematics Preparedness Course
Quiz 3 – Solutions

1. Solve for x: $\ln(\ln(\ln(x))) = y$

$$\ln(\ln(\ln(x))) = y$$

$$\ln(\ln(x)) = e^y$$

$$\ln(x) = e^{e^y}$$

$$x = e^{e^{e^y}}$$

2. Solve for x: $e^{3-2x} = 4$

$$\ln(e^{3-2x}) = \ln 4$$

$$(3-2x)\ln e = \ln 4$$

$$(3-2x) = \ln 4$$

$$3 - \ln 4 = 2x$$

$$\frac{3-\ln 4}{2} = x$$

$$\frac{3-\ln 4}{2} = x$$

Or

$$\frac{3-\ln 2^2}{2} = x \rightarrow \frac{3}{2} - \ln 2$$

3. Expand the expression: $\log x(x-1)^2$

$$\log x(x-1)^2 = \log x + \log(x-1)^2$$
$$\log x + 2\log(x-1)$$

4. What is the domain of $f(x) = \ln(x - x^2)$
We have $x - x^2 > 0$

$$x(1-x) > 0$$

Zeroes are at $x = 0, x = 1$

	$(-\infty, 0)$	$(0,1)$	$(1, \infty)$
x	-	+	+
$1-x$	+	+	-
$x(1-x)$	-	+	-

Domain: $(0,1)$

5. The inverse of $f(x) = \frac{2^x}{1+2^x}$

$$y = \frac{2^x}{1+2^x}$$

$$x = \frac{2^y}{1+2^y}$$

$$x(1+2^y) = 2^y$$
$$x + x2^y = 2^y$$
$$x = 2^y - x2^y$$
$$x = 2^y(1-x)$$
$$\frac{x}{1-x} = 2^y$$
$$\ln(\frac{x}{1-x}) = \ln 2^y$$

$$\ln(\frac{x}{1-x}) = y \ln 2$$

$$\frac{\ln(\frac{x}{1-x})}{\ln 2} = y$$

$$\ln_2\left(\frac{x}{1-x}\right) = y$$

6. Given that $f(x) = \ln x$; $g(x) = \frac{1}{1+x}$

a) Find $f\big(g(x)\big)$ and its domain

$$f\left(\frac{1}{1+x}\right) = \ln\left(\frac{1}{1+x}\right)$$

Domain is $\frac{1}{1+x} > 0; 1 + x > 0 \rightarrow x > -1$

Domain of $g(x)$ is $x \neq -1$

$$(-1, \infty)$$

b) Find $g(f(x))$ and its domain

$$g(\ln x) = \frac{1}{1+\ln x}$$

Domain of this function is $1 + \ln x \neq 0; \ln x \neq -1 \rightarrow x \neq e^{-1} \rightarrow x \neq \frac{1}{e}$

Domain of $f(x)$ is $(0, \infty)$

Domain is $(0, \frac{1}{e}) \cup (\frac{1}{e}, \infty)$

7. The reference (principal) angle for $780°$ is $60°$ and is in the I quadrant

8. Find the length of an arc of a circle with radius 10 m that subtends a central angle of $30°$

$$s = r\theta$$
$$s = \frac{(10)30\pi}{180} = \frac{5\pi}{3} m$$

9. Prove the identity: $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$

   LHS:

$$\frac{2\tan x}{1+\tan^2 x} = \frac{\frac{2\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}}$$

$$\frac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}}$$

$$\frac{\frac{2\sin x}{\cos x}}{\frac{\cos^2 x+\sin^2 x}{\cos^2 x}}$$

$$\frac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}} = \frac{2\sin x}{\cos x} \times \frac{\cos^2 x}{1} = 2\sin x cos\, x = \sin 2x = RHS$$