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Empirical Games of Market Entry and Spatial Competition in Retail Industries

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Abstract

We survey the recent empirical literature on structural models of market entry and spatial competition in oligopoly retail industries. We start with the description of a framework that encompasses various models that have been estimated in empirical applications. We use this framework to discuss important specification assumptions in this literature: firm heterogeneity; specification of price competition; structure of spatial competition; firms' information; dynamics; multi-store firms; and structure of unobservables. We next describe different types of datasets that have been used in empirical applications. Finally, we discuss econometric issues that researchers should deal with in the estimation of these models, including multiple equilibria and unobserved market heterogeneity. We comment on the advantages and limitations of alternative estimation methods, and how these methods relate to identification restrictions. We conclude with some issues and topics for future research.

Keywords: Retail industries; Market entry and exit; Spatial competition; Econometrics of discrete choice games.

JEL codes: L11, L13, L81, R30.

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1. INTRODUCTION

Competition in retail markets is often characterized by the importance of geographic location. The distance from a store to potential customers, wholesalers, and competitors can have substantial effects on demand and costs, and consequently on prices, quantities, profits, and consumer welfare. Brick-and-mortar retailers usually sell their products to consumers who physically visit their stores. Firms need to choose store locations carefully so that they are accessible to many potential customers, who are spatially dispersed. Opening a store in such attractive locations is typically more expensive (e.g., higher land prices) and it can be associated with stronger competition. Retailers should consider this trade-off when choosing the best store location. The study of the determinants of when and where to open retail stores is necessary to inform public policy and business debates such as the value of a merger between retail chains, zoning laws, spatial pre-emption, cannibalization between stores of the same chain, or the magnitude of economies of density. Therefore, it is not surprising that models of market entry, store location, and spatial competition have played a fundamental role in the theory of Industrial Organization at least since the work of Harold Hotelling (1929). However, empirical work on structural estimation of these models has been much more recent and it has followed the seminal work by Bresnahan and Reiss (1990, 1991a).

In a model of entry in a retail market, the key endogenous variables are firms' decisions to operate or not stores. In some entry models, the set of endogenous variables may also include store geographic locations, prices and quantities, store capacity, product quality, or some product characteristic(s) that provide horizontal product differentiation. Every firm makes these decisions to maximize expected profit or, if the model is dynamic, expected intertemporal profit. Empirical games of market entry in retail markets share as common features that the payoff of being active in the market depends on market size, entry cost, and the number and characteristics of other active firms. The set of structural parameters of the model varies considerably across models and applications, but it typically includes parameters that represent the entry cost and the strategic interactions between firms (competition effects). Given these common characteristics, there are substantial differences between the models that have been proposed and estimated in empirical applications. The discussion of these different specification assumptions is one of the goals of this survey.

In empirical applications of games of market entry, structural parameters are estimated using data on firms' entry decisions in a sample of markets. The estimated model is used to answer empirical questions on the nature of competition and the structure of costs in an industry, and to make predictions about the effects of changes in structural parameters or of counterfactual public policies affecting firms' profits, e.g., subsidies, taxes, or zoning laws. Entry costs, and their structure in terms of market and firm characteristics, are important primitives of these models because they play a key role in the determination of the number of stores active in a market, their characteristics, and their spatial configuration. These costs cannot be identified from the estimation of demand equations, production functions, or marginal conditions of optimality for prices or quantities. Instead, in a structural entry model, entry costs are identified using the *principle of revealed preference*: if we observe a firm operating in a market it is because its value in that market is greater than the value of shutting down and putting its assets in alternative uses. Under this principle, firms' entry and exit decisions reveal information about the underlying or latent profit function. Empirical games of market entry can be also useful to identify strategic interactions between firms that occur through variable profits. In empirical applications where a sample variation in prices is very small but there is a substantial variation in entry decisions, an entry model can provide more information about demand substitution between stores and products than the standard approach of using prices and quantities to estimate demand. Furthermore, data on prices and quantities at the store level are typically difficult to obtain, while data on store locations and entry/exit decisions are more commonly available.

Due to space constraints, this chapter focuses on issues that are directly related to empirical games of entry in retail industries. In particular, we do not cover the related literature on empirical models of spatial demand and spatial competition over prices and product assortments in retail markets where store locations are taken as exogenously given.[2] We explain in Sections 2 and 4 how these empirical models can be combined with models of entry to relax some identification assumptions and to improve estimation efficiency. Also, this survey does not cover the empirical literature that follows the influential *bounds approach* proposed by Sutton (1991) to test for *Endogenous Sunk Cost* by looking at the relationship between market size and entry.[3] Finally, we do not cover the important literature on non-structural empirical applications that study the causal effects of market entry, such as: Basker (2005), Basker and Noel (2009), and Matsa (2011) on the identification of the effects of Wal-Mart entry on competition, prices, and inventories; Watson (2009) who studies the relationship between local competition and product assortments; or Busso and Galiani (2014) who implement a randomized experiment in the Dominican Republic to obtain the first experimental evidence on the effect of increased competition on prices and quality in retail markets.[4]

[2] See Pinkse, Slade, and Brett (2002), Smith (2004), Thomadsen (2005), Davis (2006), Draganska et al. (2009), Watson (2009), Houde (2012), Wang (2010), and Miller and Osborne (2014), among others. Pinkse et al. (2002) propose a semiparametric model of spatial price competition and apply it to study competition in US gasoline markets. Smith (2004) estimates a spatial model of demand and competition between supermarkets in the UK. Thomadsen (2005) analyzes price competition between the stores of McDonalds and Burger King in Santa Clara County, California. Davis (2006) estimates a spatial model of demand of movie theaters. Draganska et al. (2009) estimate product assortment decisions of ice cream in local supermarkets. Houde (2012) studies spatial demand and price competition between gas stations. Miller and Osborne (2014) estimate a structural model of competition in US cement industry that incorporates spatial differentiation and price discrimination.

[3] Some studies within this framework are Ellickson (2007) for the US supermarket industry, Campbell and Hopenhayn (2005) for thirteen different retail industries in US metropolitan areas, and Berry and Waldfogel (2010) who study the relationship between market size and product quality in the restaurant industry and in the daily newspapers industry using also data from US metropolitan areas.

[4] The literature on Wal-Mart's impact is surveyed in Carden and Courtemanche (2015).

In Section 2, we start with the description of a comprehensive framework that encompasses various models that have been estimated in empirical applications. We use this framework to discuss important specification assumptions in this literature: firm heterogeneity; specification of price competition; structure of spatial competition; firms' information; dynamics; multi-store firms; and structure of unobservables. Section 3 describes different types of datasets that have been used in empirical applications. In Section 4, we discuss the main econometric issues that researchers should deal with in the estimation of these models, including multiple equilibria and unobserved market heterogeneity. We comment on the advantages and limitations of alternative estimation methods, and how these methods relate to identification restrictions. We conclude with some issues and topics for future research.

2. MODELS

2.1. Framework

(a) Static game with single-store firms

We start with the description of a static entry game between single-store firms. Later, we extend this framework to incorporate dynamics and multi-store firms. There are *N* retail firms that are potential entrants in a market. We index firms by $i \in \{1, 2, \ldots, N\}$. From a geographic point of view, the market is a compact set $\mathbb{C}$ in the Euclidean space $\mathbb{R}^2$, and it contains *L* locations where firms can operate stores. These locations are exogenously given and they are indexed by $\ell \in \{1, 2, \ldots, L\}$. Firms play a two-stage game. In the first stage, firms make their entry and store location decisions. Each firm decides whether to be active or not in the market, and if active, the location of its store. We can represent a firm's decision using an *L-dimension* vector of binary variables, $\boldsymbol{a}_i \equiv \{a_{i\ell} : \ell = 1, 2, \ldots, L\}$, where $a_{i\ell} \in \{0,1\}$ is the indicator of the event "firm i has a store in location ℓ". For single-store firms, there is at most one component in the vector $\boldsymbol{a}_i$ that is equal to one while the rest of the binary variables must be zero. In the second stage they compete in prices (or quantities) taking entry decisions as given. The equilibrium in the second stage determines equilibrium prices and quantities at each active store.

The market is populated by consumers. Each consumer is characterized by his preference for the products that firms sell and by his geographical location or home address *h* that belongs to the set of consumer home addresses $\{1, 2, \ldots, H\}$. The set of consumer home addresses and the set of feasible business locations may be different.[5] Following Smith (2004), Davis (2006), or Houde (2012), aggregate consumer demand comes from a discrete choice model of differentiated products where both product characteristics and transportation costs affect demand. For instance, in a spatial logit model, the demand for firm *i* with a store in location ℓ is:

[5] For instance, the set of home addresses may consist of the *centroids* of all the census tracks in the market, and the set of business locations may be equal to that set, a thinner partition of each census tract, or a subset of it due to, for example, zoning restrictions.

$$q_{i\ell} = \sum_{h=1}^{H} \left[\frac{a_{i\ell}\, exp\{x_i\, \beta \;-\; \alpha\, p_{i\ell} - \tau(d_{h\ell})\}}{1 + \sum_{j=1}^{N} \sum_{\ell'=1}^{L} a_{j\ell'}\, exp\{x_j\, \beta \;-\; \alpha\, p_{j\ell'} - \tau(d_{h\ell'})\}} \right] M(h) \tag{1}$$

where $q_{i\ell}$ and $p_{i\ell}$ are the quantity sold and the price, respectively, at store (i, ℓ); $M(h)$ represents the mass of consumers living in address h; the term within the square brackets is the market share of store (i, ℓ) among consumers living in address h; x_i is a vector of observable characteristics (other than price) of the product of firm i; and β is the vector of marginal utilities of these characteristics; α is the marginal utility of income; $d_{h\ell}$ represents the geographic distance between home address h and business location ℓ; and $\tau(.)$ is an increasing real-valued function that represents consumer transportation costs.

Given this demand system, active stores compete in prices à la Nash-Bertrand to maximize their respective variable profits, $(p_{i\ell} - c_{i\ell})q_{i\ell}$, where $c_{i\ell}$ is the marginal cost of store (i, ℓ), that is exogenously given. The solution of the system of best response functions can be described as a vector of equilibrium prices for each active firm/store.[6] Let $p_i^*(\ell, \boldsymbol{a}_{-i})$ and $q_i^*(\ell, \boldsymbol{a}_{-i})$ represent the equilibrium price and quantity for firm i given that this firm has a store at location ℓ and that the rest of the firms' entry/location decisions are represented by the vector $\boldsymbol{a}_{-i} \equiv \{\boldsymbol{a}_j : j \neq i\}$. Similarly, we can define the equilibrium (indirect) variable profit,

$$VP_i^*(\ell, \boldsymbol{a}_{-i}) = [p_i^*(\ell, \boldsymbol{a}_{-i}) - c_{i\ell}]\; q_i^*(\ell, \boldsymbol{a}_{-i}) \tag{2}$$

Consider now the entry stage of the game. The profit of firm i if it has a store in location ℓ is:

$$\pi_i(\ell, \boldsymbol{a}_{-i}\,) \;= VP_i^*(\ell, \boldsymbol{a}_{-i}) - EC_{i\ell} \tag{3}$$

where $EC_{i\ell}$ represents the entry cost of firm i at location ℓ, that for the moment is exogenously given.[7] The profit of a firm that is not active in the market is normalized to zero, i.e., $\pi_i(0, \boldsymbol{a}_{-i}\,) = 0$.[8]

[6] For a detailed description of oligopoly models of spatial price competition based on the logit demand model and its variants, see chapter 9 in Anderson, De Palma, and Thisse (1992).

[7] In an entry model with multi-store firms, the entry cost of a firm in a location may depend on the distance to other stores of the retail chain. In that case, the entry cost is endogenous because it depends on the firm's entry decisions in other locations. We describe that model below.

[8] This is a standard and innocuous normalization assumption in static discrete choice models. However, in dynamic models of market entry imposing this restriction on incumbents is not innocuous, especially when the researcher is interested in counterfactual experiments that modify the stochastic process of the state variables or the time discount factor. See Aguirregabiria and Suzuki (2014).

The description of an equilibrium in this model depends on whether firms have complete or incomplete information about other firms' costs. The empirical literature on entry games has considered both cases. In the *complete information model*, a Nash equilibrium is an N-tuple $\{\boldsymbol{a}_i^*: i = 1, 2, \dots, N\}$ such that for every firm i the following best response condition is satisfied:

$$a_{i\ell}^* = 1\{ \pi_i(\ell, \boldsymbol{a}_{-i}^*) \geq \pi_i(\ell', \boldsymbol{a}_{-i}^*) \;\; for\ any\ \ell' \neq \ell \} \tag{4}$$

where $1\{.\}$ is the indicator function. In equilibrium, each firm is maximizing its own profit given the entry and location decisions of the other firms.

In a *game of incomplete information*, there is a component of a firm's profit that is private information of the firm. For instance, suppose that the entry cost of firm i is $EC_{i\ell} = ec_{i\ell} + \varepsilon_{i\ell}$, where $ec_{i\ell}$ is public information for all the firms, and $\varepsilon_{i\ell}$ is private information of firm i. These private cost shocks can be correlated across locations for a given firm, but they are independently distributed across firms, i.e., $\boldsymbol{\varepsilon}_i \equiv \{\varepsilon_{i\ell}: \ell = 1, 2, \dots, L\}$ is independently distributed across firms with a distribution function F_i that is continuously differentiable over $\mathbb{R}^L$ and common knowledge to all the firms. A firm's strategy is an *L-dimension* mapping $\boldsymbol{\alpha}_i(\boldsymbol{\varepsilon}_i) \equiv \{\alpha_{i\ell}(\boldsymbol{\varepsilon}_i) : \ell = 1, 2, \dots, L\}$ where $\alpha_{i\ell}(\boldsymbol{\varepsilon}_i)$ is a binary-valued function from the set of possible private information values, $\mathbb{R}^L$, into $\{0, 1\}$ such that $\alpha_{i\ell}(\boldsymbol{\varepsilon}_i) = 1$ means that firm i enters location ℓ when the value of private information is $\boldsymbol{\varepsilon}_i$. A firm has uncertainty about the actual entry decisions of other firms because it does not know the realization of other firms' private information. Therefore, firms maximize expected profits. Let $\pi_i^e(\ell, \boldsymbol{\alpha}_{-i})$ be the *expected profit* of firm i if it has a store at location ℓ and the other firms follow their respective strategies in $\boldsymbol{\alpha}_{-i} \equiv \{\alpha_j: j \neq i\}$. By definition, $\pi_i^e(\ell, \boldsymbol{\alpha}_{-i}) \equiv E_{\boldsymbol{\varepsilon}_{-i}}[\, \pi_i(\ell, \boldsymbol{\alpha}_{-i}(\boldsymbol{\varepsilon}_{-i}))\,]$, where $E_{\boldsymbol{\varepsilon}_{-i}}$ represents the expectation over the distribution of the private information of firms other than i. A (Bayesian) Nash equilibrium in this game of incomplete information is an N-tuple of strategy functions $\{\alpha_i^*: i = 1, 2, \dots, N\}$ such that every firm maximizes its *expected profit*: for any $\boldsymbol{\varepsilon}_i$,

$$\alpha_{i\ell}^*(\boldsymbol{\varepsilon}_i) = 1\{ \pi_i^e(\ell, \boldsymbol{\alpha}_{-i}^*) \geq \pi_i^e(\ell', \boldsymbol{\alpha}_{-i}^*) \;\; for\ any\ \ell' \neq \ell \} \tag{5}$$

In an entry game of incomplete information, firms' strategies (and therefore, a Bayesian Nash Equilibrium) can be described also using firms' probabilities of market entry, instead of the strategy functions $\alpha_i(\boldsymbol{\varepsilon}_i)$. In Sections 2.2(a) and 2.2(d), we present examples of this representation in the context of more specific models.

(b) Multi-store firms

Multi-store firms, or retail chains, have become prominent in many retail industries such as supermarkets, department stores, apparel, electronics, fast food restaurants, or coffee shops, among others.[9] Cannibalization and economies of scope between stores of the same chain are two important factors in the entry and location decisions of a multi-store firm. The term *cannibalization* refers to the business stealing effects between stores of the same chain. Economies of scope may appear if some operating costs are shared between stores of the same retail chain such that these costs are not duplicated when the number of stores in the chain increases. For instance, some advertising, inventory, personnel, or distribution costs can be shared among the stores of the same firm. These economies of scope may become quantitatively more important when store locations are geographically closer to each other. This type of economies of scope is called *economies of density*. The recent empirical literature on retail chains has emphasized the importance of these economies of density, i.e., Holmes (2011), Jia (2008), Ellickson, Houghton, and Timmins (2013), and Nishida (forthcoming). For instance, the transportation cost associated with the distribution of products from wholesalers to retail stores can be smaller if stores are close to each other. Also, geographic proximity can facilitate sharing inventories and even personnel across stores of the same chain. We now present an extension of the basic framework that accounts for multi-store firms.

A multi-store firm decides its number of stores and their locations. We can represent a firm's entry decision using the *L-dimension* vector $\boldsymbol{a}_i \equiv \{a_{i\ell} : \ell = 1, 2, \dots, L\}$, where $a_{i\ell} \in \{0,1\}$ is still the indicator of the event "firm i has a store in location ℓ".[10] In contrast to the case with single-store firms, now the vector $\boldsymbol{a}_i$ can take any value within the choice set $\{0,1\}^L$. The demand system still can be described using equation (1). The variable profit of a firm is the sum of variable profits over every location where the firm has stores, $\sum_{\ell=1}^{L} a_{i\ell}\,(p_{i\ell} - c_{i\ell})q_{i\ell}$. Firms compete in prices taking their store locations as given. A retail chain may choose to have a uniform price across all its stores, or to charge a different price at each store.[11] In the Bertrand pricing game with spatial price discrimination (i.e., different prices at each store), the best response of firm i can be characterized by the first-order conditions:

$$q_{i\ell} + (p_{i\ell} - c_{i\ell})\frac{\partial q_{i\ell}}{\partial p_{i\ell}} + \sum_{\ell' \neq \ell}(p_{i\ell'} - c_{i\ell'})\frac{\partial q_{i\ell'}}{\partial p_{i\ell}} = 0 \qquad (6)$$

The first two terms represent the standard marginal profit of a single-store firm. The last term represents the effect on the variable profits of all other stores within the firm, and it captures how the pricing decision of the firm internalizes the cannibalization effect among its own stores. A Nash-Bertrand equilibrium is a solution in prices to the system of best response equations in (6). The equilibrium (indirect) variable profit of firm i is:

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[9] This development is discussed in more detail by Foster et al. (2015).

[10] We consider here that a firm can have at most one store at each location ℓ. However, the model can be trivially extended to allow for multiple stores per location, i.e., $a_{i\ell} \in \{0, 1, \dots, n\}$.

[11] For empirical studies on the trade-offs between uniform pricing and spatial price discrimination in retail chains, see Chintagunta, Dube, and Singh (2003) on supermarkets, and Thomadsen (2005) in fast food restaurants.

$$VP_i^*(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) = \sum_{\ell=1}^{L} a_{i\ell} \, [p_i^*(\ell, \boldsymbol{a}_{-i}) - c_{i\ell}] \, q_i^*(\ell, \boldsymbol{a}_{-i}) \tag{7}$$

where $p_i^*(\ell, \boldsymbol{a}_{-i})$ and $q_i^*(\ell, \boldsymbol{a}_{-i})$ represent Bertrand equilibrium prices and quantities, respectively.

The total profit of the retail chain is equal to total variable profit minus total entry cost: $\pi_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) = VP_i^*(\boldsymbol{a}_i, \boldsymbol{a}_{-i}) - EC_i(\boldsymbol{a}_i)$. The entry costs of a retail chain may depend on the number of stores (i.e., (dis)economies of scale) and on the distance between the stores (e.g., economies of density). In Section 2.2(e) below, we provide examples of specifications of entry costs for multi-store retailers.

The description of an equilibrium in this game of entry between retail chains is similar to the game between single-store firms. With *complete information*, a Nash equilibrium is an N-tuple $\{\boldsymbol{a}_i^*: i = 1, 2, \dots, N\}$ that satisfies the following best response conditions:

$$\pi_i(\boldsymbol{a}_i^*, \boldsymbol{a}_{-i}^*) \geq \pi_i(\boldsymbol{a}_i, \boldsymbol{a}_{-i}^*) \quad for\ any\ \boldsymbol{a}_i \neq \boldsymbol{a}_i^* \tag{8}$$

With *incomplete information*, a Bayesian Nash equilibrium is an N-tuple of strategy functions $\{\boldsymbol{\alpha}_i^*: i = 1, 2, \dots, N\}$ such that every firm maximizes its *expected profit*: for any $\boldsymbol{\varepsilon}_i$:

$$\pi_i^e(\boldsymbol{\alpha}_i^*(\boldsymbol{\varepsilon}_i), \boldsymbol{\alpha}_{-i}^*) \geq \pi_i^e(\boldsymbol{a}_i, \boldsymbol{\alpha}_{-i}^*) \quad for\ any\ \boldsymbol{a}_i \neq \boldsymbol{\alpha}_i^*(\boldsymbol{\varepsilon}_i) \tag{9}$$

(c) Dynamic game

Opening (or closing) a store is a forward-looking decision with significant non-recoverable entry costs, mainly due to capital investments which are both firm and location-specific. The sunk cost of setting up new stores, and the dynamic strategic behavior associated with them, are potentially important forces behind the configuration of the spatial market structure that we observe in retail markets. We now present an extension of the previous model that incorporates these dynamic considerations.[12]

Time is discrete and indexed by $t \in \{\dots, 0, 1, 2, \dots\}$. At the beginning of period t a firm's network of stores is represented by the vector $\boldsymbol{a}_{it} \equiv \{a_{i\ell t} : \ell = 1, 2, \dots, L\}$, where $a_{i\ell t}$ is the number of stores that firm i operates in location ℓ at period t. For simplicity, we maintain the assumption that a firm can have at most one store in a location, such that $a_{i\ell t} \in \{0,1\}$. The market structure at period t is represented by the vector $\boldsymbol{a}_t \equiv \{\boldsymbol{a}_{it} : i = 1, 2, \dots, N\}$ capturing the store network of all firms. Following the structure in the influential work on dynamic games of oligopoly competition by Ericson and Pakes (1995) and Pakes and McGuire (1994), at every period t the

[12] For a detailed description of the model see Aguirregabiria and Vicentini (2012).

model has two stages, similar to the ones described in the static game above. In the second stage, taking the vector of firms' store networks $\boldsymbol{a}_t$ as given, retail chains compete in prices in exactly the same way as in the Bertrand model described in Section 2.1(b) above. The equilibrium in this Bertrand game determines the indirect variable profit function, $VP_i^*(\boldsymbol{a}_t; \boldsymbol{z}_t)$, where $\boldsymbol{z}_t$ is a vector of exogenous state variables in demand and costs. Some components of $\boldsymbol{z}_t$ may be random variables, and their future values may not be known at the current period. In the first stage, every firm decides its network of stores next period, $\boldsymbol{a}_{it+1}$, and pays at period t the entry and exit costs associated to opening and closing stores.[13] The period profit of a firm is $\pi_i(\boldsymbol{a}_{it+1}, \boldsymbol{a}_t, \boldsymbol{z}_t) = VP_i^*(\boldsymbol{a}_t; \boldsymbol{z}_t) - FC_i(\boldsymbol{a}_{it}; \boldsymbol{z}_t) - AC_i(\boldsymbol{a}_{it+1}, \boldsymbol{a}_{it})$, where FC_i is the fixed cost of operating the network, and AC_i is the cost of adjusting the network from $\boldsymbol{a}_{it}$ to $\boldsymbol{a}_{it+1}$, i.e., costs of opening and closing stores. A firm chooses its new network $\boldsymbol{a}_{it+1}$ to maximize the sum of its discounted expected future profits.

A *Markov Perfect Equilibrium* of this dynamic game is an N-tuple of strategy functions $\{\alpha_i^*(\boldsymbol{a}_t, \boldsymbol{z}_t): i = 1, 2, \ldots, N\}$ such that every firm maximizes its *expected intertemporal profit*:

$$\alpha_i^*(\boldsymbol{a}_t, \boldsymbol{z}_t) = argmax_{\{\boldsymbol{a}_{it+1}\}}\left[\,\pi_i(\boldsymbol{a}_{it+1}, \boldsymbol{a}_t, \boldsymbol{z}_t) + \delta\, E\left(V_i(\boldsymbol{a}_{it+1}, \alpha_{-i}^*(\boldsymbol{a}_t, \boldsymbol{z}_t), \boldsymbol{z}_{t+1}; \alpha_{-i}^*)\right)\right] \tag{10}$$

where $\delta \in (0,1)$ is the discount factor, and $V_i(\boldsymbol{a}_t, \boldsymbol{z}_t; \alpha_{-i}^*)$ is the value of firm i when firms' networks are equal to $\boldsymbol{a}_t$, the value of exogenous state variables is $\boldsymbol{z}_t$, and the other firms follow strategies α_{-i}^*.[14]

2.2. Specification assumptions

The games of entry in retail markets that have been estimated in empirical applications have imposed different types of restrictions on the framework that we have presented in Section 2.1, e.g., restrictions on firm and market heterogeneity, firms' information, spatial competition, multi-store firms, dynamics, or the form of the structural functions. The motivations for these restrictions are diverse. Some restrictions are imposed to achieve identification or precise enough estimates of the parameters of interest, given the researcher's limited information on the characteristics of markets and firms. For instance, as we describe in Section 3, prices and quantities at the store level are typically not observable to the researcher, and most sample information comes from firms' entry decisions. These limitations in the available data have motivated researchers to use simple specifications for the indirect variable profit function. Other

[13] In this version of the model, we assume that it takes one period to open or close a store, i.e., *time-to-build* assumption. It is straightforward to modify the model to eliminate time-to-build.

[14] Given arbitrary strategy functions for firms other than i, say $\alpha_{-i}(.)$, the value function of firm i is implicitly defined as the unique solution to the Bellman equation:

$$V_i(\boldsymbol{a}_t, \boldsymbol{z}_t; \alpha_{-i}) = max_{\{\boldsymbol{a}_{it+1}\}}\left[\,\pi_i(\boldsymbol{a}_{it+1}, \boldsymbol{a}_t, \boldsymbol{z}_t) + \delta\, E\left(V_i(\boldsymbol{a}_{it+1}, \alpha_{-i}(\boldsymbol{a}_t, \boldsymbol{z}_t), \boldsymbol{z}_{t+1}; \alpha_{-i})\right)\right]$$

restrictions are imposed for computational convenience in the solution and estimation of the model, e.g., to obtain closed form solutions, to guarantee equilibrium uniqueness as it facilitates the estimation of the model, or to reduce the dimensionality of the space of firms' actions or states. In this subsection, we review some important models in this literature and discuss their main identification assumptions. We have organized these models in an approximate chronological order.

(a) Homogeneous firms

Work in this field was pioneered by Bresnahan and Reiss. In Bresnahan and Reiss (1991a), they study several retail and professional industries in US, i.e., pharmacies, tire dealers, doctors, and dentists. The main purpose of the paper is to estimate the "nature" or "degree" of competition for each of the industries: how fast variable profits decline when the number of firms in the market increases. More specifically, the authors are interested in estimating how many entrants are needed to achieve an oligopoly equilibrium equivalent to the competitive equilibrium, i.e., hypothesis of *contestable markets* (Baumol, 1982). For each industry, their dataset consists of a cross-section of *M* small "isolated markets". In Section 3, we discuss the empirical motivation and implementation of the "isolated markets" restriction. For the purpose of the model, a key aspect of this restriction is that the *M* local markets are independent in terms of demand and competition such that the equilibrium in one market is independent of the one in the other markets. The model also assumes that each market consists of a single location, i.e., *L=1*, such that spatial competition is not explicitly incorporated in the model. For each local market, the researcher observes the number of active firms (n), a measure of market size (s), and some exogenous market characteristics that may affect demand and/or costs (**x**). Given this limited information, the researcher needs to restrict firm heterogeneity. Bresnahan and Reiss propose a static game between single-store firms where all the potential entrants in a market are identical and have complete information on demand and costs. The profit of a store is $\pi(n) = s * vp(\mathbf{x}, n) - EC(\mathbf{x}) - \varepsilon$, where $vp(\mathbf{x}, n)$ represents variable profit per capita (per consumer) that depends on the number of active firms n, and $EC(\mathbf{x}) + \varepsilon$ is the entry cost, where ε is unobservable to the researcher.[15] The form of competition between active firms is not explicitly modelled. Instead, the authors consider a flexible specification of the variable profit per capita, that is strictly decreasing but nonparametric in the number of active stores. Therefore, the specification is consistent with a general model of competition between homogeneous firms, or even between symmetrically differentiated firms.

Given these assumptions, the equilibrium in a local market can be described as a number of firms n^* that satisfies two conditions: (1) every active firm is maximizing profits by being active in the market, i.e., $\pi(n^*) \geq 0$; and (2) every inactive firm is maximizing profits by being out of the market, i.e., $\pi(n^* + 1) < 0$. That is, every firm is making its best response given the actions of

[15] In fact, to have a more robust test of the contestable markets hypothesis, Bresnahan and Reiss allow entry cost to depend on the number of active stores, i.e., $EC(\mathbf{x}, \mathrm{n})$.

the others. Since the profit function is strictly decreasing in the number of active firms, the equilibrium is unique and it can be represented using the following expression: for any value $n \in \{0, 1, 2, \ldots\}$,

$$\begin{aligned}\{n^* = n\} \;&\Leftrightarrow\; \{\pi(n) \geq 0 \;\; and \;\; \pi(n+1) < 0\,\} \\ &\Leftrightarrow\; \{s * \; vp(\mathbf{x}, n+1) - EC(\mathbf{x}) < \varepsilon \leq s * \; vp(\mathbf{x}, n) - EC(\mathbf{x})\}\end{aligned} \tag{11}$$

And this condition implies that the distribution of the equilibrium number of firms given exogenous market characteristics is:[16]

$$\Pr(n^* = n \mid s, \mathbf{x}) = F\big(s \;\, vp(\mathbf{x}, \mathrm{n}) - EC(\mathbf{x})\big) - F\big(s \;\, vp(\mathbf{x}, \mathrm{n}+1) - EC(\mathbf{x})\big) \tag{12}$$

where $F(.)$ is the CDF of ε. This representation of the equilibrium as an ordered discrete choice model is convenient for estimation.

In the absence of price and quantity data, the separate identification of the variable profit function and the entry cost function is based on the exclusion restrictions that variable profit depends on market size and on the number of active firms while the entry cost does not depend on these variables.[17]

The previous model can be slightly modified to allow for firms' private information. This variant of the original model maintains the property of equilibrium uniqueness and most of the simplicity of the previous model. Suppose that now the entry cost of a firm is $EC(\mathbf{x}) + \varepsilon_i$, where ε_i is private information of firm i and it is independently and identically distributed across firms with a CDF $F(.)$. There are N potential entrants in the local market. The presence of private information implies that, when potential entrants make entry decisions, they do not know ex-ante the actual number of firms that will be active in the market. Instead, each firm has beliefs about the probability distribution of the number of other firms that are active. We represent these beliefs, for say firm i, using the function $G_i(n) \equiv \Pr\big(n^*_{(-i)} = n \mid s, \mathbf{x}\big)$, where $n^*_{(-i)}$ represents the number of firms other than i that are active in the market. Then, the expected profit of a firm if active in the market is:

[16] If there is a maximum number of potential entrants, N, then $\Pr(n^* = N \mid s, \mathbf{x}) = F\big(s \;\, v(\mathbf{x}, N) - EC(\mathbf{x})\big)$.

[17] In Bresnahan and Reiss (1991a) the separate identification of variable profit and entry cost is based only on the restriction that the first depends on market size and the second does not. However, most of the subsequent empirical applications in the literature have imposed also the restriction that entry costs do not depend on the number of active stores.

$$\pi_i^e = \left[\sum_{n=0}^{N-1} G_i(n)\, s\ \ vp(\mathbf{x}, \mathrm{n}+1)\right] - EC(\mathbf{x}) - \varepsilon_{\mathrm{i}} \tag{13}$$

The best response of a firm is to be active in the market if and only if its expected profit is positive or zero, i,e., $a_i = 1\{\pi_i^e \geq 0\}$. Integrating this best response function over the distribution of the private information ε_i we obtain the *best response probability of being active* for firm i, i.e., $P_i \equiv F([\sum_{n=0}^{N-1} G_i(n)\, s\ \ vp(\mathbf{x}, \mathrm{n}+1)\,] - EC(\mathbf{x}))$. Since all firms are identical, up to their independent private information, it seems reasonable to impose the restriction that in equilibrium they all have the same beliefs and, therefore, the same best response probability of entry. Therefore, in equilibrium, firms' entry decisions $\{a_i\}$ are independent Bernoulli random variables with probability P, and the number of firms active other than i in the market has a Binomial distribution with argument $(N-1, P)$ such that $\Pr\left(n_{(-i)}^* = n\right) = B(n|N-1, P) \equiv \binom{N-1}{n} (P)^n (1-P)^{N-1-n}$. In equilibrium, the beliefs function $G(n)$ should be consistent with firms' best response probability P. Therefore, a Bayesian Nash Equilibrium in this model can be described as a probability of market entry P^* that is the best response probability when firms' beliefs about the distribution of other firms active in the market are $G(n) = B(n|N-1, P^*)$. We can represent this equilibrium condition using the following equation:[18]

$$P^* = F\left(\left[\sum_{n=0}^{N-1} B(n|N-1, P^*)\, s\ \ vp(\mathbf{x}, \mathrm{n}+1)\right] - EC(\mathbf{x})\right) \tag{14}$$

When the variable profit $vp(\mathbf{x}, \mathrm{n})$ is a decreasing function in the number of active stores, the right hand side in equation (14) is also a decreasing function in the probability of entry P, and this implies equilibrium uniqueness. In contrast to the complete information model in Bresnahan and Reiss (1991), this incomplete information model does not have a closed form solution for the equilibrium distribution of the number of active firms in the market. However, the numerical solution of the fixed point problem in equation (14) is computationally very simple, and so are the estimation and comparative statistics using this model.

Given that the only difference between the two models described in Section 2.2(a) is in their assumptions about firms' information, it seems reasonable to consider whether these models are observationally different or not. In other words, does the assumption on complete versus incomplete information have implications on the model predictions on competition? Grieco (2014) investigates this question in the context of an empirical application to local grocery markets. In Grieco's model firms are heterogeneous in terms of (common knowledge)

[18] Equation (14) defines a fixed point problem in the compact probability space [0,1]. Given that the equilibrium function in the right-hand-side of equation (14) is continuous, Brower's Theorem implies the existence of an equilibrium.

observable variables, and this observable heterogeneity plays a key role in his approach to empirically distinguish between firms' public and private information. Note that the comparison of equilibrium conditions in equations (12) and (14) shows other testable difference between the two models. In the game of incomplete information, the number of potential entrants N has an effect on the whole probability distribution of the number of active firms: a larger number of potential entrants implies a shift to the right in the whole distribution of the number of active firms. In contrast, in the game of complete information, the value of N affects only the probability $\Pr(n^* = N \mid s, \mathbf{x})$ but not the distribution of the number of active firms at values smaller than N. This empirical prediction has relevant economic implications: with incomplete information, the number of potential entrants has a positive effect on competition even in markets where this number is not binding.[19]

(b) Entry with endogenous product choice

Mazzeo (2002) studies market entry in the motel industry using local markets along U.S. interstate highways.[20] A local market is defined as a narrow region around a highway exit. Mazzeo's model maintains most of the assumptions in Bresnahan and Reiss (1991a), such as no spatial competition (i.e., $L = 1$), ex-ante homogeneous firms, complete information, no multi-store firms, and no dynamics. However, he extends Bresnahan-Reiss model in an interesting dimension: it introduces endogenous product differentiation. More specifically, firms not only decide whether to enter in a market but they also choose the type of product: low quality product E (i.e., economy hotel), or high quality product H (i.e., upscale hotel).[21] Product differentiation makes competition less intense, and it can increase firms' profits. However, firms have also an incentive to offer the type of product for which demand is stronger.

The profit of an active hotel of type $T \in \{E, H\}$ is:

$$\pi_T(n_E, n_H) = s \; v_T(\mathbf{x}, n_E, n_H) - EC_T(\mathbf{x}) - \varepsilon_T \tag{15}$$

where n_E and n_H represent the number of active hotels with low and high quality, respectively, in the local market. Similarly to Bresnahan-Reiss model, $v_T(.)$ is the variable profit per capita and $EC_T(\mathbf{x}) + \varepsilon_T$ is the entry cost for type T hotels, where ε_T is unobservable to the researcher. Mazzeo solves and estimates his model under two different equilibrium concepts: Stackelberg

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[19] We can test for pure complete information using a Kolmogorov-Smirnov test of the equality of distributions for two similar subsets of local markets with different number of potential entrants. A potential issue in the implementation of this type of test is that the number of potential entrants in a local market is typically difficult to observe.

[20] Although hotels are not technically a retail sector, the method and insights from this analysis apply to many retail sectors as well.

[21] Mazzeo uses American Automobile Association (AAA) hotel categories, from 1 to 4 diamonds, to determine the quality of a hotel.

and what he terms a "two-stage game."[22] A computational advantage of the two-stage game is that under the assumptions of the model the equilibrium is unique. In the first stage, the total number of active hotels, $n \equiv n_E + n_H$, is determined in a similar way as in Bresnahan-Reiss model. Hotels enter the market as long as there is some configuration (n_E, n_H) where both low quality and high quality hotels make positive profits. Define the *first-stage profit function* as:

$$\Pi(n) \equiv max_{\{n_E, n_H:\, n_E + n_H = n\}} \{min[\, \pi_E(n_E, n_H)\,,\, \pi_H(n_E, n_H)]\} \tag{16}$$

Then, the equilibrium number of hotels in the first stage is the value n^* that satisfies two conditions: (1) every active firm wants to be in the market, i.e., $\Pi(n^*) \geq 0$; and (2) every inactive firm prefers to be out of the market, i.e., $\Pi(n^* + 1) < 0$.[23] If the profit functions π_E and π_H are strictly decreasing functions in the number of active firms (n_E, n_H), then $\Pi(n)$ is also a strictly decreasing function, and the equilibrium number of stores in the first stage, n^*, is unique. In the second stage, active hotels choose simultaneously their type or quality level. In this second stage, an equilibrium is a pair (n_E^*, n_H^*) such that every firm chooses the type that maximizes its profit given the choices of the other firms: low quality firms are not better off by switching to high quality, and *vice versa*,

$$\pi_E(n_E^*, n_H^*) \geq\, \pi_H(n_E^* - 1, n_H^* + 1) \;\; and \;\; \pi_H(n_E^*, n_H^*) \geq\, \pi_E(n_E^* + 1, n_H^* - 1) \tag{17}$$

Mazzeo shows that the equilibrium pair (n_E^*, n_H^*) in this second stage is also unique.

Using these equilibrium conditions, it is possible to obtain a closed form expression for the (quadrangle) region in the space of the unobservables $(\varepsilon_E, \varepsilon_H)$ that generate a particular value of the equilibrium pair (n_E^*, n_H^*). Let $R_\varepsilon(n_E, n_H; s, \mathbf{x})$ be the quadrangle region in $\mathbb{R}^2$ associated with the pair (n_E, n_H) given exogenous market characteristics $(s, \mathbf{x})$, and let $F(\varepsilon_E, \varepsilon_H)$ be the CDF of the unobservable variables. Then, we have that

$$\Pr(n_E^* = n_E, n_H^* = n_H | s, \mathbf{x}) = \int 1\{(\varepsilon_E, \varepsilon_H) \in R_\varepsilon(n_E, n_H; s, \mathbf{x})\}\; dF(\varepsilon_E, \varepsilon_H). \tag{18}$$

In the empirical application, Mazzeo finds that hotels have strong incentives to differentiate from their rivals to avoid nose-to-nose competition.

Ellickson and Misra (2008) estimate a game of incomplete information for the US supermarket industry where supermarkets choose the type of "pricing strategy": 'Everyday Low Price'

[22] In fact, it is a three stage game where the third stage is the (implicit) price or quantity competition between active stores.

[23] Note that a weakness of Mazzeo's "two-stage equilibrium" concept is that some type of potential entrants may want to be in the market even when $\Pi(n^* + 1) < 0$, as long as π_H or π_E is positive.

(EDLP) versus 'High-Low' pricing.[24] The choice of pricing strategy can be seen as a form of horizontal product differentiation. The authors find evidence of strategic complementarity between supermarkets pricing strategies: firms competing in the same market tend to adopt the same pricing strategy not only because they face the same type of consumers but also because there are positive synergies in the adoption of the same strategy. From an empirical point of view, this result is more controversial than Mazzeo's finding of firms' incentive to differentiate from each other. In particular, the existence of unobservables that are positively correlated across firms but are not fully accounted in the econometric model, may generate a spurious estimate of positive spillovers in the adoption of the same strategy. Vitorino (2012) estimates a game of store entry in shopping centers that allows for incomplete information, positive spillover effects among stores, and also unobserved market heterogeneity for the researcher that is common knowledge to firms. Her empirical results show that, after controlling for unobserved market heterogeneity, firms face business stealing effects but also significant incentives to collocate, and that the relative magnitude of these two effects varies substantially across store types.

(c) Firm heterogeneity

Bresnahan and Reiss (1990) estimate a static model of entry in the US automobile dealers industry. They focus on small, geographically-isolated markets with at most two entrants. Their model ignores spatial competition (i.e., $L = 1$), multi-store firms, and dynamics, but it allows for firm heterogeneity in entry costs. Each local market has two potential entrants, which we index with $i, j \in \{1,2\}$. The profit function of firm i if active in the market is:

$$\pi_i(a_j) = s \; v(\mathbf{x}, a_j) - EC(\mathbf{x}) - \varepsilon_i \tag{19}$$

where $a_j \in \{0,1\}$ represents the entry decision of the other potential entrant. Therefore, $\pi_i(0)$ is the profit of firm i under monopoly, and $\pi_i(1)$ is its profit under duopoly. Note that all the exogenous observable variables $(s, \mathbf{x})$ are common to the two firms, but the unobservable ε_i is firm specific, but public information for both firms.

A Nash equilibrium in this game is a pair of actions (a_i^*, a_j^*) such that every firm maximizes profits taking the other firm's action as given:

$$a_i^* = 1\{\pi_i(a_j^*) \geq 0\} \quad and \quad a_j^* = 1\{\pi_j(a_i^*) \geq 0\} \tag{20}$$

Given this description of an equilibrium, we can derive the quadrangle regions in the space of the unobservables $(\varepsilon_i, \varepsilon_j)$ associated with the different equilibrium outcomes. Define the following threshold values in the space of ε: the threshold for entry as a monopolist, $\Delta^M(\mathrm{s}, \mathbf{x}) \equiv$

[24] Strictly speaking, this paper is not a study on entry decisions as the model takes the stores' entry decisions as exogenously given. We nonetheless mention this paper here as it is an interesting applied work on endogenous characteristics of retail stores that estimates a static game of incomplete information.

$s\ v(\mathbf{x},0) - EC(\mathbf{x})$; and the threshold for entry as a duopolist, $\Delta^D(\mathrm{s},\mathbf{x}) \equiv s\ v(\mathbf{x},1) - EC(\mathbf{x})$. Competition implies that $\Delta^M(\mathrm{s},\mathbf{x}) \geq \Delta^D(\mathrm{s},\mathbf{x})$. Then,

$$\begin{aligned}
(a_i^*, a_j^*) = (0,0) &\iff \{\varepsilon_i > \Delta^M(\mathrm{s},\mathbf{x})\ \ and\ \ \varepsilon_j > \Delta^M(\mathrm{s},\mathbf{x})\} \\
(a_i^*, a_j^*) = (0,1) &\iff \{\varepsilon_i > \Delta^D(\mathrm{s},\mathbf{x})\ \ and\ \ \varepsilon_j \leq \Delta^M(\mathrm{s},\mathbf{x})\} \\
(a_i^*, a_j^*) = (1,0) &\iff \{\varepsilon_i \leq \Delta^M(\mathrm{s},\mathbf{x})\ \ and\ \ \varepsilon_j > \Delta^D(\mathrm{s},\mathbf{x})\} \\
(a_i^*, a_j^*) = (1,1) &\iff \{\varepsilon_i \leq \Delta^D(\mathrm{s},\mathbf{x})\ \ and\ \ \varepsilon_j \leq \Delta^D(\mathrm{s},\mathbf{x})\}
\end{aligned} \tag{21}$$

This model has multiple equilibria. For values of $(\varepsilon_i, \varepsilon_j)$ within the square region $[\Delta^D(\mathrm{s},\mathbf{x}), \Delta^M(\mathrm{s},\mathbf{x})] \times [\Delta^D(\mathrm{s},\mathbf{x}), \Delta^M(\mathrm{s},\mathbf{x})]$, outcomes $(0,1)$ and $(1,0)$ are both Nash equilibria. In this empirical application where all the exogenous state variables are common market characteristics and the two firms are identical in terms of observable characteristics and structural parameters, the type of multiplicity of equilibria in this model does not generate serious estimation problems. The two outcomes $(0,1)$ and $(1,0)$ imply the same number of entrants, i.e., a monopoly market. Therefore, the model implies the unique distribution of the number of entrants:

$$\begin{aligned}
&\Pr(n^* = 2 \mid s, \mathbf{x}) = F(\Delta^D, \Delta^D) \\
&\Pr(n^* = 1 \mid s, \mathbf{x}) = F(\Delta^M, +\infty) + F(+\infty, \Delta^M) - F(\Delta^M, \Delta^M) - F(\Delta^D, \Delta^D) \\
&\Pr(n^* = 0 \mid s, \mathbf{x}) = 1 - F(\Delta^M, +\infty) - F(+\infty, \Delta^M) + F(\Delta^M, \Delta^M)
\end{aligned} \tag{22}$$

where $F(.)$ is the distribution function of the unobservables. As shown by Bresnahan and Reiss (1990, 1991b), the restrictions in equation (22), together with a known parametric distribution of the unobservables, provide the identification of the variable profit and the entry cost functions of the model.

The problem of multiple equilibria becomes more serious when firms are heterogeneous in terms of observable variables and the researcher is interested in the identity, or the characteristics, of firms active in the market. In this context, the indeterminacy associated with multiple equilibria can generate non-trivial problems in the estimation of these models. This issue has generated a substantial literature in the Econometrics of Games during the last decade. We provide a discussion on this issue in Section 4. The survey papers by Berry and Tamer (2006), Bajari, Hong, and Nekipelov (2013), and De Paula (2013) provide detailed discussions on this issue.

(d) Entry and spatial competition

How do market power and profits of a retail firm depend on the location of its store(s) relative to the location of competitors? How important is spatial differentiation to explain market power? These are important questions in the study of competition in retail markets. Seim (2006) studies these questions in the context of the video rental industry. Seim's work is the first study that endogenizes store locations and introduces spatial competition in a game of market entry. Her model has important similarities with the static game with single-store firms and incomplete information that we have presented above in Section 2.1(a). The main difference is that Seim's model does not include an explicit model of spatial consumer demand and price competition. Instead, she considers a "semi-structural" specification of a store's profit that captures the idea that the profit of a store declines when competing stores get closer in geographic space. The specification seems consistent with the idea that consumers face transportation costs and therefore spatial differentiation between stores can increase profits.

As described in the framework in Section 2.1(a), a local market has L business locations, indexed by ℓ. For every business location point, Seim defines B concentric rings around that point: a first ring of radius d_1 (e.g., half a mile), a second ring of radius d_2 (e.g., one mile), a third ring of radius d_3, and so on, where $d_1 < d_2 < \cdots < d_B$. The profit of a store in location ℓ depends on the number of other stores located within each of the B rings. Closer stores have stronger negative effects on profits. The specification of the profit function is:

$$\pi_{i\ell} = \mathbf{x}_\ell \, \beta + \sum_{\mathrm{b}=1}^{\mathrm{B}} \gamma_\mathrm{b} \, n_{\mathrm{b}\ell} + \xi_\ell + \varepsilon_{i\ell} \tag{23}$$

where $\beta, \gamma_1, \gamma_2, \dots, \gamma_\mathrm{B}$ are parameters; $\mathbf{x}_\ell$ is a vector of observable exogenous characteristics that affect profits in location ℓ; $n_{\mathrm{b}\ell}$ is the number of stores in ring b around location ℓ; ξ_ℓ represents exogenous characteristics of location ℓ that are unobserved to the researcher but common and observable to firms; and $\varepsilon_{i\ell}$ is a component of the profit of firm i in location ℓ that is private information to this firm. The profit from not being active in the market is equal to $\pi_{i0} = \varepsilon_{i0}$. Private information variables $\{\varepsilon_{i\ell}\}$ are assumed i.i.d. over firms and locations with the Type 1 Extreme Value distribution. The parameters $\gamma_1, \gamma_2, \dots, \gamma_\mathrm{B}$ capture the effect of spatial differentiation. We expect these parameters to be negative and decline in absolute value with the index of the ring radius b.

A firm does not know other firms' private information, and therefore, the number-of active stores at different ring-locations $\{n_{\mathrm{b}\ell}\}$ is unknown to this firm. Instead, the firm has a belief or expectation on these values. Let $\{n^e_{\mathrm{b}\ell}\}$ be a firm's expectation about the number of stores active at ring-location (b, ℓ). Given its beliefs, a firm chooses the entry-location decision that maximizes its expected profit. Its best response function is:

$$a_{i\ell} = 1\left\{ \mathbf{x}_\ell \beta + \sum_{\mathrm{b}=1}^{\mathrm{B}} \gamma_\mathrm{b} \, n^e_{\mathrm{b}\ell} + \xi_\ell + \varepsilon_{i\ell} \geq \mathbf{x}_{\ell'} \beta + \sum_{\mathrm{b}=1}^{\mathrm{B}} \gamma_\mathrm{b} \, n^e_{\mathrm{b}\ell'} + \xi_{\ell'} + \varepsilon_{i\ell'} \quad \forall \ell' \neq \ell \right\} \tag{24}$$

Integrating this best response function over the distribution of the private information variables, we obtain a probabilistic representation. Given the Extreme Value assumption on the distribution of private information, the probability that the best response of a firm is to have a store in location ℓ has the following logit form:

$$\mathrm{P}_{\ell} = \frac{exp\left\{\mathbf{x}_{\ell}\beta + \sum_{\mathrm{b}=1}^{\mathrm{B}} \gamma_{\mathrm{b}}\, n_{\mathrm{b}\ell}^{e} + \xi_{\ell}\right\}}{1 + \sum_{\ell'=1}^{L} exp\left\{\mathbf{x}_{\ell'}\beta + \sum_{\mathrm{b}=1}^{\mathrm{B}} \gamma_{\mathrm{b}}\, n_{\mathrm{b}\ell'}^{e} + \xi_{\ell'}\right\}} \tag{25}$$

In equilibrium, firms' beliefs/expectations must be consistent with other firms' best responses. This implies the following relationship between the expected number of firms $n_{\mathrm{b}\ell}^{e}$ and the vector of choice probabilities, $\mathbf{P} = \{\mathrm{P}_{\ell}: \ell = 1, 2, \ldots, L\}$:

$$n_{\mathrm{b}\ell}^{e} = N \left[\sum_{\ell'=1}^{L} D_{\ell\ell'}^{b}\ \mathrm{P}_{\ell'}\right] \tag{26}$$

where $D_{\ell\ell'}^{b}$ is a binary indicator of the event "ℓ' belongs to ring b around ℓ". Plugging equation (26) into (25), we obtain a system of L equations with L unknowns that define a fixed point mapping in the space of the vector of entry probabilities $\mathbf{P} = \{\mathrm{P}_{\ell}: \ell = 1, 2, \ldots, L\}$. An equilibrium of the model is a fixed point of this mapping. By Brower's Theorem an equilibrium exists. The equilibrium may not be unique. Seim shows that if the γ parameters are not large, they decline fast enough with b, and locations are not very heterogeneous, then the equilibrium is unique.

In their empirical study on competition between big-box discount stores in US (i.e., Kmart, Target and Wal-Mart), Zhu and Singh (2009) extend Seim's entry model by introducing firm heterogeneity. The model allows competition effects to be asymmetric across three different chains. The model can incorporate a situation where, for example, the impact on the profit of Target of a Wal-Mart store 10 miles away is stronger than the impact of a Kmart store located 5 miles away. The specification of the profit function of a store of chain *i* at location ℓ is:

$$\pi_{i\ell} = \mathbf{x}_{\ell}\,\beta_{i} + \sum_{j \neq i} \sum_{\mathrm{b}=1}^{\mathrm{B}} \gamma_{bij}\, n_{b\ell j} + \xi_{\ell} + \varepsilon_{i\ell} \tag{27}$$

where $n_{b\ell j}$ represents the number of stores that chain j has within the b-ring around location ℓ. Despite the paper studies competition between retail chains, it still makes similar simplifying assumptions as in Seim's model that ignores important aspects of competition between retail chains. In particular, the model ignores economies of density, and firms' concerns on cannibalization between stores of the same chain. It assumes that the entry decisions of a retail chain are made independently at each location. Under these assumptions, the equilibrium of the model can be described as a vector of $N * L$ entry probabilities, one for each firm and location, that solves the following fixed point problem:

$$\mathrm{P}_{i\ell} = \frac{exp\{\mathbf{x}_{\ell}\,\beta_i + \sum_{j\neq i}\sum_{\mathrm{b}=1}^{\mathrm{B}}\gamma_{bij}\,N\big[\sum_{\ell'=1}^{L} D_{\ell\ell'}^{b}\;\;\mathrm{P}_{j\ell'}\big]\;+\xi_{\ell}\}}{1+\sum_{\ell'=1}^{L} exp\{\mathbf{x}_{\ell'}\,\beta_i + \sum_{j\neq i}\sum_{\mathrm{b}=1}^{\mathrm{B}}\gamma_{bij}\,N\big[\sum_{\ell''=1}^{L} D_{\ell'\ell''}^{b}\;\;\mathrm{P}_{j\ell''}\big]\;+\xi_{\ell'}\}} \tag{28}$$

The authors find substantial heterogeneity in the competition effects between these three big-box discount chains, and in the pattern of how these effects decline with distance. For instance, Wal-Mart's supercenters have a very substantial impact even at large distance.

Datta and Sudhir (2013) estimate an entry model of grocery stores that endogenizes both location and product type decisions. Their main interests are the consequence of zoning on market structure. Zoning often reduces firms' ability to avoid competition by locating remotely each other. Theory suggests that in such a market firms have a stronger incentive to differentiate their products. Their estimation results support this theoretical prediction. The authors also investigate different impacts of various types of zoning ("centralized zoning", "neighborhood zoning", "outskirt zoning") on equilibrium market structure.[25]

(e) Multi-store firms

As we have mentioned above, economies of density and cannibalization are potentially important factors in store location decisions of retail chains. A realistic model of competition between retail chains should incorporate this type of spillover effects. Taking into account these effects requires a model of competition between multi-store firms similar to the one in Section 2.1(b). The model takes into account the joint determination of a firm's entry decisions at different locations. A firm's entry decision is represented by the *L-dimension* vector $\boldsymbol{a}_i \equiv \{a_{i\ell} : \ell = 1, 2, \ldots, L\}$, with $a_{i\ell} \in \{0,1\}$, such that the set of possible actions contains 2^L elements. For instance, Jia (2008) studies competition between two chains (Wal-Mart and Kmart) over 2,065 locations (US counties). The number of possible decisions of a retail chain is 2^{2065}, which is larger than 10^{621}. It is obvious that, without further restrictions, computing firms' best responses is intractable.

Jia (2008) proposes and estimates a game of entry between Kmart and Wal-Mart over more than two thousand locations (counties). Her model imposes restrictions on the specification of firms' profits that imply the supermodularity of the game and facilitate substantially the computation of an equilibrium. Suppose that we index the two firms as i and j. The profit function of a firm, say i, is $\Pi_i = VP_i(\boldsymbol{a}_i, \boldsymbol{a}_j) - EC_i(\boldsymbol{a}_i)$, where VP_i is the variable profit function such that

$$VP_i(\boldsymbol{a}_i, \boldsymbol{a}_j) = \sum_{\ell=1}^{L} a_{i\ell}\,[\mathbf{x}_{\ell}\,\beta_i + \gamma_{ij}\,a_{j\ell}], \tag{29}$$

[25] Schivardi and Pozzi (2015) provide a broader discussion of zoning and its impact on entry.

and EC_i is the entry cost function such that

$$EC_i(\boldsymbol{a}_i) = \sum\nolimits_{\ell=1}^{L} a_{i\ell} \left[\theta_{i\ell}^{EC} - \frac{\theta^{ED}}{2} \sum_{\ell' \neq \ell} \frac{a_{i\ell'}}{d_{\ell\ell'}} \right]. \tag{30}$$

where $\mathbf{x}_\ell$ is a vector of market/location characteristics; γ_{ij} is a parameter that represents the effect on the profit of firm i of competition from a store of chain j; $\theta_{i\ell}^{EC}$ is the entry cost that firm i would have in location ℓ in the absence of economies of density (i.e., if it were a single-store firm); θ^{ED} is a parameter that represents the magnitude of the economies of density and is assumed to be positive; and $d_{\ell\ell'}$ is the distance between locations ℓ and ℓ'. Jia further assumes that the entry cost $\theta_{i\ell}^{EC}$ consists of three parts: $\theta_{i\ell}^{EC} = \theta_i^{EC} + (1-\rho)\xi_\ell + \varepsilon_{i\ell}$, where θ_i^{EC} is chain-fixed effects, ρ is a scale parameter, ξ_ℓ is a location random effect, and $\varepsilon_{i\ell}$ is a firm-location error term. Both $\{\xi_\ell\}$ and $\{\varepsilon_{i\ell}\}$ are i.i.d. draws from the standard normal distribution and known to all the players when making decisions. To capture economies of density, the presence of the stores of the same firm at other locations is weighted by the inverse of the distance between locations, $1/d_{\ell\ell'}$. This term is multiplied by one-half to avoid double counting in the total entry cost of the retail chain.

The specification of the profit function in equations (29) and (30) imposes some important restrictions. Under this specification, locations are interdependent only through economies of density. In particular, there are no cannibalization effects between stores of the same chain at different locations. Similarly, there is no spatial competition between stores of different chains at different locations. In particular, this specification ignores the spatial competition effects between Kmart, Target, and Wal-Mart that Zhu and Singh (2009) find in their study. The specification also rules out cost savings that do not depend on store density such as lower wholesale prices due to strong bargaining power of chain stores. The main motivation for these restrictions is to have a supermodular game that facilitates very substantially the computation of an equilibrium, even when the model has a large number of locations.

In a Nash equilibrium of this model, the entry decisions of a firm, say i, should satisfy the following L optimality conditions:

$$a_{i\ell} = 1\left\{ \mathbf{x}_\ell \beta_i + \gamma_{ij}\, a_{j\ell} - \theta_{i\ell}^{EC} + \frac{\theta^{ED}}{2} \sum_{\ell' \neq \ell} \frac{a_{i\ell'}}{d_{\ell\ell'}} \geq 0 \right\} \tag{31}$$

These conditions can be interpreted as the best response of firm i in location ℓ given the other firm's entry decisions, and given also firm i's entry decisions at locations other than ℓ. We can write this system of conditions in a vector form as $\boldsymbol{a}_i = \mathrm{br}_i(\boldsymbol{a}_i, \boldsymbol{a}_j)$. Given $\boldsymbol{a}_j$, a fixed point of the mapping $\text{br}_i(., \boldsymbol{a}_j)$ is a (full) best response of firm i to the choice $\boldsymbol{a}_j$ by firm j. With $\theta^{ED} > 0$ (i.e., economies of density), it is clear from equation (31) that the mapping br_i is increasing in $\boldsymbol{a}_i$. By Topkis's Theorem, this increasing property implies that: (i) the mapping has at least one fixed point solution; (ii) if it has multiple fixed points they are ordered from the lowest to the largest; and (iii) the smallest (largest) fixed point can be obtained by successive iterations in the mapping br_i using as starting value $\boldsymbol{a}_i = \mathbf{0}$ ($\boldsymbol{a}_i = \mathbf{1}$). Given these properties, Jia shows that the following algorithm provides the Nash Equilibrium that is most profitable for firm i: (Step $[i]$) Given the lowest possible value for $\boldsymbol{a}_j$, i.e., $\boldsymbol{a}_j = (0,0, \dots 0)$, we apply successive iterations with respect to $\boldsymbol{a}_i$ in the fixed point mapping $\text{br}_i(., \boldsymbol{a}_j = \mathbf{0})$ starting at $\boldsymbol{a}_i = (1,1, \dots 1)$. These iterations converge to the largest best response of firm i, that we denote by $\boldsymbol{a}_i^{(1)} = BR_i^{(High)}(\mathbf{0})$. (Step $[j]$) Given $\boldsymbol{a}_i^{(1)}$, we apply successive iterations with respect to $\boldsymbol{a}_j$ in the fixed point mapping $\text{br}_j(., \boldsymbol{a}_i^{(1)})$ starting at $\boldsymbol{a}_j = \mathbf{0}$. These iterations converge to the lowest best response of firm j, that we denote by $\boldsymbol{a}_j^{(1)} = BR_j^{(Low)}(\boldsymbol{a}_i^{(1)})$. Then, we keep iterating in (Step $[i]$) and (Step $[j]$) until convergence. At any iteration, say k, given $\boldsymbol{a}_j^{(k-1)}$ we first apply (Step $[i]$) to obtain $\boldsymbol{a}_i^{(k)} = BR_i^{(HIgh)}\left(\boldsymbol{a}_j^{(k-1)}\right)$, and then we apply (Step $[j)$ to obtain $\boldsymbol{a}_j^{(k)} = BR_j^{(Low)}\left(\boldsymbol{a}_i^{(k)}\right)$. The supermodularity of the game assures the convergence of this process and the resulting fixed point is the Nash equilibrium that most favors firm i. Jia combines this solution algorithm with a simulation of unobservables to estimate the parameters of the model using the method of simulated moments (MSM).

In his empirical study of convenience stores in Okinawa Island of Japan, Nishida (forthcoming) extends Jia's model in two directions. First, a firm is allowed to open multiple stores (up to four) in the same location. Second, the model explicitly incorporates some form of spatial competition: a store's revenue is affected not only by other stores in the same location but also by those in adjacent locations.

Although the approach used in these two studies is elegant and useful, its use in other applications is somewhat limited. First, supermodularity requires that the own network effect on profits is monotonic, i.e., the effect of $\sum_{\ell' \neq \ell} \frac{a_{i\ell'}}{d_{\ell\ell'}}$ is either always positive ($\theta^{ED} > 0$) or always negative ($\theta^{ED} < 0$). This condition rules out situations where the net effect of cannibalization and economies of density varies across markets. Second, the number of (strategic) players must be equal to two. For a game to be supermodular, players' strategies must be strategic complements. In a model of market entry, players' strategies are strategic substitutes. However, when the number of players is equal to two, any game of strategic substitutes can be transformed into one of strategic complements by changing the order of strategies of one player (e.g., use zero for entry and one for no entry). This trick no longer works when we have more than two players.[26]

Ellickson et al. (2013, EHT hereafter) propose an alternative estimation strategy and apply it to data of U.S. discount store chains. Their estimation method is based on a set of inequalities that arise from the best response condition of a Nash equilibrium. Taking its opponents' decisions as given, a chain's profit associated with its observed entry decision must be larger than the profit of any alternative entry decision. EHT consider particular deviations that relocate one of the observed stores to another location. Let $\boldsymbol{a}_i^*$ be the observed vector of entry decisions of firm i, and suppose that in this observed vector the firm has a store in location ℓ but not in location ℓ'. Consider the alternative (hypothetical) choice $\boldsymbol{a}_i^{(\ell\to\ell')}$ that is equal to $\boldsymbol{a}_i^*$ except that the store in location ℓ is closed and relocated to location ℓ'. Revealed preference implies that $\pi_i(\boldsymbol{a}_i^*) \geq \pi_i(\boldsymbol{a}_i^{(\ell\to\ell')})$. EHT further simplify this inequality by assuming that there are no economies of scope or density (e.g., $\theta^{ED} = 0$), and that there are no firm-location-specific factors unobservable to the researcher, i.e., $\varepsilon_{i\ell} = 0$. Under these two assumptions, the inequality above can be written as the profit difference between two locations

$$[\mathbf{x}_\ell - \mathbf{x}_{\ell'}]\beta_i + \sum_{j\neq i} \gamma_{ij} \left[a_{j\ell}^* - a_{j\ell'}^*\right] + [\xi_\ell - \xi_{\ell'}] \geq 0 \tag{32}$$

Now, consider another chain, say k, that has an observed choice $\boldsymbol{a}_k^*$ with a store in location ℓ' but not in location ℓ. For this chain, we consider the opposite (hypothetical) relocation decision that for firm i above: the store in location ℓ' is closed and a new store is open in location ℓ. For this chain, revealed preference implies that $[\mathbf{x}_{\ell'} - \mathbf{x}_\ell]\beta_k + \sum_{j\neq k} \gamma_{kj} \left[a_{j\ell'}^* - a_{j\ell}^*\right] + [\xi_{\ell'} - \xi_\ell] \geq 0.$ Summing up the inequalities for firms i and k, we generate an inequality that is free from location fixed effects ξ_ℓ.

$$[\mathbf{x}_\ell - \mathbf{x}_{\ell'}](\beta_i - \beta_k) + \sum_{j\neq i} \gamma_{ij} \left[a_{j\ell}^* - a_{j\ell'}^*\right] + \sum_{j\neq k} \gamma_{kj} \left[a_{j\ell'}^* - a_{j\ell}^*\right] \geq 0. \tag{33}$$

EHT construct a number of inequalities of this type and obtain estimates of the parameters of the model by using a smooth maximum score estimator (Manski 1975, Horowitz, 1992, Fox, 2010).

Unlike the lattice theory approach of Jia and Nishida, the approach applied by EHT can accommodate more than two players, allows the researcher to be agnostic about equilibrium selections, and is robust to the presence of unobserved market heterogeneity. Their model, however, rules out any explicit interdependence between stores in different locations, including

[26] See Fudenberg and Tirole (2000) for further details on this topic.

spatial competition, cannibalization and economies of density. Although incorporating such inter-locational interdependencies does not seem to cause any fundamental estimation issue, doing so can be difficult in practice as it considerably increases the amount of computation. Another possible downside of this approach is the restriction it imposes on unobservables. The only type of structural errors that this model includes are the variables ξ_ℓ that are common for all firms. Therefore, to accommodate observations that are incompatible with inequalities in (33) above, the model requires non-structural errors, which may be interpreted as firms' optimization errors.

(f) Dynamics with single-store firms

When the entry cost is partially sunk, firms' entry decisions depend on their incumbency status, and dynamic models become more relevant. The role of sunk entry costs in shaping market structure in an oligopoly industry was first empirically studied by Bresnahan and Reiss (1993). They estimate a two-period model using panel data of the number of dentists. Following recent developments in the econometrics of dynamic games of oligopoly competition,[27] several studies have estimated dynamic games of market entry-exit in different retail industries.

Aguirregabiria and Mira (2007) estimate dynamic games of market entry and exit for five different retail industries: restaurants, bookstores, gas stations, shoe shops, and fish shops. They use annual data from a census of Chilean firms created for tax purposes by the Chilean Internal Revenue Service during the period 1994-1999. The estimated models show significant differences in fixed costs, entry costs, and competition effects across the five industries, and these three parameters provide a precise description of the observed differences in market structure and entry-exit rates between the five industries. Fixed operating costs are a very important component of total profits of a store in the five industries, and they range between 59% (in restaurants) to 85% (in bookstores) of the variable profit of a monopolist in a median market. Sunk entry costs are also significant in the five industries, and they range between 31% (in shoe shops) and 58% (in gas stations) of a monopolist variable profit in a median market. The estimates of the parameter that measures competition effect show that restaurants are the retailers with the smallest competition effects, that might explained by a higher degree of horizontal product differentiation in this industry.

Suzuki (2012) examines the consequence of tight land use regulation on market structure of hotels through its impacts on entry costs and fixed costs. He estimates a dynamic game of entry-exit of mid-scale hotels in Texas that incorporates detailed measures of land use regulation into cost functions of hotels. The estimated model shows that imposing stringent regulation increases costs considerably and has substantial effects on market structure and hotel profits. Consumers also incur a substantial part of the costs of regulation in the form of higher prices.

[27] See Aguirregabiria and Nevo (2013) for a recent survey of this literature.

Dunne et al. (2013) estimate a dynamic game of entry and exit in the retail industries of dentists and chiropractors in US, and use the estimated model to evaluate the effects on market structure of subsidies for entry in small geographic markets, i.e., markets that were designated by the government as Health Professional Shortage Areas (HPSA). The authors compare the effects of this subsidy with those of a counterfactual subsidy on fixed costs, and they find that subsidies on entry costs are cheaper, or more effective for the same present value of the subsidy.

Yang (2014) extends the standard dynamic game of market entry-exit in a retail market by incorporating information spillovers from incumbent firms to potential entrants.[28] In his model, a potential entrant does not know a market-specific component in the level of profitability of a market (e.g., a component of demand or operating costs). Firms learn about this profitability only when they actually enter that market. In this context, observing incumbents stay in this market is a positive signal for potential entrants about the quality of this market. Potential entrants use these signals to update their beliefs about the profitability of the market (i.e., Bayesian updating). These information spillovers from incumbents may contribute to explain why we observe retail clusters in some geographic markets. Yang estimates his model using data from the fast food restaurant industry in Canada, which goes back to the initial conditions of this industry in Canada. He finds significant evidence supporting the hypothesis that learning from incumbents induces retailers to herd into markets where others have previously done well in, and to avoid markets where others have previously failed in.

(g) Dynamics and spatial competition between multi-store firms

A structural empirical analysis of economies of density, cannibalization, or spatial entry deterrence in retail chains requires the specification and estimation of models that incorporate dynamics, multi-store firms, and spatial competition. Some recent papers present contributions on this research topic.

Holmes (2011) studies the temporal and spatial pattern of store expansion by Wal-Mart during the period 1971-2005. He proposes and estimates a dynamic model of entry and store location by a multi-store firm similar to the one that we have described in Section 2.1(c) above. The model incorporates economies of density and cannibalization between Wal-Mart stores, though it does not model explicitly competition from other retailers or chains (e.g., Kmart or Target), and therefore it abstracts from dynamic strategic considerations such as spatial entry deterrence. The model also abstracts from price variation and assumes that Wal-Mart sets constant prices across all stores and over time. However, Holmes takes into account three different types of stores and plants in Wal-Mart retail network: regular stores that sell only general merchandise; supercenters, that sell both general merchandise and food; and distribution centers, which are the

[28] Yang's paper is motivated by the empirical results in Toivanen and Waterson (2005) who find substantial amount of clustering in the stores of McDonalds and Burger King in UK. The authors suggest that informational spillovers might be a possible explanation for this finding.

warehouses in the network, and that have also two different types, i.e., general and food distribution centers. The distinction between these types of stores and warehouses is particularly important to explain the evolution of Wal-Mart retail network over time and space. In the model, every year Wal-Mart decides the number and the geographic location of new regular stores, supercenters, and general and food distribution centers. Economies of density are channeled through the benefits of stores being close to distribution centers. The structural parameters of the model are estimated using the Moment Inequalities estimation method in Pakes et al. (2014). More specifically, moment inequalities are constructed by comparing the present value of profits from Wal-Mart's actual expansion decision with the present value from counterfactual expansion decisions which are slight deviations from the observed ones. Holmes finds that Wal-Mart obtains large savings in distribution costs by having a dense store network.

Igami and Yang (2014) study the trade-off between cannibalization and spatial pre-emption in the fast-food restaurant industry, e.g., McDonalds, Burger King, etc. Consider a chain store that has already opened its first store in a local market. Opening an additional store increases this chain's current and future variable profits by, first, attracting more consumers and, second, preventing its rivals' future entries (preemption). However, the magnitude of this increase could be marginal when the new store steals customers from its existing store (cannibalization). Whether opening a new store economically makes sense or not depends on the size of the entry cost. Igami and Yang estimate a dynamic structural model and find the quantitative importance of preemptive motives. However, they do not model explicitly spatial competition, or allow for multiple geographic locations within their broad definition of geographic market.

Schiraldi, Smith, and Takahashi (2013) study store location and spatial competition between UK supermarket chains. They propose and estimate a dynamic game similar to the one in Aguirregabiria and Vicentini (2012) that we have described in Section 2.1(c). A novel and interesting aspect of this application is that the authors incorporate the regulator's decision to approve or reject supermarkets' applications for opening a new store in a specific location. The estimation of the model exploits a very rich dataset from the U.K. supermarket industry on exact locations and dates of store openings/closings, applications for store opening, approval/rejection decisions by the regulator, as well as rich data of consumer choices and consumer locations. The estimated model is used to evaluate the welfare effects of factual and counterfactual decision rules by the regulator.

3. DATA

The datasets that have been used in empirical applications of structural models of entry in retail markets consist of a sample of geographic markets with information on firms' entry decisions and consumer socio-economic characteristics over one or several periods of time. In these applications, the number of firms and time periods is typically small such that statistical inference (i.e., the construction of sample moments and the application of law of large numbers and central limit theorems) is based on a "large" number of markets. In most applications, the number of geographic markets is between a few hundred and a few thousand. Within these common features, there is substantial heterogeneity in the type of data that have been used in empirical applications.

In this section, we concentrate on four features of the data that are particularly important because they have substantial implications on the type of model that can be estimated, the empirical questions that we can answer, and the econometric methods to use. These features are: (a) the selection of geographic markets; (b) presence or not of within market spatial differentiation; (c) information on prices, quantities, or sales at the store level; and (d) information on potential entrants. [29]

(a) Selection of geographic markets

In a seminal paper, Bresnahan and Reiss (1990) use cross-sectional data from 149 small U.S. towns to estimate a model of entry of automobile dealerships. For each town, the dataset contains information on the number of stores in the market, demographic characteristics such as population and income, and input prices such as land prices. The selection of the 149 small towns is based on the following criteria: the town belongs to a county with fewer than 10,000 people; there is no other town with a population of over 1000 people within 25 miles of the central town; and there is no large city within 125 miles. These conditions for the selection of a sample of markets are typically described as the "isolated small towns" market selection. This approach has been very influential and has been followed in many empirical applications of entry in retail markets. The main motivation for using this sample selection is in the assumptions of spatial competition in the Bresnahan-Reiss model described in Section 2. That model assumes that the location of a store within a market does not have any implication on its profits or in the degree of competition with other stores. This assumption is plausible only in small towns where the possibilities for spatial differentiation are very limited. If this model were estimated using a sample of large cities, we would spuriously find very small competition effects simply because there is negligible or no competition at all between stores located far away of each other within the city. The model also assumes that there is no competition between stores located in different markets. This assumption is plausible only if the market under study is not geographically close to other markets; otherwise the model would ignore relevant competition from stores outside the market.

Although the "isolated small towns" approach has generated a good number of important applications, it has some limitations. The extrapolation to urban markets of the estimation results obtained in these samples of rural markets is in general not plausible. Focusing on rural areas

[29] A comprehensive review of retail datasets is beyond the scope of this paper. See Hwang (2015) for more detail on datasets used in retail research.

makes the approach impractical for many interesting retail industries that are predominantly urban. Furthermore, when looking at national retail chains, these rural markets account for a very small fraction of these firms' total profits.

(b) Within market spatial differentiation

The limitations of the "isolated small towns" approach have motivated the development of empirical models of entry in retail markets that take into account the spatial locations and differentiation of stores within a city market. The work by Seim (2006) was seminal in this evolution of the literature. In Seim's model, a city is partitioned into many small locations or blocks, e.g., census tracts, or a uniform grid of square blocks. A city can be partitioned into dozens, hundreds, or even thousands of these contiguous blocks or locations. In contrast to the "isolated small towns" approach, these locations are not isolated, and the model allows for competition effects between stores at different locations. The datasets in these applications contain information on the number of stores, consumer demographics, and input prices at the block level. This typically means that the information on store locations should be geocoded, i.e., the exact latitude and longitude of each store location. Information on consumer demographics is usually available at a more aggregate geographic level.

The researcher's choice for the size of a block depends on multiple considerations, including the retail industry under study, data availability, specification of the unobservables, and computational cost. In principle, the finer is the grid the more flexible can be the model to measure spatial substitution between stores. The computational cost of estimating the model can increase rapidly with the number of locations. The assumption on the distribution of the unobservables across locations is also important, too. A common approach is to use a definition of a block/location at which demographic information is available, e.g., the set of locations is equal to the set of census tracts within the city.[30] While convenient, a drawback of this approach is that some blocks, especially those in the periphery of a city, tend to be very large. These large blocks are often problematic because (i) within-block spatial differentiation seems plausible, and (ii) the distance to other blocks becomes highly sensitive to choices of block centroids. In particular, a mere use of geometric centroids in these large blocks can be quite misleading as the spatial distribution of population is often quite skewed. To avoid this problem, Seim (2006) uses population weighted centroids rather than (unweighted) geometric centroids. An alternative approach to avoid this problem is to draw a square grid on the entire city and use each square as a possible location, as in Datta and Sudhir (2013) and Nishida (forthcoming). The value of consumer demographics in a square block is equal to the weighted average of the demographics at the census tracts that overlap with the square. The advantage of this approach is that each submarket has a uniform shape. In practice, implementation of this approach requires the removal of certain squares where entry cost is prohibitive. These areas include those with some

[30] Gowrisankaran and Krainer (2011) take an alternative approach. In their study on entries of ATMs, the authors use convenience stores, grocery stores and banks as possible locations of ATMs.

particular natural features (e.g., lakes, mountains and wetlands) or where commercial space is prohibited by zoning. For example, Nishida (forthcoming) excludes areas with zero population, and Datta and Sudhir (2013) remove areas that do not have any big box store as these areas are very likely to be zoned for either residential use or small stores.

So far, all the papers that have estimated this type of model have considered a sample of cities (but not locations within a city) that is still in the spirit of Bresnahan-Reiss isolated small markets approach. For instance, Seim selects U.S. cities with population between 40,000 and 150,000 people, and without other cities with more than 25,000 people within 20 miles. The main reason for this is to avoid the possibility of outside competition at the boundaries of a city. It is interesting that in the current generation of these applications, statistical inference is based on the number of cities and not on the number of locations. A relevant question is whether this model can be estimated consistently using data from a single city with many locations, i.e., the estimator is consistent when the number of locations goes to infinity. This type of application can be motivated by the fact that city characteristics that are relevant for these models, such as the appropriate measure of geographic distance, transportation costs, or land use regulations and zoning, can be city specific. Xu (2014) studies an empirical game of market entry for a single city (network) and presents conditions for consistency and asymptotic normality of estimators as the number of locations increases. As far as we know, there are not yet empirical applications following that approach.

(c) Information on prices, quantities, or sales at the store level

Most applications of models of entry in retail markets do not use data on prices and quantities due to the lack of such data. The most popular alternative is to estimate the structural (or semi-structural) parameters of the model using market entry data only, e.g., Bresnahan and Reiss (1990), Mazzeo (2002), Seim (2006), or Jia (2008), among many others. Typically, these studies either do not try to separately identify variable profits from fixed costs, or they do it by assuming that the variable profit is proportional to an observable measure of market size. Data on prices and quantities at store level can substantially help the identification of these models. In particular, it is possible to consider a richer specification of the model that distinguishes between demand, variable cost, and fixed cost parameters, and includes unobservable variables into each of these components of the model.

A sequential estimation approach is quite convenient for the estimation of this type of model. In a first step, data on prices and quantities at the store level can be used to estimate a spatial demand system as in Davis (2006) for movie theatres or Houde (2012) for gas stations. Note that, in contrast to standard applications of demand estimation of differentiated products, the estimation of demand models of this class should deal with the endogeneity of store locations. In other words, in these demand models, not only prices are endogenous but also the set of "products" or stores available at each location is potentially correlated with unobserved errors in the demand system. In a second step, variable costs can be estimated using firms' best response functions in Bertrand or Cournot model.[31] Finally, in a third step, we estimate fixed cost parameters using the entry game and information of firms' entry and store location decisions. It is important to emphasize that the estimation of a demand system of spatial differentiation in the first step provides the structure of spatial competition effects between stores at different locations, such that the researcher does not need to consider other type of semi-reduced form specifications of strategic interactions, as in Seim (2006) among others.

In some applications, price and quantity are not available, but there is information on revenue at the store level (e.g., Ellickson and Misra, 2012, Aguirregabiria, Clark, and Wang, 2013, Suzuki, 2013). This information can be used to estimate a (semi reduced form) variable profit function in a first step, and then in a second step the structure of fixed costs is estimated.

(d) Information on potential entrants

An important modelling decision in empirical entry games is to define the set of potential entrants. In most cases, researchers have limited information on the number of potential entrants, let alone their identity. This problem is particularly severe when entrants are mostly independent small stores (e.g., mom-and-pop stores). A practical approach is to estimate the model under different numbers of potential entrants and examine how estimates are sensitive to these choices, e.g., Seim (2006) and Jia (2014). The problem is less severe when most entrants belong to national chains (e.g., big box stores) because the names of these chains are often obvious and the number is typically small.

4. ESTIMATION

The estimation of games of entry and spatial competition in retail markets should deal with some common issues in the econometrics of games and dynamic structural models. Here we do not try to present a detailed discussion of this econometric literature. Instead, we provide a brief description of the main issues, with an emphasis on aspects that are particularly relevant for empirical applications in retail industries.[32]

4.1. Multiple equilibria

Entry models with heterogeneous firms often generate more than one equilibria for a given set of parameters. Multiple equilibria pose challenges to the researcher for two main reasons. First, standard maximum likelihood estimation no longer works because the likelihood of certain

[31] Given space limitations, we do not discuss estimation of demand and variable costs. See Ackerberg et al. (2007) and Mortimer (2015) for the details of demand estimation.

[32] For recent surveys specifically dealing with the econometrics of these models, see Berry and Tamer (2006), Bajari, Hong, and Nekipelov (2013), and De Paula (2013) for static games, and Aguirregabiria and Mira (2010), and Aguirregabiria and Nevo (2013), for dynamic games.

outcomes is not well-defined without knowing the equilibrium selection mechanism. Second, without further assumptions, some predictions or counterfactual experiments using the estimated model are subject to an identification problem. These predictions depend on the type of equilibrium that is selected in an hypothetical scenario not included in the data.

Several approaches have been proposed to estimate an entry game with multiple equilibria. Which method works the best depends on assumptions imposed in the model, especially its information structure. In a game of complete information, there are at least four approaches. The simplest approach is to impose some particular equilibrium selection rule beforehand and estimate the model parameters under this rule. For instance, Jia (2008) estimates the model of competition between big-box chains using the equilibrium that is most preferable to K-mart. She also estimates the same model under alternative equilibrium selection rules to check for the robustness of some of her results. The second approach is to construct a likelihood function for some endogenous outcomes of the game that are common across all the equilibria. Bresnahan and Reiss (1991) estimate their model by exploiting the fact that, in their model, the total number of entrants is unique in all the equilibria.

A third approach is to make use of inequalities that are robust to multiple equilibria. One example is the profit inequality approach of Ellickson et al. (2013), which we described in Section 2.2(e) above. Another example is the method of moment inequality estimators proposed by Ciliberto and Tamer (2009). They characterize the lower and upper bounds of the probability of a certain outcome that are robust to any equilibrium selection rule. Estimation of structural parameters relies on the set of probability inequalities constructed from these bounds. In the first step, the researcher nonparametrically estimates the probabilities of equilibrium outcomes conditional on observables. The second step is to find a set of structural parameters such that the resulting probability inequalities are most consistent with the data. The application of Ciliberto and Tamer's approach to a spatial entry model may not be straightforward. In models of this class, the number of possible outcomes (i.e., market structures) is often very large. For example, consider a local market consisting of ten sub-blocks. When two chains decide whether they enter into each of these sub-blocks, the total number of possible market structures is 1,024 (=2^{10}). Such a large number of possible outcomes makes it difficult to implement this approach for two reasons. The first stage estimate is likely to be very imprecise even when a sample size is reasonably large. The second stage estimation can be computationally intensive because one needs to check, for a given set of parameters, whether each possible outcome meets the equilibrium conditions or not.

A fourth approach proposed by Bajari, Hong and Ryan (2010) consists in the specification of a flexible equilibrium selection mechanism and in the joint estimation of the parameters in this mechanism and the structural parameters in firms' profit functions. Together with standard exclusion restrictions for the identification of games, the key specification and identification assumption in this paper is that the equilibrium selection function depends only on firms' profits.

In empirical games of incomplete information, the standard way to deal with multiple equilibria is to use a two-step estimation method (Aguirregabiria and Mira, 2007, and Bajari, Hong, Krainer and Nekipelov, 2010).[33] In the first step, the researcher estimates the probabilities of firms' entry conditional on market observables (called policy functions) in a nonparametric way, e.g., a sieves estimator. The second step is to find a set of structural parameters that are most consistent with the observed data and these estimated policy functions. A key assumption for the consistency of this approach is that, in the data, two markets with the same observable characteristics do not select different types of equilibria, i.e., same equilibria conditional on observables. Without this assumption, the recovered policy function in the first stage would be a weighted sum of firms' policies under different equilibria, making the second-stage estimates inconsistent. Several authors have recently proposed extensions of this method to allow for multiplicity of equilibria in the data for markets with the same observable characteristics.[34]

4.2. Unobserved Market Heterogeneity

Some market characteristics affecting firms' profits may not be observable to the researcher. For example, consider local attractions that spur the demand for hotels in a particular geographic location. Observing and controlling for all the relevant attractions are often impossible to the researcher. This demand effect implies that markets with such attractions should have more hotels than those without such attractions but with equivalent observable characteristics. Therefore, without accounting for this type of unobservables, researchers may wrongly conclude that competition boosts profits, or under-estimate the negative effect of competition on profits.

Unobserved market heterogeneity usually appears as an additive term (ω_ℓ) in the firm's profit function ($\pi_{i\ell}$) where ω_ℓ is a random effect from a distribution known up to some parameters.[35] The most common assumption (e.g., Seim, 2006, Zhu and Singh, 2009, Datta and Sudhir, 2013) is that these unobservables are common across locations in the same local market (i.e., $\omega_\ell = \omega$ for all ℓ). Under this assumption the magnitude of unobserved market heterogeneity matters whether the firm enters some location in this market but not which location. Orhun (2013) relaxes this assumption by allowing unobserved heterogeneity to vary across locations in the same market.

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[33] In principle, Ciliberto and Tamer's approach can be applied to a game of incomplete information as well. However, this approach may not be practical because the computation of the probability bounds requires the derivation of all the equilibria for each trial value of the structural parameters.

[34] See De Paula and Tang (2012), Aguirregabiria and Mira (2013), and Xiao (2014).

[35] This random effect assumption is not necessarily restrictive. For instance, it is easy to allow correlation between these unobservables and market characteristics using a correlated random effects model.

In a game of complete information, accommodating unobserved market heterogeneity does not require a fundamental change in the estimation process. In a game of incomplete information, however, unobserved market heterogeneity introduces an additional challenge. Consistency of the two-step method requires that the initial nonparametric estimator of firms' entry probabilities in the first step should account for the presence of unobserved market heterogeneity. A possible solution is to use a finite mixture model. In this model, every market's ω_ℓ is drawn from a distribution with finite support. Aguirregabiria and Mira (2007) show how to accommodate such market-specific unobservables into their nested pseudo likelihood (NPL) algorithm. Arcidiacono and Miller (2011) propose an expectation-maximization (EM) algorithm in a more general environment. An alternative way to deal with this problem is to use panel data with a reasonably long time horizon. In that way, we can incorporate market fixed effects as parameters to be estimated. This approach is popular when estimating a dynamic game (e.g., Ryan, 2012, and Suzuki, 2013).[36] A necessary condition to implement this approach is that every market at least observes some entries during the sample period.[37] Dropping markets with no entries from the sample may generate a selection bias.

4.3. Computation

The number of geographic locations, L, introduces two dimensionality problems in the computation of firms' best responses in games of entry with spatial competition. First, in a static game, a multi-store firm's set of possible actions includes all the possible spatial configurations of its store network. The number of alternatives in this set is equal to 2^L, and this number is extremely large even with modest values of L, such as a few hundred geographic locations. Without further assumptions, the computation of best responses becomes impractical. This is an important computational issue that has deterred some authors to account for multi-store retailers in their spatial competition models, e.g., Seim (2006), or Zhu and Singh (2009), among many others. As we have described in Section 2.2(e), two approaches that have been applied to deal with this issue are: (a) impose restrictions that guarantee supermodularity of the game (i.e., only two players, no cannibalization effects); (b) avoid the exact computation of best responses and use instead inequality restrictions implied by these best responses.

Looking at the firms' decision problem as a sequential or dynamic problem helps also to deal with the dimensionality in the space of possible choices. In a given period of time (e.g., year, quarter, month), we typically observe that a retail chain makes small changes in its network of stores, i.e., it opens a few new stores, or closes a few existing stores. Imposing these small changes as a restriction on the model implies a very dramatic reduction in the dimension of the action space such that the computation of best responses becomes practical, at least in a "myopic" version of the sequential decision problem.

[36] We are not aware of any study that estimates a static game using panel data.
[37] When estimating a dynamic game, every market needs to undergo at least one turnover instead.

However, to fully take into account the sequential or dynamic nature of a firm's decision problem, we also need to acknowledge that firms are forward looking. In the firm's dynamic programming problem, the set of possible states is equal to all the possible spatial configurations of a store network, and it has 2^L elements. Therefore, by going from a static model to a dynamic-forward-looking model, we have just "moved" the dimensionality problem from the action space into the state space. Recent papers propose different approaches to deal with this dimensionality problem in the state space. Arcidiacono et al. (2013) present a continuous-time dynamic game of spatial competition in a retail industry and propose an estimation method of this model. The continuous-time assumption eliminates the curse of dimensionality associated to integration over the state space. Aguirregabiria and Vicentini (2012) propose a method of spatial interpolation that exploits the information provided by the (indirect) variable profit function.

5. CONCLUDING REMARKS

We conclude with some ideas for further research.

Spillovers between different retail sectors. Existing applications of games of entry and spatial competition in retail markets concentrate on a single retail industry. However, there are also interesting spillover effects between different retail industries. Some of these spillovers are positive, e.g., good restaurants can make a certain neighborhood more attractive for shopping. There are also negative spillovers effects through land prices, i.e., retail sectors with high value per unit of space (e.g., jewellery stores) are willing to pay higher land prices that supermarkets that have low markups and are intensive in the use of land. The consideration and measurement of these spillover effects is interesting in itself, and it can help to explain the turnover and reallocation of industries in different parts of a city. Relatedly, endogenizing land prices would also open the possibility of using these models for the evaluation of specific public policies at the city level.

Richer datasets with store level information on prices, quantities, inventories. The identification and estimation of competition effects based mainly on data of store locations has been the rule more than the exception in this literature. This approach typically requires strong restrictions in the specification of demand and variable costs. The increasing availability of datasets with rich information on prices and quantities at product and store level should create a new generation of empirical games of entry and spatial competition that relax these restrictions. Also, data on store characteristics such as product assortments or inventories will allow to introduce these important decisions as endogenous variables in empirical models of competition between retail stores.

Measuring spatial pre-emption. So far, all the empirical approaches to measure the effects of spatial pre-emption are based on the comparison of firms' actual entry with firms' behavior in a counterfactual scenario characterized by a change in either (i) a structural parameter (e.g., a store exit value), or (ii) firms' beliefs (e.g., a firm believes that other firms' entry decisions do not respond to this firm's entry behavior). These approaches suffer the serious limitation that they do not capture only the effect of pre-emption and are contaminated by other effects. The development of new approaches to measure the pure effect of pre-emption would be a methodological contribution with relevant implications in this literature

Geography. Every local market is different in its shape and its road network. These differences may have important impacts on the resulting market structure. For example, the center of a local market may be a quite attractive location for retailers when all highways go through there. However, it may not be the case anymore when highways encircle the city center (e.g., Beltway in Washington D.C.). These differences may affect retailers' location choices and the degree of competition in an equilibrium. The development of empirical models of competition in retail markets that incorporate, in a systematic way, these idiosyncratic geographic features will be an important contribution in this literature.

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