ECO 3901
EMPIRICAL INDUSTRIAL ORGANIZATION
Lecture 5
Dynamic Spatial Competition

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Lecture 5: Dynamic Spatial Competition Outline

1. **Introduction: Firms' spatial location**
2. **Static games of location choice**
3. **Holmes (2011) on the spatial diffusion of Walmart**

1. Introduction
Firms' Spatial Location

Firms' Spatial Location: Introduction

- We study a firm's decision of **where to locate its stores / plants**.

- Different factors can play an important role:

  - **Demand**: what is the consumer traffic at different locations.

  - **Location-specific costs**. Rental price.

  - **Location of competitors**. Spatial differen. Positive spillovers.

  - **Location of own stores**. Cannibalization. Econ of scope/density.

Beyond Geographic Space

- With slight changes, models for the geographic location of stores can be applied to study **firms' decisions on product design**.
- We need to replace the 2D (or 3D) geographic space with the KD **space of product characteristics**, and define the relevant distance in that space.
- Similar factors play an important role in firms' product design:
  - Consumer demand for a bundle of product characteristics.
  - Cost of entry in a bundle of characteristics.
  - Location of competitors in the product space.
  - Cannibalization of own preexisting products.
  - Economies of scope in producing similar products,

Empirical Questions

- **Spatial differentiation**. How do profits increase with distance to competitors?
- **Cannibalization**. To what extend a multi-product firm is concerned with competition between its own products?
- **Economies of scope**. Do the costs of a new store/product decline with the number of other stores/products the firm has?
- **Economies of density**. Do the costs of a new store/product decline with the spatial proximity to other stores/products the firm has?
- **Entry preemption / deterrence motives**. Firms may have incentives to proliferate stores / products in a region to deter/preempt the entry of competitors.

2. Static Games of Firms' Location Choice

Space of feasible store locations (the city)

- A market (city) is a set, for instance **a rectangle**, in the space $\mathbb{R}^2$.
- We divide the city/rectangle into L small squares, each one with its center.
- Each of these squares is a submarket (or neighborhood, or location).
- A market/city can have hundreds or thousands of these submarkets/locations.
- We index these locations by $\ell =\in \{1, 2, ..., L\}$

The city: Space of feasible store locations

		Longitude							
		1	2	3	4	5	6	7	8
	1	●	●	●	●	●	●	●	●
	2	●	●	●	●	●	●	●	●
	3	●	●	●	●	●	●	●	●
Latitude	4	●	●	●	●	●	●	●	●
	5	●	●	●	●	●	●	●	●
	6	●	●	●	●	●	●	●	●
	7	●	●	●	●	●	●	●	●
	8	●	●	●	●	●	●	●	●

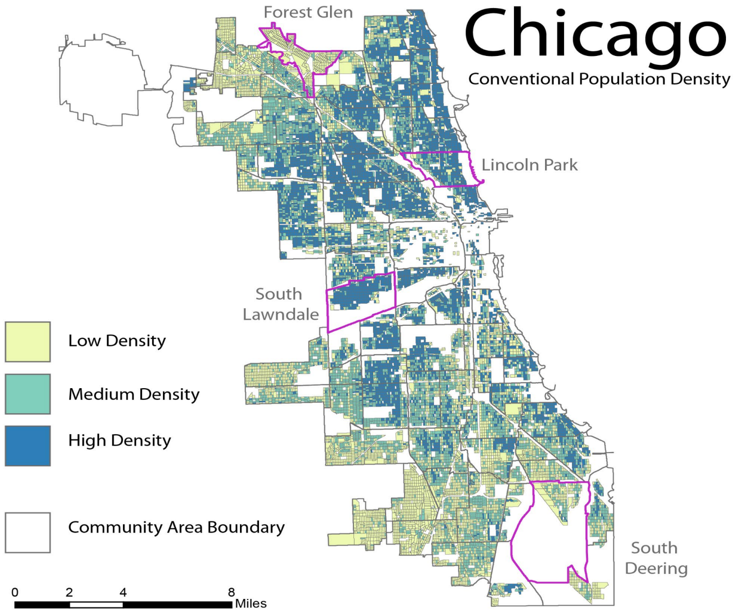
Space of feasible store (product) locations

- Each location has some exogenous characteristics that can affect demand and costs of a firm in that location:

  - Population; demographics; rental prices.

- Exogenous characteristics of location ℓ: vector $\mathbf{x}_\ell$.

- Therefore, we can see a market as a landscape of the characteristics $\mathbf{x}_\ell$ over the L locations.


Chicago
Conventional Population Density
Forest Glen
Lincoln Park
South Lawndale
South Deering
Low Density
Medium Density
High Density
Community Area Boundary
0
2
4
8
Miles

Model: Firms

- There are N potential entrants indexed by $i \in \{1, 2, ..., N\}$ that can open stores in this market, e.g., WalMart and Kmart.

- The decision variable for firm i is:

$$\boldsymbol{a}_i = (a_{i1},\ a_{i2},\ ...,\ a_{iL})$$

where $a_{i\ell} = 1\{\text{Firm } i \text{ opens a store in location } \ell\} \in \{0, 1\}$

- $\boldsymbol{a}_i = \mathbf{0}$ represents "no entry".

Model: Profit function

- We could consider a model of consumer spatial demand and price competition between active firms; obtain the Bertrand equilibrium of that game, and the corresponding equilibrium profits (see Aguirregabiria & Vicentini, JIE 2016).

- This approach requires having data on prices and quantities at every location.

- Instead, Seim (RAND, 2006) and Jia (ECMA, 2008) consider a convenient shortcut.

- Their models do not specify (explicitly) consumer choices and price competition, but incorporate the idea that geographic distance to competitors (spatial differentiation) can increase a firm's profit.

Model: Profit function [2]

- This is based on the profit function in Jia (2008):

$$\Pi_i = \sum_{\ell=1}^{L} a_{i\ell} \left[x_\ell \beta_i + \xi_\ell + \theta_i^{COM} \left(\sum_{\ell'=1}^{L} \frac{a_{j\ell'}}{d_{\ell\ell'}} \right) + \theta_i^{CAN} \left(\sum_{\ell' \neq \ell} \frac{a_{i\ell'}}{d_{\ell\ell'}} \right) + \varepsilon_{i\ell} \right]$$

  where $d_{\ell\ell'}$ = distance between ℓ and ℓ'.

- θ_i^{COM} captures competition ($\theta_i^{COM} < 0$) or spillovers ($\theta_i^{COM} > 0$) from rivals.

- θ_i^{CAN} captures cannibalization ($\theta_i^{CAN} < 0$) or economies of scope/density ($\theta_i^{CAN} > 0$) from own stores.

- Seim (2006) considers more flexible specification: B bands.

Best responses & Equilibrium

- The space of the vector $\boldsymbol{a}_i = (a_{i1}, a_{i2}, ..., a_{iL})$ has 2^L possible points.
- For instance, Jia (2008) studies competition between in entry/location between Walmart and Kmart in $L = 2,065$ locations (US counties). This implies $2^L = 2^{2065} \simeq 10^{621}$.
- The computation of an equilibrium in this model is computationally very costly.
- Researchers have consider different approaches to deal with this issue.
  (a) Moment inequalities based on restrictions on the unobservables: Ellickson, Houghton, and Timmins (RAND, 2013)
  (b) Lattice theory approach: Jia (Econometrica, 2008); Nishida (Marketing Science, 2014)

Ellickson, Houghton, and Timmins (RAND, 2013)

- Consider a game between N multi-store firms but ignore for the moment cannibalization and economies of scope/density such that:

$$\Pi_i = \sum_{\ell=1}^{L} a_{i\ell} \left[\mathbf{x}_\ell \ \beta_i + \sum_{j \neq i} \gamma_{ij} \ a_{j\ell} + \varepsilon_{i\ell} \right]$$

- They assume $\varepsilon_{i\ell} = \alpha_i + \xi_\ell$, and complete information.

- By revealed preference, the profit of the observed action of firm i, $\boldsymbol{a}_i$, should be larger than the profit of any alternative action, $\boldsymbol{a}_i'$:

$$\Pi_i \left(\boldsymbol{a}_i \right) - \Pi_i \left(\boldsymbol{a}_i' \right) \geq 0 \quad \text{for any } \boldsymbol{a}_i' \neq \boldsymbol{a}_i$$

- EHT (2013) consider hypothetical choices $\boldsymbol{a}_i'$ that difference out the error term such that we do not need to integrate over a space of 2^L unobservables.

Ellickson, Houghton, and Timmins [2]

- Suppose that the observed choice of firm i, $\boldsymbol{a}_i$, is such that:

$$a_{i\ell} = 1 \textit{ and } a_{i\ell'} = 0.$$

- Consider the hypothetical choice $\boldsymbol{a}_i^*$ that consists in the relocation of a store from ℓ into ℓ', such that:

$$a_{i\ell}^* = 0 \textit{ and } a_{i\ell'}^* = 1$$

- Then:

$$\Pi_i\left(\boldsymbol{a}_i\right) - \Pi_i\left(\boldsymbol{a}_i^*\right) =$$

$$\left[\mathbf{x}_\ell - \mathbf{x}_{\ell'}\right]\beta_i + \sum_{j\neq i}\gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + \left[\xi_\ell - \xi_{\ell'}\right] \geq 0$$

Ellickson, Houghton, and Timmins [3]

- Now, suppose that for a different firm, $k \neq i$, the observe choice, $\boldsymbol{a}_k$, is such that:

$$a_{k\ell} = 0 \textit{ and } a_{k\ell'} = 1.$$

- Consider the hypothetical choice $\boldsymbol{a}_k^*$ that consists in the relocation of a store from ℓ' into ℓ, such that:

$$a_{k\ell}^* = 1 \textit{ and } a_{k\ell'}^* = 0$$

- Then, for firm k we have:

$$\Pi_k\left(\boldsymbol{a}_k\right) - \Pi_k\left(\boldsymbol{a}_k^*\right) =$$

$$\left[\mathbf{x}_{\ell'} - \mathbf{x}_{\ell}\right]\beta_k + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] + \left[\xi_{\ell'} - \xi_{\ell}\right] \geq 0$$

Ellickson, Houghton, and Timmins [4]

- Adding up the two positive inequalities, we get:

$$[\mathbf{x}_{\ell} - \mathbf{x}_{\ell'}]\,\beta_i + \sum_{j \neq i} \gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + [\xi_{\ell} - \xi_{\ell'}] \geq 0$$

$$[\mathbf{x}_{\ell'} - \mathbf{x}_{\ell}]\,\beta_k + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] + [\xi_{\ell'} - \xi_{\ell}] \geq 0$$

- We have, inequality restrictions that involve only data and parameters of interest (no unobservables):

$$[\mathbf{x}_{\ell} - \mathbf{x}_{\ell'}]\ \ [\beta_i - \beta_k] + \sum_{j \neq i} \gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] \geq 0$$

Ellickson, Houghton, and Timmins [5]

- Using different pairs of locations and/or firms, we can construct many different inequalities like

$$[\mathbf{x}_{\ell} - \mathbf{x}_{\ell'}] \; [\beta_i - \beta_k] + \sum_{j \neq i} \gamma_{ij} \left[a_{j\ell} - a_{j\ell'}\right] + \sum_{j \neq k} \gamma_{kj} \left[a_{j\ell'} - a_{j\ell}\right] \geq 0$$

- Using these inequalities, we can estimate the parameters β (up to a normalization) and γ using a **Maximum Score estimator (MSE)** (Manski, 1975; Horowitz, 1992; Fox, 2010).

- If we describe these inequalities as $\mathbf{z}_{ik\ell\ell'} \; \boldsymbol{\theta} \geq 0$, the **score function** is

$$S(\boldsymbol{\theta}) = \sum_{i,k,\ell,\ell'} 1\{\mathbf{z}_{ik\ell\ell'} \; \boldsymbol{\theta} \geq 0\}$$

  and the MSE is the value of θ that maximizes $S(\boldsymbol{\theta})$.

- EHT (RAND, 2013) apply this approach to study competition in entry/location between department store chains in US.

4. Holmes (2011): The Diffusion of Walmart and Economies of Density

Motivation

- For a retail chain, what is the optimal location of its new stores?
- Tradeoff between **cannibalization** and **economies of density**.
- **Cannibalization:** A proportion of customers for the new stores come from the chain's pre-existing stores.
- **Economies of density:** Costs savings due to proximity of stores.
  — Logistics of deliveries and inventories: — Saving trucking costs.
  — Management: Single regional manager; labor relocation. ...
- **Entry deterrence of competitors** can be also an important motive. This paper paper abstracts from this.

Motivation [2]

- The main purpose of this paper is to **identify the magnitude of cannibalization and economies of density for Walmart**, using very limited data on Walmart's store opennings.
- Why is this and important economic question?
- Understand the main forces behind the **geographic diffusion of a new business format** [using the largest retail chain in US].
- Understand the (increasing) **agglomeration of economic activity**.
- **Antitrust:** To measure some costs of **divesting Walmart**.

Empirical Strategy

- First, measure cannibalization effects from the estimation of a **consumer demand system** for the choice of department store [without data on prices or quantities !!!].
- Second, measure Walmart's costs (variable, fixed, and entry costs) and the impact of economies of density in these costs from the estimation of a **dynamic structural model of market entry and store location decisions**.
- Putting it simply: given cannibalization from demand system, revealed preference identifies economies of density.

Data

- **Store-level data** from year 2005 on Annual Sales, Employment, and Store Size (Source: AC Nielsen).

  - Key for estimating demand system and cannibalization.

- **Geographic location and opening date** of every Walmart store and every Walmart distribution center: 1962-2005.

  - Key for estimating dynamic store location model.

- **Population census socio-economics** at the census block level: Population density; Per capita income; Age distribution; Ethnic composition; Wages; Rents.

  - Key to account for location heterogeneity in demand and costs.

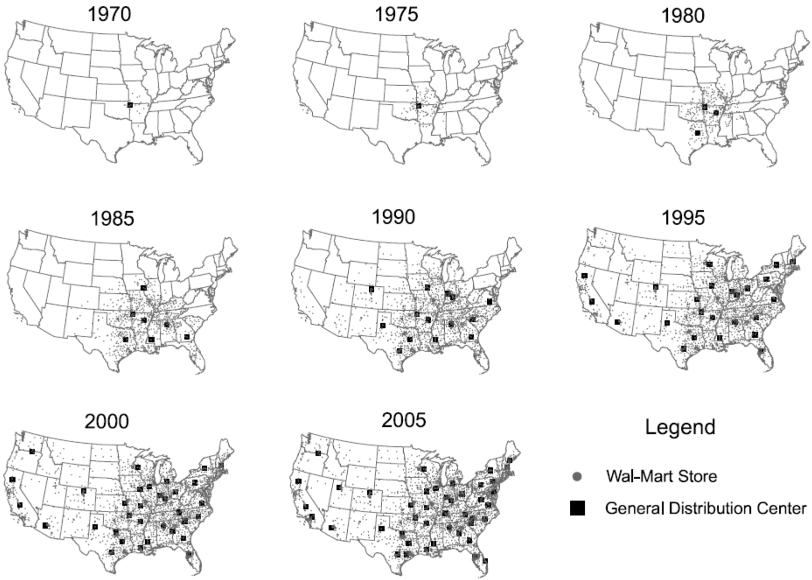
Data [2]

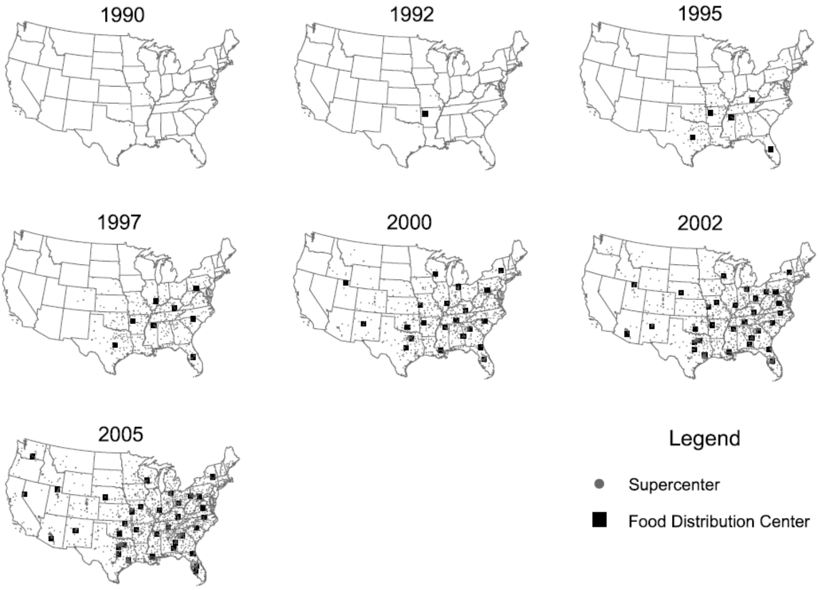
- The 2006 annual report also provides a **Walmart's own estimate of cannibalization effects**:

  "As we continue to add new stores in the United States, we do so with an understanding that additional stores may take sales away from existing units. We estimate that in fiscal years 2004, 2003, 2002 sales of pre-existing stores were negatively impacted by the opening of new stores by **approximately 1%***"*

- Importantly, cannibalization is endogenous and varies over space and time as the store network became more dense.

- Holmes uses this 1% in 2005 to validate its on estimation based on the demand system.


1970
1975
1980
1985
1990
1995
2000
2005
Legend
Wal-Mart Store
General Distribution Center


1990
1992
1995
1997
2000
2002
2005
Legend
Supercenter
Food Distribution Center

Model: Consumer Demand

- Consumers are distributed geographically in L locations (census blocks) indexed by ℓ.
- $H_{\ell t} = \#$ consumers living in location ℓ at period t. Located at the *centroid* of the block.
- Consumers buy two types of product categories: groceries (*gro*), and general merchandises (*gen*).
- Each consumer expends (in \$) λ^{gro} and λ^{gen} in these product categories.
- A consumer chooses where to purchase these two products.
- This discrete choice is modeled as a Nested Logit demand model.

Model: Nested Logit Demand Model

- The set of choice alternative for consumers in (ℓ, t) are:
  - All Walmart stores within 25 miles of block ℓ. (indexed by j)
  - An outside composite choice that represents all the other retail alternatives ($j = 0$).

- Utility for consumer i if choosing the outside alternative is:

$$u_{i0\ell t} = \gamma_0 + \gamma_1 \ \ln(m_{\ell t}) + \gamma_z' \ z_{\ell t} + \varepsilon_{i0\ell t}^{(1)}$$

- Utility for consumer i if choosing Walmart store j is:

$$\begin{aligned} u_{ij\ell t} &= -\xi_0 \text{distance}\,(\ell, j) - \xi_1 \text{distance}\,(\ell, j) * \ln(m_{\ell t}) \\ &\quad + \gamma_x' \ x_j + \varepsilon_{i\ell t}^{(1)} + (1 - \sigma) \ \varepsilon_{ij\ell t}^{(2)} \end{aligned}$$

- $m_{\ell t}$ represents population density within 5 mile radius. The utility from the outside alternative increases with density.

- Note: no prices.

Model: Aggregate demand and revenue

- Walmart stores can be regular (only general merchandise) or supercenters (also groceries).

- Aggregate revenue Walmart **regular store**:

$$R_{jt} = R_{jt}^{gen} = \lambda^{gen} \sum_{\ell \in B(j)} H_{\ell t} \; s_{j\ell t}^{gen}$$

- Aggregate revenue for Walmart **supercenter store** j:

$$R_{jt} = R_{jt}^{gen} + R_{jt}^{gro} = \sum_{\ell \in B(j)} H_{\ell t} \left[\lambda^{gen} \; s_{j\ell t}^{gen} + \lambda^{gro} \; s_{j\ell t}^{gro} \right]$$

  $s_{j\ell t}^{gen}$ and $s_{j\ell t}^{gro}$ are the market shares of store j for consumers living in ℓ.

- These market shares capture the cannibalization effect: they decline with the density of Walmart stores in the region.

Estimated Demand Parameters

PARAMETER ESTIMATES FOR DEMAND MODEL

Parameter	Definition	Unconstrained	Constrained (Fits Reported Cannibalizatio
λ^g	General merchandise spending per person (annual in \$1,000)	1.686 (.056)	1.938 (.043)
λ^f	Food spending per person (annual in \$1,000)	1.649 (.061)	1.912 (.050)
ξ_0	Distance disutility (constant term)	.642 (.036)	.703 (.039)
ξ_1	Distance disutility (coefficient on $\ln(\underline{\text{Popden}})$)	−.046 (.007)	−.056 (.008)
α	Outside alternative valuation parameters		
	Constant	−8.271 (.508)	−7.834 (.530)
	$\ln(\underline{\text{Popden}})$	1.968 (.138)	1.861 (.144)
	$\ln(\underline{\text{Popden}})^2$	−.070 (.012)	−.059 (.013)
	Per capita income	.015 (.003)	.013 (.003)
	Share of block group black	.341 (.082)	.297 (.076)
	Share of block group young	1.105 (.464)	1.132 (.440)
	Share of block group old	.563 (.380)	.465 (.359)
γ	Store-specific parameters		
	Store age 2 + dummy	.183 (.024)	.207 (.023)
σ^2	Measurement error	.065 (.002)	.065 (.002)
N		3,176	3,176
Sum of squared error		205.117	206.845

Estimated Demand: Parameter estimates

- Outside good is better in more dense areas.
- Utility decreases in distance traveled to a Walmart.
- See Table in next slide for the magnitude of the effects of Distance and Pop density.
- Non-linearity of the effect of distance: from 5 to 10 miles.
- Estimates of λ^{gen} and λ^{gen} can be compared to aggregate statistics from national consumer surveys.
  - In 2005, $\$1,800$ per capita in general merchandise (NAICS 452).
  - In 2005, $\$1,800$ per capita in food & beverages (NAICS 445).

Effect of "Distance to Closest Walmart" & "Pop Dens"

* Benchmark (Distance = 0 and Density = 1): Rural household besides a Walmart store.

COMPARATIVE STATICS WITH DEMAND MODEL[a]

Distance (Miles)	Population Density (Thousands of People Within a 5-Mile Radius) 1	5	10	20	50	100	250
0	.999	.989	.966	.906	.717	.496	.236
1	.999	.979	.941	.849	.610	.387	.172
2	.997	.962	.899	.767	.490	.288	.123
3	.995	.933	.834	.659	.372	.206	.086
4	.989	.883	.739	.531	.268	.142	.060
5	.978	.803	.615	.398	.184	.096	.041
10	.570	.160	.083	.044	.020	.011	.006

Estimated Demand: Implied Cannibalization

- Calculate what sales would be in a particular year for preexisting stores if no new stores were opened in the year: $\widehat{Sales}_t(without\ \ new\ \ stores)$.

- Calculate predicted sales to preexisting stores with the actual new store openings $\widehat{Sales}_t(with\ \ actual\ \ new\ \ stores)$.

- Define:

$$Cannibalization\ \ Rate_t = 100 * \left[\frac{\widehat{Sales}_t(without\ \ new\ \ stores)}{\widehat{Sales}_t(with\ \ actual\ \ new\ \ stores)} - 1 \right]$$

- The estimate demand model (unrestricted) does a good job in generating cannibalization rates close to Walmart's self-reported 1%.

- By Revealed Preference, the larger the Cann. rate Walmart is willing to tolerate, the larger the estimated Econ of Density. To get a lower bound on Econ. Dens., Holmes restricts Cann. Rate = 1% in 2005.

Cannibalization from Estimated Demand

CANNIBALIZATION RATES, FROM ANNUAL REPORTS AND IN MODEL[a]

Year	From Annual Reports	Demand Model (Unconstrained)	Demand Model (Constrained)
1998	n.a.	.62	.48
1999	n.a.	.87	.67
2000	n.a.	.55	.40
2001	1	.67	.53
2002	1	1.28	1.02
2003	1	1.38	1.10
2004	1	1.43	1.14
2005	1	1.27	1.00[b]

[a]Source: Estimates from the model and Wal-Mart Stores, Inc. (1971–2006) (Annual Reports 2004, 2006).

[b]Cannibalization rate is imposed to equal 1.00 in 2005.

Model: Variable Profit

- Variable profit of a store if located in ℓ:

$$VP_{\ell t} = R_{\ell t} - VC_{\ell t} = R_{\ell t} - [CMer_{\ell t} + CLabor_{\ell t} + CLand_{\ell t}]$$

- $CMer_{\ell t}$ = Cost of Merchandise. Assumption:

$$\frac{R_{\ell t} - CMer_{\ell t}}{R_{\ell t}} = \frac{p - c}{p} = \mu \quad \text{(Gross margin)}$$

- $CLabor_{\ell t}$ = Cost of Labor. Assumption. The number of workers needed is proportional to the store's revenue.

$$CLabor_{\ell t} = w_{\ell t} \; L_{\ell t} = w_{\ell t} \; \upsilon_{Labor} \; R_{\ell t}$$

where υ_{Labor} is the number of workers per \$ of revenue.

- $CLandr_{\ell t}$ = Rental cost of land.

$$CLandr_{\ell t} = \upsilon_{Land} * ValueLand_{\ell t}$$

where υ_{Land} is the rental price as % of value of land.

Model: Variable Profit [2]

- Putting these pieces together, we have:

$$VP_{\ell t} = \left(\mu - \upsilon_{Labor} \; w_{\ell t} - \upsilon_{Land} \; r_{\ell t} \right) \; R_{\ell t}$$

where $r_{\ell t} = \dfrac{ValueLand_{\ell t}}{R_{\ell t}}$.

- Holmes has data on $w_{\ell t}$ and $r_{\ell t}$ at the store level, and he calibrates parameters μ, υ_{Labor}, and υ_{Land} using data from WalMart's annual reports.

$$\begin{cases} \mu = & \text{Gross margin} = & 24\% \\ \upsilon_{Labor} = & \text{\# workers per million \$ sales} = & 3.61 \\ \upsilon_{Land} = & \text{Rental cost as \% of property value} = & 20\% \end{cases}$$

Model: Fixed Costs & Economies of Density

- The fixed cost of store has two components:

$$FC_{\ell t} = f_{\ell t} + \tau \ d_{\ell t}^{DC}$$

- $f_{\ell t}$: exogenous fixed and does not depend on economies of scope:

$$f_{\ell t} = \omega_0 + \omega_1 \ \ln \left(m_{\ell t} \right) + \omega_2 \ \ln \left(m_{\ell t} \right)^2$$

- $\tau \ d_{\ell t}^{DC}$ is the distribution cost and it depends on:

$$d_{\ell t}^{DC} \ = \ \text{Distance to the nearest distribution center}$$

Model: Fixed Costs & Economies of Scale

- We should expect a component of the fixed cost to depend on the number of stores.
- This component is implicit in all the analysis of this paper. However, the paper does not study, specifies, or estimate economies of scale.
- We will see later how the estimation approach avoids dealing with (dis)economies of scale in the total number of stores.

Model: Entry and Store Location Decisions

- Let $a_{\ell t}^{g} \in \{0,1\}$ = indicator "Walmart has store type g in block ℓ at t". Every period t, Walmart decides $\boldsymbol{a}_t$ to maximize its value:

$$\sum_{s=0}^{T} \beta^{s} \left[\sum_{\ell=0}^{L} a_{\ell t}^{gen} \, \Pi_{\ell t}^{gen} + a_{\ell t}^{gro} \, \Pi_{\ell t}^{gro} \right]$$

- **Store opening decisions are irreversible** (very large exit cost). This is the only source of dynamics in this model. Therefore, a key restriction is:

$$\boldsymbol{a}_t \geq \boldsymbol{a}_{t-1}$$

- Walmart's strategy is a function σ such that ($\boldsymbol{z}_t$ is the vector with exogenous location characteristics):

$$\boldsymbol{a}_t = \sigma\left(\boldsymbol{a}_{t-1}, \boldsymbol{z}_t\right)$$

such that σ maximizes the Walmart's value at any state $(\boldsymbol{a}_{t-1}, \boldsymbol{z}_t)$.

Model: Dimensionality of the Problem

- The dimension of the set of $\boldsymbol{a}_{t-1}$ – both the state space and the action space – is 2^L, where $L \simeq 11$ million is the number of census blocks in US: $2^L \simeq 10^{1,000,000}$.

- Solving exactly this DP problem is impractical.

- Holmes uses an approach in the same spirit as EHT (RAND, 2013), but applied to a dynamic decision model.

- Let $V_{\sigma_{obs}}$ be the Value of Walmart under its actual strategy σ_{obs} observed in the data. Then, under any other hypothetical / counterfactual strategy σ^*, we should have that:

$$V_{\sigma_{obs}} \geq V_{\sigma^*}$$

to estimate the structural parameters in costs.

Estimation of Fixed Cost Parameters from Dyn. Model

- The remaining parameters to estimate are the fixed cost parameters: ω_0, ω_1, and τ.
- Holmes uses a **moment inequality approach**.
- Holmes represents a strategy σ as the sequence of store opening choices from $t = 1$:

$$\sigma = \{\boldsymbol{a}_t : t = 1, 2, ..., \infty\}$$

- σ^{obs} represents the actual observed strategy (evolution), and σ is any other alternative strategy.
- If $V_t(\sigma)$ is the value of Walmart at period t, optimal behavior implies that for any $\sigma \neq \sigma^{obs}$:

$$V_t(\sigma^{obs}) - V_t(\sigma) \geq 0$$

Estimation of Fixed Cost Parameters [2]

- We can use the inequalities $V_t(\sigma^{obs}) - V_t(\sigma) \geq 0$ to form moment inequalities that provide partial (set) identification of structural parameters.

- In this model:

$$V_t(\sigma^{obs}) - V_t(\sigma) = y_t(\sigma) - \mathbf{x}_t(\sigma)\ \boldsymbol{\theta}$$

with $y_t(\sigma) = V_t^{\pi}(\sigma^{obs}) - V_t^{\pi}(\sigma)$:

$$V_t^{\pi}(\sigma) = \sum_{s=0}^{T} \beta^s \left[\sum_{\ell=0}^{L} a_{\ell t}^{gen}(\sigma)\ \Pi_{\ell t}^{gen}(\sigma) + a^{gro}(\sigma)\ \Pi_{\ell t}^{gro}(\sigma) \right]$$

Estimation of Fixed Cost Parameters [3]

- $\boldsymbol{\theta} = (\tau, \omega_0, \omega_1)$. And:

$$\mathbf{x}_t(\sigma) = \Big[V_t^d(\sigma^{obs}) - V_t^d(\sigma), \; V_t^{c1}(\sigma^{obs}) - V_t^{c1}(\sigma), V_t^{c2}(\sigma^{obs}) - V_t^{c2}(\sigma)$$

- with:

$$\begin{aligned}
V_t^d(\sigma) &= \sum_{s=0}^{T} \beta^s \left[\sum_{\ell=0}^{L} a_{\ell t}(\sigma) \; d_{\ell t}^{DC}(\sigma) \right] \\
V_t^{c1}(\sigma) &= \sum_{s=0}^{T} \beta^s \left[\sum_{\ell=0}^{L} a_{\ell t}(\sigma) \; \ln\left(m_{\ell t}\right) \right] \\
V_t^{c2}(\sigma) &= \sum_{s=0}^{T} \beta^s \left[\sum_{\ell=0}^{L} a_{\ell t}(\sigma) \; \ln\left(m_{\ell t}\right)^2 \right]
\end{aligned}$$

Estimation of Fixed Cost Parameters [4]

- Let $\{z_{kt} : k = 1, 2, ..., K\}$ be K instruments with $z_{kt} \geq 0$ (e.g., predetermined state variables). For any (k, σ), at the true $\boldsymbol{\theta}^0$:

$$m_{k,\sigma}(\boldsymbol{\theta}^0) = \mathbb{E}\left(z_{kt}\left[y_t(\sigma) - \mathbf{x}_t(\sigma)\ \boldsymbol{\theta}^0\right]\right) \geq 0$$

- Let $\widehat{m}_{k,\sigma}(\boldsymbol{\theta})$ be the sample counterpart of $m_{k,\sigma}(\boldsymbol{\theta})$:

$$\widehat{m}_{k,\sigma}(\boldsymbol{\theta}) = \left[\frac{1}{T}\sum_{t=1}^{T} z_{kt}\ y_t(\sigma)\right] - \left[\frac{1}{T}\sum_{t=1}^{T} z_{kt}\ \mathbf{x}_t(\sigma)\right]\boldsymbol{\theta}$$

- The estimation of the identified set Θ_I is:

$$\widehat{\Theta}_I = \text{set of } \arg\min \sum_{k,\sigma} \min\left\{0\ ;\ \widehat{m}_{k,\sigma}(\boldsymbol{\theta})\right\}^2$$

Selection of Deviation Policies sigma

- How to choose the policies σ that deviate from σ^{obs}?
- It is important to "design" these alternative policies in a way that they can be as informative as possible about the structural parameters ω_0, ω_1, and τ.
- Holmes considers the following deviation σ_s.
- Restrict attention to **pairwise resequencing**: opening dates of pairs of stores are reordered.
  * If store number 1 actually opened in 1962 and number 2 opened in 1964, a pairwise resequencing would be to open store number 2 in 1962, store number 1 in 1964, leaving everything else the same.

Selection of Deviation Policies sigma [2]

- Holmes consider 12 deviations σ^s that belong to three different "groups" – according to the intuition for the target identification
  Store density decreasing; Store density increasing; Population density changing

- **"Store density decreasing" deviations**.
  - Actual choice: at some early time period (t) there was a new store (j) near the pre-existing stores; at a later period (t') there was a store opening (j') that at period t would have been far away from the cluster of preexisting store.
  Deviation: swap the opening of j and j': that is, j' is opened at period t, and j is opened at period t'.

- This deviation reduces the density of Walmart stores between periods t and t'.

Estimation Dynamic Model: Alternative Policies

SUMMARY STATISTICS OF DEVIATIONS BY DEVIATION GROUP

Deviation Group	Brief Description of Group	Number of Deviations	Mean Values: $\Delta\tilde{\Pi}$ (Millions of 2005 Dollars)	ΔD (Thousands of Miles)	ΔC_1 (log Popden)	ΔC_2 (log Popden^2)
	Store density decreasing					
1	$-.75 \leq \Delta D < 0$	64,920	−2.7	−.4	−.6	−3.0
2	$-1.50 \leq \Delta D < -.75$	61,898	−3.6	−1.1	−1.5	−9.0
3	$\Delta D < -1.50$	114,588	−4.7	−3.0	−3.4	−22.2
	Store density increasing					
4	$0 < \Delta D \leq .75$	158,208	3.0	.3	−1.9	−17.2
5	$.75 < \Delta D \leq 1.50$	34,153	3.7	1.0	−3.6	−28.9
6	$1.50 < \Delta D$	16,180	5.9	2.1	−4.8	−37.7
	Population density changing					
7	Class 4 to class 3	7,048	1.2	.0	3.2	31.1
8	Class 3 to class 2	10,435	3.7	.0	3.4	25.7
9	Class 2 to class 1	14,399	5.3	−.1	3.5	19.3
10	Class 1 to class 2	12,053	−2.4	.0	−3.4	−19.3
11	Class 2 to class 3	14,208	.6	−.1	−3.9	−29.4
12	Class 3 to class 4	14,877	2.5	.0	−4.6	−44.9
All	Weighted mean	522,967	−.2	−.6	−2.1	−15.6

Estimation Dynamic: Distribution Costs

TABLE AI

BASELINE ESTIMATED BOUNDS ON DISTRIBUTION COST τ[a]

	Specification 1 Basic Moments (12 Inequalities)		Specification 2 Basic and Level 1 (84 Inequalities)		Specification 3 Basic and Levels 1, 2 (336 Inequalities)	
	Lower	Upper	Lower	Upper	Lower	Upper
Point estimate	3.33	4.92	3.41	4.35	3.50	3.67
Confidence thresholds						
With stage 1 error correction						
PPHI inner (95%)	2.69	6.37	2.89	5.40	3.01	4.72
PPHI outer (95%)	2.69	6.41	2.86	5.45	2.97	5.04
No stage 1 correction						
PPHI inner (95%)	2.84	5.74	2.94	5.11	3.00	4.62
PPHI outer (95%)	2.84	5.77	2.93	5.13	2.99	4.97

[a] Units are in thousands of 2005 dollars per mile year; number of deviations $M = 522{,}967$; number of store locations $N = 3{,}176$.

Estimation Dynamic: Distribution Costs [2]

- Baseline estimate of $\tau = \$3,500$ per mile, store, and year.
- If all 5,000 Walmart stores were each **100 miles farther** from their distribution centers, Walmart's costs would increase by almost **$2 billion per year**.
- Based on information on trucking costs (and back of the envelope calculation), Holmes estimate that this $\tau = \$3,500$ is approximately four times as large as the savings in trucking costs alone.
- Holmes interprets the additional component of τ as coming from the value of just-in-time inventory management (flexibility to respond to demand shocks), and managerial economies of density.

Summary & Conclusions

- Estimates of this paper show that public policies that would substantially constrain Walmart's store density would result in significant cost increases.
- The analysis does not take explicit account of the location of competitors but it is very implausible that competition explains Walmart's geographic pattern of expansion: Kmart, the leader in the 1970s and 80s ...
- More interestingly, the analysis ignores Walmart's preemption motive. This may play a role in Walmart's pattern of geographic expansion.