Mathematics Skill Development - Module 3
Assessment Test

The following questions are will evaluate the student's understanding of materials covered in Module 3. Topics Include Euclidean distance between two points, slope-intercept representations of lines, intersection points between two curves and identification of various curves as either circles, ellipses, hyperbolas or parabolas.

1. Let $A = (3, 4)$. Find the point(s) B on the line $y = 10$ for which the distance between A and B is 7 units.
Solution:

Denoting the points $A = (x_1, y_1) = (3, 4)$ and $B = (x_2, y_2) = (x_2, 10)$, we use the Euclidean distance formula to equate

$$\begin{aligned}
7 = D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
7^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\
7^2 &= (x_2 - 3)^2 + (10 - 4)^2 \\
13 &= (x_2 - 3)^2 \\
x_2 - 3 &= \pm\sqrt{13} \\
x_2 &= -3 \pm \sqrt{13}.
\end{aligned}$$

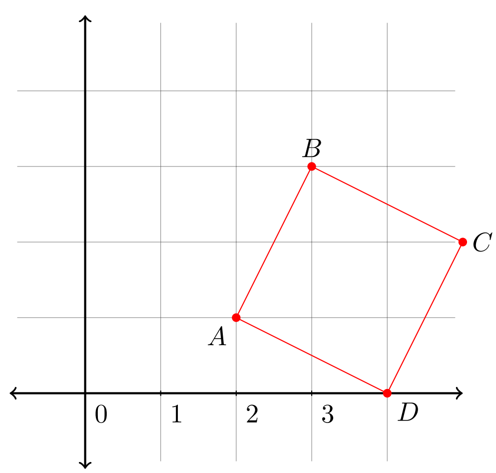
Thus, the points on $y = 10$ which are of distance 7 units from the point$A = (3, 4)$ are given by $B = (-3 + \sqrt{13}, 10), (-3 - \sqrt{13}, 10)$.

$\square$

2. Compute the area of the rectangle in the Euclidian plane whose vertices are $A = (2, 1), B = (3, 3), C = (5, 2)$ and $D = (4, 0)$.

Solution:

To find the area of the rectangle, we need to find the lengths of two of its adjacent sides. To check that the two lengths we select are indeed adjacent, we sketch the rectangle:



For example, AB and AD are adjacent and we can use the Euclidean distance formula to compute the lengths of AB and AD. We use the notation that $|AB|$ represents the length of the line segment AB. Computing

$$|AB| = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{2^2 + q^2} = \sqrt{5}$$

and

$$|AD| = \sqrt{(0-1)^2 + (4-2)^2} = \sqrt{1^2 + 2^2} = \sqrt{5},$$

we find that this rectangle is in fact a square with side length $\sqrt{5}$. Its area is given by

$$|AB| \cdot |AD| = \sqrt{5} \cdot \sqrt{5} = 5.$$

□

3. Find the points of intersection between the line $y = x + 1$ and the circle of radius 2 which is centered at the point (1,1).

Solution:

The equation of the circle of radius 2 which is centered at (1,1) is given by

$$(x-1)^2 + (y-1)^2 = 2^2.$$

To find the points of intersection, we substitute the equation $y = x + 1$ and solve for x

$$\begin{aligned} (x-1)^2 + ((x+1)-1)^2 &= 2^2 \\ x^2 - 2x + 1 + x^2 &= 4 \\ 2x^2 - 2x - 3 &= 0 \end{aligned}$$

This is a quadratic equation which can be solved using the quadratic formula.

$$\begin{aligned} x &= \frac{2 \pm \sqrt{4 - 4(2)(-3)}}{4} \\ &= \frac{2 \pm \sqrt{28}}{4} \\ &= \frac{1 \pm \sqrt{7}}{2}. \end{aligned}$$

These are the x coordinates of the 2 intersection points. To find the corresponding y coordinates, we substitute these values into the equation $y = x + 1$ to obtain.

$$\left(\frac{1+\sqrt{7}}{2}, \frac{3+\sqrt{7}}{2}\right) \text{ and } \left(\frac{1-\sqrt{7}}{2}, \frac{3-\sqrt{7}}{2}\right).$$

□

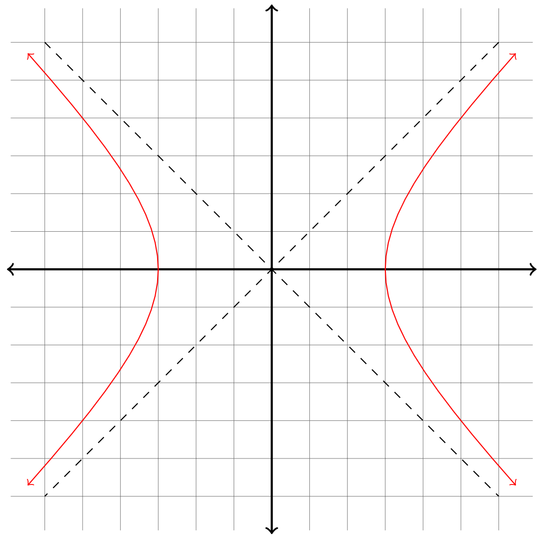
4. Identify and sketch the curve given by $3y^2 = 3x^2 - 27$.

Solution:

Rearranging this equation to the standard form $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$, we have

$$\begin{aligned} 3y^2 &= 3x^2 - 27 \\ 3x^2 - 3y^2 &= 27 \\ \frac{x^2}{9} - \frac{y^2}{9} &= 1 \end{aligned}$$

which is the equation of a hyperbola. This hyperbola has a horizontal transverse axis, vertices at $(\pm a, 0) = (\pm 3, 0)$ and asymptotes $y = \pm \frac{b}{a} x = \pm x$. We sketch this curve:



$\square$