Mathematics Skill Development - Module 4
Assessment Test

The following questions will evaluate the student's ability to classify basic functions, find the domain and range of a given function and sketch algebraic transformations of functions.

1. Determine if the following are functions, if so, state their domain and classify them as either a power function, root function, polynomial function, rational function, logarithmic function, exponential function or algebraic function.
(a) $f(x) = \frac{x^2-4}{x-2}$
(b) $2x^2 - 2y^2 = 8$
(c) $f(x) = \sqrt{x} - \sqrt{-x}$.

Solution:

(a) This is a rational function whose numerator can be factored as $x^2 - 4 = (x-2)(x+2)$. If $x \neq 2$, we can cancel out the term $x - 2$ expression from the numerator and denominator to get $f(x) = x + 2$ whenever $x \neq 2$. When $x = 2$, we cannot make sense of the expression defining f since we cannot divide by 0. Therefore f is a function with domain $\{x \in \mathbb{R} : x \neq 2\}$.

□

(b) We may rewrite this expression in the standard form

$$\left(\frac{x}{2}\right)^2 - \left(\frac{y}{2}\right)^2 = 1,$$

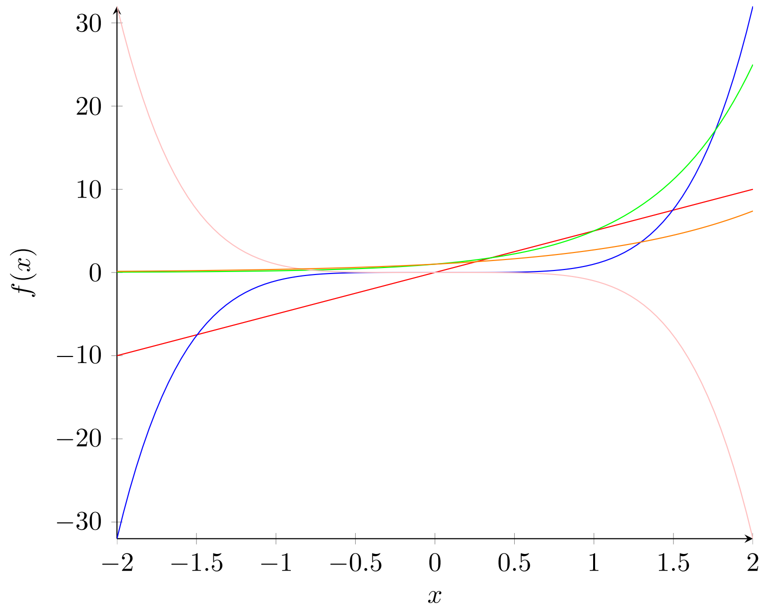
which is a hyperbola. Thus, it does not pass the vertical line test and is not a function. Alternatively, if $x = 3$, then the values $y = \pm\sqrt{5}$ both satisfy the relation. That is, there are two y-values for a given x-value and this cannot be a function.

□

(c) This is the difference of two root functions and is algebraic. The function $\sqrt{x}$ is well-defined only when $x \geq 0$, while $\sqrt{-x}$ is defined only when $x \leq 0$. The domain of f is the intersection of these domains $\{x \leq 0\} \cap \{x \geq 0\} = \{x = 0\}$.

□

2. Match each equation with its graph. Explain your choices.
(a) $f(x) = 5x$
(b) $f(x) = 5^x$
(c) $f(x) = x^5$
(d) $f(x) = (-x)^5$
(e) $f(x) = e^x$.



□

Solution:

The red function is linear and hence must be $f(x) = 5x$. The green and orange functions grow exponentially and tend to zero as x tends to the negative axis. Thus, these functions must be either $f(x) = 5^x$ or $f(x) = e^x$. Since the growth rate is faster for the green function, we conclude that it is $f(x) = 5^x$ and hence the orange function must be $f(x) = e^x$. Only the pink and blue functions remain with possible choices of $f(x) = x^5$ and $f(x) = (-x)^5$ (which are vertical reflections of one another). Note that if $x > 0$, then $x^5 > 0$ while $(-x)^5 < 0$, therefore the blue function is $f(x) = x^5$ and the pink function is $f(x) = (-x)^5$.

□

3. Let $f(x)$ be an arbitrary function. How would you interpret the transformation $f(-x)$? How about $-f(-x)$? Can you think of a function which satisfies the relationship

$$-f(-x) = f(x)?$$

How about the relationship

$$f(-x) = f(x)?$$

Solution:

Given a function $f(x)$, we may interpret $f(-x)$ as a horizontal reflection of f about the y-axis. Then $-f(-x)$ would be a horizontal reflection followed by a vertical reflection.

A function which satisfies the relationship $-f(-x) = f(x)$ is called an *odd function*. One such example is $f(x) = x^3$.

A function which satisfies the relationship $f(-x) = f(x)$ is called an *even function*. One such example is $f(x) = x^2$.

□