Industrial Organization II (ECO 3901)
Winter 2022. Victor Aguirregabiria

Final (Take Home) Exam
Due on Monday, April 11, 2022, before 10:00am

INSTRUCTIONS.
1. Please, answer all the questions.
2. Write your answers electronically in a word processor.
3. Convert the document with your answers to PDF format and submit the PDF online via Quercus.
4. You should submit your exam before 10:00am on Monday, April 11th, 2022.
5. The exam should be written individually.

The total number of marks is 100.

MODEL / CONTEXT

- Consider a dynamic duopoly game of price competition. Time is discrete and indexed by t, firms are indexed by $j \in \{1, 2\}$. These firms compete in M separate geographic markets, that we indexed by m.

- Consumer demand in every market is static and it has a standard Logit structure. This logit demand depends only on firms' prices, p_{1mt} and p_{2mt}, and firms' (local market) qualities represented by variables ξ_{1mt} and ξ_{2mt}. These qualities follow a first order Markov process, e.g., a linear VAR(1) process.

- In each market, a firm can charge only two possible prices: either a high price (or regular price), or a low price (or promotion price). The values for the "low" and "high" prices can be different for the two firms, but they are constant over time and across markets: (p_1^{hi}, p_1^{lo}) and (p_2^{hi}, p_2^{lo}). That is, these target prices are set by each firm's headquarters, but every period the local market managers decide which of the two prices to set in their respective markets.

- This pricing decision is dynamic because there is a fixed cost of changing the price with respect to previous period price (i.e., a menu cost). The game is of incomplete information because there is a firm-market-specific, zero-mean i.i.d. shock in the menu cost, say ε_{jmt}, and this shock is private information of each firm. We assume that ε_{1mt} and ε_{2mt} have a logistic distribution. The time-invariant component of the menu cost can vary across firms, but it is assumed constant across markets: θ_1^{menu} and θ_2^{menu}.

- Marginal costs, c_{1mt} and c_{2mt}, do not depend on the amount of output, but they vary over time and across geographic markets. Similarly as the case of qualities, these costs also follow a linear VAR(1) process.
- The researcher has a panel dataset $\{p_{1mt}, p_{2mt}, q_{1mt}, q_{2mt}, c_{1mt}, c_{2mt} : t = 1, 2, ..., T; m = 1, 2, ..., M\}$, where q represents output, M is large, and T is short.

- **QUESTION 1. [10 points]**

  (1.1) Describe the logit demand model.

  (1.2) Explain how to estimate the price coefficient in this model.

  (1.3) Propose an instrument given these data and model.

  (1.4) Explain how to recover estimates for the unobserved qualities ξ_{1mt} and ξ_{2mt}.

  For the rest of this Exam, we treat ξ_{1mt} and ξ_{2mt} as observable variables, and the parameter(s) in the demand system as known.

- **QUESTION 2. [10 points]** Describe formally the dynamic game.

  (2.1) Write the equation for the profit function, and explain the different elements, variables and parameters.

  (2.2) Which are the state variables of the model? Which state variables are observable to the researcher? Describe the stochastic process of these state variables.

- **QUESTION 3. [20 points]** Describe a Markov Perfect Equilibrium (MPE) in this dynamic game.

  (3.1) First, describe a MPE of this game in terms of pricing strategies.

  (3.2) Now, describe a MPE of this game in terms of Conditional Choice Probabilities (CCPs) of charging a high price.

- **QUESTION 4. [20 points]** Estimation.

  (4.1) Explain how to estimate the VAR(1) models for qualities and unit costs.

(4.2) Describe in detail a two-step estimation method for the menu cost parameters.

(4.3) Describe in detail a full solution estimation method for the menu cost parameters.

- **QUESTION 5. [20 points]** Counterfactuals.

  (5.1) Describe in detail a counterfactual experiment that measures how the difference in menu costs $\theta_1^{menu} - \theta_2^{menu} > 0$ contributes to explain the difference in profitability / value of the two firms.

  (5.2) Now, suppose that the dataset includes many time periods (i.e, T is large) for which the same structure applies, such that we can estimate with enough precision menu cost parameters separately for each geographic market m: $\{\theta_{1m}^{menu}, \theta_{2m}^{menu} : m = 1, 2, ..., M\}$. Suppose that managers at firm's the headquarters know the actual menu cost and that this cost does not vary across geographic markets, such that the estimated market variation can be interpreted as local managers' behavioral biases. Describe a counterfactual experiment that measures the value for a firm of centralizing at the headquarters the pricing decisions for all the local markets. Note that in this centralized decision, each local market has its own price. You can assume that the competing firm maintains its decentralized pricing.

A VARIATION OF THE MODEL: DYNAMICS IN DEMAND

- Suppose that we modify the previous model/application to incorporate dynamics in consumer demand. More specifically, the source of dynamics in demand is that the product is storable. A consumer buys the product for inventory, stores it at home, consumes it over time, and comes back to buy again the product only when its inventory is low enough.

- We represent product storability in the demand system by considering different groups of consumers depending on the time duration since their last purchase of the product. Let d be the variable that represents time duration since last purchase. This variable belongs to set $d \in \{1, 2, ..., D\}$, where D is finite. Let $s_{jmt}(d)$ be the market share of product j in market m at period t among consumers with time duration since last purchase equal to d. We consider the following logit demand system:

$$s_{jmt}(d) \;=\; \frac{\exp\left(-\alpha\ p_{jmt} + \delta\ d + \xi_{jmt}\right)}{1 + \exp\left(-\alpha\ p_{1mt} + \delta\ d + \xi_{1mt}\right) + \exp\left(-\alpha\ p_{2mt} + \delta\ d + \xi_{2mt}\right)}$$

  where α and δ are parameters. The total market share for form j in market m at period t is:

$$s_{jmt} \;=\; \sum_{d=1}^{D} w_{mt}(d)\ s_{jmt}(d)$$

  where $w_{mt}(d)$ is the proportion of consumers with time duration d.

- Now, the dataset includes also information about the variables $w_{mt}(d)$ and $s_{jmt}(d)$. That is, the researcher has a panel dataset $\{p_{1mt}, p_{2mt}, s_{1mt}(d), s_{2mt}(d), w_{mt}(d), c_{1mt}, c_{2mt} : t = 1, 2, ..., T; m = 1, 2, ..., M; d = 1, 2, ..., D\}$.

- For the rest of the questions, suppose that demand parameters α and δ have been estimated, the demand system is treated as known, and the qualities ξ_{1mt} and ξ_{2mt} are treated as observable variables.

- **QUESTION 6. [20 points]** Dynamics in demand.

  (6.1) Which are the state variables of the model? Which state variables are observable to the researcher?

  (6,2) Describe in detail the transition rule for the variables $w_{mt}(d)$.

  (6.3) Describe a Markov Perfect Equilibrium (MPE) in this dynamic game.

  (6.4) Intuitively, which are the implications of product storability for pricing dynamics in this game?

END OF FINAL EXAM