Online Mathematics Preparedness Course
Quiz 1 – Solutions

1. The set of whole numbers included in the interval $(-1,1)\ U\ [2,7]$ are $0, 2, 3, 4, 5, 6, 7$ (note that the open brackets indicate that the numbers -1 and 1 are not be included.

2. For what values of a does the following polynomial $x^2 - 16x + a$ have no roots?
   We need to find the discriminant $D = b^2 - 4ac$ as that determines how many roots a quadratic equation can have. Having $D < 0$ would mean that there would be no solutions
$$D < 0 \rightarrow (-16)^2 - 4(1)(a) < 0$$
$$256 - 4a < 0$$
$$256 < 4a$$
$$\frac{256}{4} < a$$
$$64 < a$$
$$(64, \infty)$$

3. $\frac{x^2+1}{x+1}$ <u>does not</u> simplify to x

4. Define $|ax + b|$ as an absolute value function

   We have $|ax+b| = \begin{cases} ax+b & if\ ax+b \geq 0 \rightarrow x \geq -\frac{b}{a} \\ -(ax+b) & if\ ax+b < 0 \rightarrow x < -\frac{b}{a} \end{cases}$

5. Let $A = [-6,1), B = Z, C = \{0\}$, and $D = [-5,5]$. Express $(A \cap D) \cap C$ in an interval notation.
$$(A \cap D) = [-5,1)$$
$$(A \cap D) \cap C = [-5,1) \cap \{0\} = \{0\}$$

6. Given the inequality, $|x-3| + |x+2| < 11$, pick the option that includes all the intervals that are of interest
   We have $|x-3| = \begin{cases} x-3 & if\ x \geq 3 \\ -(x-3) & if\ x < 3 \end{cases}$

   We have $|x+2| = \begin{cases} x+2 & if\ x \geq -2 \\ -(x+2) & if\ x < -2 \end{cases}$

   Intervals of interest are: $(-\infty, -2), (-2,3), (3, \infty)$

7. Given the inequality, $|x-3|+|x+2|<11$, give the solution set that satisfies the inequality

$(-\infty,-2)$	$(-2,3)$	$(3,\infty)$
$-(x-3)-(x+2)<11$ $-x+3-x-2<11$ $-2x+1<11$ $-2x<10$ $x>-5$ Okay, as it is within the given interval	$-(x-3)+(x+2)<11$ $-x+3+x+2<11$ $5<11$ True mathematically	$(x-3)+(x+2)<11$ $x-3+x+2<11$ $2x-1<11$ $2x<12$ $x<6$ Works

Combining the three cases we see that the inequality is satisfied when $-5<x<6$.
The solution set is $(-5,6)$

8. Factor: $3x^{\frac{3}{2}}-9x^{\frac{1}{2}}+6x^{-\frac{1}{2}}$

$$3x^{\frac{1}{2}}(x-3+\frac{2}{x})$$

9. Solve: $2x(4-x)^{-\frac{1}{2}}-3\sqrt{4-x}=0$

$$2x(4-x)^{-\frac{1}{2}}-3(4-x)^{\frac{1}{2}}=0$$
$$(4-x)^{-\frac{1}{2}}(2x-3(4-x))=0$$
$$(4-x)^{-\frac{1}{2}}(2x-12+3x)=0$$
$$(4-x)^{-\frac{1}{2}}(5x-12)=0$$
$$5x=12$$
$$x=\frac{12}{5}, \text{provided that } x\neq 4$$

10. Simplify:

$$\frac{x^2}{x-3}-\frac{-2x+15}{x-3}$$
$$\frac{x^2+2x-15}{x-3}$$
$$\frac{(x-3)(x+5)}{x-3}=x+5, x\neq 3$$

11. In factoring $-4x+12x^2+3$, the product and sum respectively are: $36, -4$
It is integral that you rearrange the polynomial in ax^2+bx+c, as product is $a\times c$, and sum is b

12. Given the expression $x^5y^2-xy^6$, the fully factored version is

$$x^5y^2-xy^6=xy^2(x^4-y^4)=xy^2(x^2-y^2)(x^2+y^2)=xy^2(x+y)(x-y)(x^2+y^2)$$

13. In simplifying $\frac{\frac{x}{y}+1}{1-\frac{y}{x}}$, there are no common factors that are canceled between the numerator and the denominator

$$\frac{\frac{x}{y}+1}{1-\frac{y}{x}} = \frac{\frac{x+y}{y}}{\frac{x-y}{x}} = \frac{x+y}{y} \times \frac{x}{x-y} = \frac{x(x+y)}{y(x-y)}$$

True. No terms were cancelled, as there are no common factors