Mathematics Skill Development - Module 2
Assessment Test

The following questions will evaluate the student's understanding of the material in module 2. Topics include radical expressions, inequalities, absolute value equations and the quadratic formula.

1. Simplify $\sqrt{32}+\sqrt{18}$.

Solution:

$$\begin{aligned}\sqrt{32}+\sqrt{18} &= \sqrt{4^2\cdot 2}+\sqrt{3^2\cdot 2}\\ &= 4\sqrt{2}+3\sqrt{2}\\ &= 7\sqrt{2}.\end{aligned}$$

□

2. Solve $-3 < \frac{x-1}{3} - \frac{2x-1}{5} \leq 2$.

Solution:

$$-3 < \frac{x-1}{3} - \frac{2x-1}{5} \leq 2$$
$$-45 < 5x-5-(6x-3) \leq 30$$
$$-45 < -x-2 \leq 30$$
$$-30 \leq x+2 < 45$$
$$-32 \leq x < 43.$$

□

3. Solve

$$x-4=\frac{3}{x}+2.$$

Solution:

We multiply throughout by x then compute

$$\begin{aligned}x-4 &= \frac{3}{x}+2\\ x^2-4x &= 3+2x\\ x^2-6x-3 &= 0.\end{aligned}$$

From here, we invoke the quadratic formula to obtain

$$x=\frac{6\pm\sqrt{36-4\cdot(-3)}}{2}=3\pm 2\sqrt{3}.$$

□

4. Solve $|x^2 - 1| = 1$.

Solution:

If $x^2 - 1 \geq 0$, then $|x^2 - 1| = x^2 - 1$ and we solve

$$\begin{aligned} x^2 - 1 &= 1 \\ x^2 &= 2 \\ x &= \pm\sqrt{2}. \end{aligned}$$

Both $x = +\sqrt{2}$ and $x = -\sqrt{2}$ are possible solutions to the original equations. We check that these numbers satisfy our ansatz: $(\pm\sqrt{2})^2 - 1 = 2 - 1 > 0$ and conclude that both numbers are solutions.

If $x^2 - 1 < 0$, then $|x^2 - 1| = -(x^2 - 1)$ and we solve

$$\begin{aligned} -x^2 + 1 &= 1 \\ -x^2 &= 0 \\ x &= 0. \end{aligned} \tag{1}$$

We check that $x = 0$ satisfies our ansatz: $0^2 - 1 = -1 < 0$. Therefore $x = 0$ is another solution of this equation.

To summarize, this equation has 3 solutions $x = +\sqrt{2}$, $x = -\sqrt{2}$ and $x = 0$.

$\square$

5. Solve $4 + \sqrt{x + 2} = x$.

Solution: We isolate the square root and square both sides to obtain

$$\begin{aligned} 4 + \sqrt{x+2} &= x \\ \sqrt{x+2} &= x - 4 \\ (\sqrt{x+2})^2 &= (x-4)^2 \\ x + 2 &= x^2 - 8x + 16 \\ 0 &= x^2 - 9x + 14 \\ 0 &= (x-7)(x-2). \end{aligned}$$

Therefore $x = 7$ and $x = 2$ are possible solutions of the original equation. However, when we perform operations such as squaring both sides of an equation, we must be to careful to check that we have not introduced spurious solutions. In this case, when $x = 7$:

$$4 + \sqrt{7+2} = 4 + 3 = 7,$$

while when $x = 2$,

$$4 + \sqrt{2+2} = 4 + 2 = 6 \neq 4.$$

Therefore only $x = 7$ solves the original equation.

$\square$