ECO 2901
EMPIRICAL INDUSTRIAL ORGANIZATION

Lecture 3:
Market entry and spatial competition

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Lecture 3: Models of Firms' Spatial Location: Outline

1. Introduction
2. Models with single-store (product) firms
3. Models with multiple-store (product) firms

1. Firms' Spatial Location Introduction

Firms' Spatial Location: Introduction

- Consider the decision of a firm (e.g., coffee shop, restaurant, supermarket) of where to open a new store within a city.
- Different factors can play an important role:
  - Demand: what is the consumer traffic at different locations;
  - Rental prices
  - Location of competitors
  - Location of its own existing stores (cannibalization)
- Geographic distance can be an important source of product differentiation. Ceteris paribus, a firm's profit increases with its distance to competitors.

Space: Beyond geographic location of stores

- Models for the geographic location of stores can be applied to study firms' decisions on product design.
- We need to replace geographic space with the space of product characteristics, and define the relevant distance in that space.
- Similar factors play an important role in firms' product location decisions:
  - Consumer demand at different locations.
  - Cost of entry in a bundle of characteristics.
  - Location of competitors in the product space.

- Cannibalization of own preexisting products.

Empirical questions

- How do profits increase with distance to competitors?
- **Cannibalization**: to what extend a multi-product firm is concerned with competition between its own products?
- **Economies of scope**: Do the costs of a new store/product decline with the number of other stores/products the firm has?
- **Economies of density:** Do the costs of a new store/product decline with the spatial proximity to other stores/products the firm has?
- Effect on competition of **a change in the geography of the city**, e.g., new neighborhoods. Similarly, effect of an expansion in the space of **technologically feasible product characteristics**.

2. Models of Firms' Spatial Location: Single product (store) firms

Space of feasible store locations (the city)

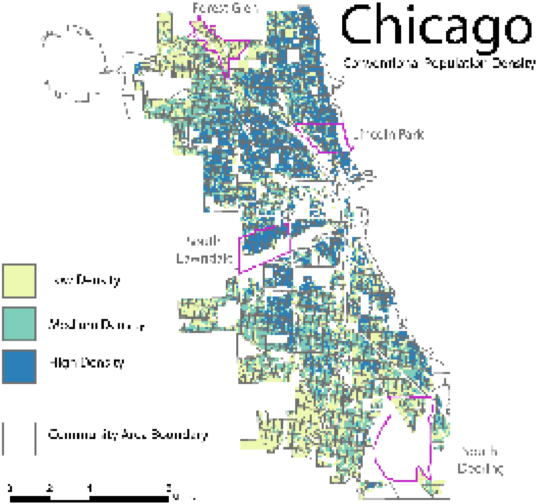
- From a geographical point of view, a market (city) is a set, for instance **a rectangle**, in the space $\mathbb{R}^2$.
- Suppose that we divide this city/rectangle into L small squares, each one with its center.
- Each of these squares is a submarket (or neighborhood, or location).
- A market/city can have hundreds or thousands of these submarkets/locations.
- We index these locations by $\ell = \in \{1, 2, ..., L\}$

The city: space of feasible store locations

Latitude \ Longitude	1	2	3	4	5	6	7	8
1	●	●	●	●	●	●	●	●
2	●	●	●	●	●	●	●	●
3	●	●	●	●	●	●	●	●
4	●	●	●	●	●	●	●	●
5	●	●	●	●	●	●	●	●
6	●	●	●	●	●	●	●	●
7	●	●	●	●	●	●	●	●
8	●	●	●	●	●	●	●	●

Space of feasible store (product) locations

- Each location has some exogenous characteristics that can affect demand and costs of a firm in that location:
  - Population; demographics; rental prices.
- Exogenous characteristics of location ℓ: vector $\mathbf{x}_\ell$.
- Therefore, we can see a city as a landscape of the characteristics $\mathbf{x}_\ell$ over the L locations.


Chicago
Forest Glen
Lincoln Park

Product Space instead of Geographic Space

- In a model of geographic location, $\ell \in \mathbb{R}^2$ (longitude, latitude).
- We can extend this mode to allow ℓ to have K dimensions: $\ell \in \mathbb{R}^K$.
- These K dimensions correspond to K observable characteristics of a product.
  - E.g., for automobiles, horse power, max speed, fuel consumption, etc.
- The space of feasible locations is a finite set within $\mathbb{R}^K$.
- We index these product locations by $\ell \in \{1, 2, ..., L\}$

Model: Firms

- There are N potential entrants in this industry and city: e.g., supermarkets in Toronto.
- In the simpler version of the model, each potential entrant has only one possible store: no multi-store firms (no chains).
- For the moment, we consider this simpler version.
- Let a_i represent the entry / location decision of firm i.

$$a_i \in \{0, 1, ..., L\}$$

- $a_i = 0$ represents "no entry";
- $a_i = \ell > 0$ represents entry in location ℓ.

Model: Profit function

- What is the profit of firm i if it opens a store in location ℓ?
- In principle, we could consider a model of consumer choice of where to purchase (e.g., logit), a model of price competition between active firms; obtain the Bertrand equilibrium of that game, and the corresponding equilibrium profits (see Aguirregabiria & Vicentini, JIE 2016).
- This approach requires having data on prices and quantities at every location [or/and the repeated solution of the equilibria of a complex two-stage game].
- Instead, Seim (2006) considers a convenient shortcut.
- Her model does not specify (explicitly) consumer choices and price competition, but it incorporates the idea that geographic distance to competitors (spatial differentiation) can increase a firm's profit.

Model: Profit function [2]

- Suppose that we draw a circle of radius d around the center point of location ℓ, e.g., a radius of 1km.
- From the point of view of a store located at ℓ, we can divide its competitors in two groups:
  - Close competitors: within the circle of radius d.
  - Far away competitors: outside the circle of radius d.
- Let $N_{\ell}(close)$ and $N_{\ell}(far)$ be the number of close and far away competitors relative to location ℓ.
- We can consider a profit function that depends on:

$$\gamma_{close}\ N_{\ell}(close) + \gamma_{far}\ N_{\ell}(far)$$

- γ_{close} and γ_{far} are parameters. We expect $\gamma_{close} < \gamma_{far} < 0$. The difference $\gamma_{far} - \gamma_{close}$ tell us how a firm's increase with spatial differentiation.

Model: Profit function [3]

- We can generalized this idea to allow for **multiple bands or radii** around the center of a location ℓ.
- Let $d_1 < d_2 < ... < d_B$ be B different radii of increasing magnitude, e.g., $d_1 = 0.2\ km$, $d_2 = 0.5\ km$, ..., $d_{10} = 20\ km$.
- We can construct the number of firms within the bands defined by these radii:

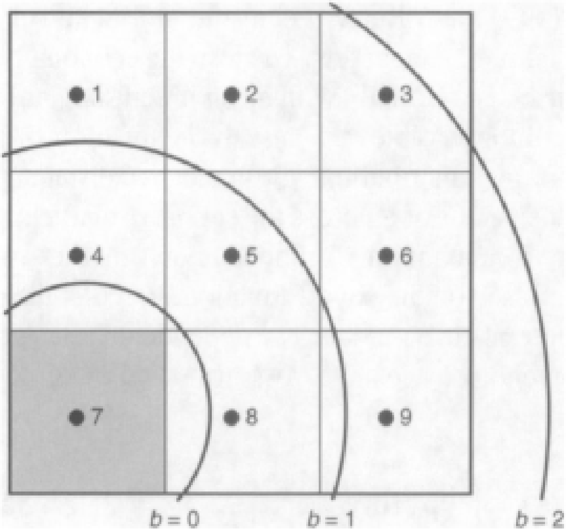
$$N_\ell(1) = \text{Number firms within the circle of radius } d_1$$
$$N_\ell(2) = \text{Number firms within band defined by } d_1 \ \& \ d_2$$

...

$$N_\ell(B) = \text{Number firms within band defined by } d_{B-1} \ \& \ d_B$$
$$N_\ell(B+1) = \text{Number firms outside circle with radius } d_B.$$

FIGURE 1

IMPACT ON PROFITS OF COMPETITORS' LOCATIONS: ILLUSTRATION



Model: Profit function [4]

- Profit of an active firm at location ℓ is:

$$\Pi_{i\ell} = x_\ell \, \beta + \xi_\ell + \sum_{b=1}^{B} \gamma_b \, N_\ell(b) + \varepsilon_{i\ell}$$

- We expect:

$$\gamma_1 < \gamma_2 < ... < \gamma_B < 0$$

- ξ_ℓ represents attributes of location ℓ which are known to firms but unobserved to the researcher.

- ε_{i1}, ε_{i2}, ..., ε_{iL} are private information of firm i assumed iid over firms and locations with extreme value distribution.

Profit function - Space of product characteristics

- We can apply this approach to the model of spatial location in product space.
- Now we have that $\ell \in \mathbb{R}^K$ is the vector of observable characteristics of hypothetical product ℓ.
- Given the B radii $d_1 < d_2 < ... < d_B$, we can define:

$$N_\ell(b) \equiv \text{Number of existing products with with characteristics } \ell' \text{ such that } d_{b-1} < \|\ell' - \ell\| \leq d_b$$

- Profit of an active firm at location ℓ is:

$$\Pi_{i\ell} = x_\ell \ \beta + \xi_\ell + \sum_{b=1}^{B} \gamma_b \ N_\ell(b) + \varepsilon_{i\ell}$$

Model: Expected Profit

- The game is of incomplete information. Firms do not know the actual numbers $N_\ell(1)$, ..., $N_\ell(B)$. Instead, a firm has beliefs for the probability that any other firm decides to enter in a location ℓ.

- Let $\mathbf{P} \equiv \{P_\ell : \ell = 1, 2, .., L\}$ be these beliefs.

- Given $\mathbf{P}$, the expected number of firms in band b around ℓ is:

$$\mathbb{E}\left[N_\ell(b)\right] \equiv N_\ell^e(b; \mathbf{P}) = \sum_{\ell'=1}^{L} 1\left\{d_{b-1} < \left\|\ell' - \ell\right\| \leq d_b\right\} \; N \; P_{\ell'}$$

- Given that a firm has beliefs $\mathbf{P}$, the firm's expected profit of entry in a location ℓ is:

$$\Pi_{i\ell}^e = x_\ell \; \beta + \xi_\ell + \sum_{b=1}^{B} \gamma_b \; N_\ell^e(b; \mathbf{P}) + \varepsilon_{i\ell}$$

and $\Pi_i(a_i = 0) = 0$.

Model: Bayesian Nash Equilibrium

- A (symmetric) BNE is an equilibrium strategy $\sigma(\varepsilon_i)$ from $\mathbb{R}^{L+1} \rightarrow \{0, 1, ..., L\}$ such that:

$$\sigma(\varepsilon_i) = \ell \quad \Leftrightarrow \quad \Pi^e_{i\ell} > \Pi^e_{i\ell'} \text{ for any } \ell' \neq \ell$$

- Integrating strategies over ε_i, we can represent a BNE as a vector $\mathbf{P} = \{P_\ell : \ell = 1, 2, .., L\}$ such that:

$$P_\ell = \frac{\exp\left\{x_\ell \ \beta + \xi_\ell + \sum_{b=1}^{B} \gamma_b \ N^e_\ell(b; \mathbf{P})\right\}}{1 + \sum_{j=1}^{L} \exp\left\{x_j \ \beta + \xi_j + \sum_{b=1}^{B} \gamma_b \ N^e_j(b; \mathbf{P})\right\}}$$

- This system of L equations with L unknowns is continuous on compact space. By Brower's Theorem, the model has at least one equilibrium.

Model: Equilibrium (3)

- In equilibrium, a change in x_ℓ in a single location affects the entry probabilities $P_{\ell'}$ at every location ℓ' in the city.
- Example: Policy that encourages entry in location 1.
  - Direct substitution effect: Keeping all $N_\ell^e(b; \mathbf{P})$ constants, the increase in $x_1\beta$ generates a substitution from other locations into location 1.
  - Indirect equilibrium effect: the increase in P_1 implies an increase in the expected number of competitors $N_\ell^e(b; \mathbf{P})$ at every location ℓ and band b that includes location 1; implies a reduction in entry probabilities in locations ℓ nearby location 1.
  - "Bullwhip" shape of the effect at different locations.

Data and Estimation

- We have data from an industry (e.g., supermarkets) in a city (or one network). We observe:

$$\text{Data} = \{x_\ell,\ n_\ell : \ell = 1, 2, ..., L\}$$

Given these data, we can construct shares: $s_\ell : \ell = 1, 2, ..., L$:

$$s_\ell = \frac{n_\ell}{N} \quad \text{and} \quad s_0 = \frac{N - n_1 - ... - n_L}{N}$$

- We distinguish three cases for the estimation of the model:

Case 1: City/industry with Large N (such that $s_\ell > 0$ at every ℓ).

Case 2: City/industry with Small N ($s_\ell = 0$ for some ℓ).

Case 3: M cities with $M \rightarrow \infty$. Small N.

Estimation: One City; Large N

- In general, the model implies that:

$$\ln\left(\frac{P_\ell}{P_0}\right) = x_\ell\ \beta + \sum_{b=1}^{B} \gamma_b\ N_\ell^e(b; \mathbf{P}) + \xi_\ell$$

- With large N (as $N \to \infty$) we have that for every location ℓ and band b:

$$s_\ell = P_\ell + o_p(1) \quad \text{and} \quad N_\ell^e(b; \mathbf{P}) = N_\ell^e(b; \mathbf{s}) + o_p(1)$$

- Therefore (up to an $o_p(1)$ estimation error), we have the equation:

$$\ln\left(\frac{s_\ell}{s_0}\right) = x_\ell\ \beta + \sum_{b=1}^{B} \gamma_b\ N_\ell^e(b; \mathbf{s}) + \xi_\ell$$

- This is a linear regression model with regressors x_ℓ, $N_\ell^e(1)$, ..., $N_\ell(B)$, and error term ξ_ℓ.

One City; Large N: OLS

$$\ln\left(\frac{s_\ell}{s_0}\right) = x_\ell\ \beta + \sum_{b=1}^{B} \gamma_b\ N_\ell^e(b;\mathbf{s}) + \xi_\ell$$

- Remember that:

$$N_\ell^e(b;\mathbf{s}) \equiv \sum_{\ell'=1}^{L} 1\left\{d_{b-1} < \left\|\ell' - \ell\right\| \leq d_b\right\}\ Ns_{\ell'}$$

  where $s_{\ell'}$ are endogenous variables.

- Regressors $N_\ell^e(b;\mathbf{s})$ are endogenous: OLS estimator is inconsistent.

- Because positive spatial correlation $\xi's$, we expect: $cov(N_\ell^e(1;\mathbf{s}), \xi_\ell) > cov(N_\ell^e(2;\mathbf{s}), \xi_\ell) > ... > 0$

- This implies that OLS estimator is upward biased: $bias(\gamma_1) > bias(\gamma_2) > ... > 0$

One City; Large N: IV

$$\ln\left(\frac{s_\ell}{s_0}\right) = x_\ell\ \beta + \sum_{b=1}^{B} \gamma_b\ N_\ell^e(b; \mathbf{s}) + \xi_\ell$$

- Model implies instruments for the endogenous regressors.
- Market characteristics $x_{\ell'}$ in locations ℓ' other than ℓ do not enter in the equation for location ℓ but affect the values $N_\ell^e(b; \mathbf{s})$.
- Let $\overline{x}_\ell(b)$ be the mean value of $x_{\ell'}$ in those locations ℓ' that belong to the band b around location ℓ.
- We can use $\overline{x}_\ell(b)$ as an instrument for $N_\ell^e(b; \mathbf{s})$.

One City; Large N

- This regression-like approach has two important advantages.
- [1] **Dealing with endogeneity.** We can deal with endogeneity using a standard IV method.
- [2] **Computational simplicity.** For the estimation of the structural parameters, we don't need to solve for an equilibrium of the model even once.

One City; Small N

- With small N, we cannot use this simple regression-like approach.
- Now, $s_\ell = \dfrac{n_\ell}{N}$ is zero for many (most) locations ℓ. Furthermore, $s_\ell = \dfrac{n_\ell}{N}$ is no longer $P_\ell + o_p(1)$.
- We can use a MLE but a key issue is how to deal with the endogeneity problem associated with the unobservables ξ_ℓ.
- We first describe the MLE under the assumption of $\xi_\ell = 0$ (**no unobserved location heterogeneity, other than the idiosyncratic** $\varepsilon_i' s$) and then we relax this assumption.

One City; Small N and epsi = 0

- Seim (2006) uses a **Maximum Likelihood method** implemented using a **Nested Fixed Point (NFXP) algorithm**.
- The application of this algorithm requires that the models has a unique equilibrium for every possible value of the structural parameters.
- Haiqing Xu (IER, 2018) proves that a sufficient condition for this model to have unique equilibrium is that $\left|\sum_{b=1}^{B} \gamma_b\right| < \dfrac{1}{N}$.
- Therefore, the MLE - NFXP method needs to impose the restriction $\left|\sum_{b=1}^{B} \gamma_b\right| < \dfrac{1}{N}$ at each iteration of the algorithm in the search for the ML estimate.

One City; Small N; and epsi = 0 [2]

- Let θ be the vector of structural parameters.
- Let $\mathbf{P}(\theta)$ be the vector of equilibrium probabilities associated with θ.
- That is, $\mathbf{P}(\theta) = \{P_\ell(\theta) : \ell = 1, 2, ..., L\}$ and this vector solves the system of equations:

$$P_\ell(\theta) = \frac{\exp\left\{x_\ell\ \beta + \sum_{b=1}^{B} \gamma_b\ N_\ell^e(b; \mathbf{P}(\theta))\right\}}{1 + \sum_{j=1}^{L} \exp\left\{x_j\ \beta + \sum_{b=1}^{B} \gamma_b\ N_j^e(b; \mathbf{P}(\theta))\right\}}$$

- Under the condition $\left|\sum_{b=1}^{B} \gamma_b\right| < \dfrac{1}{N}$, we have that $\mathbf{P}(\theta)$ is a **function of θ,** and it is continuously differentiable.
- However, **we do not have a closed-form expression for $\mathbf{P}(\theta)$**. For each trial value of θ, we need to use an algorithm (e.g., fixed point; Newton's) to compute the corresponding equilibrium $\mathbf{P}(\theta)$.

One City; Small N and epsi = 0 [3]

- According to the model,

$$[n_1, n_2, ..., n_L] \sim Multinomial\left(N;\; P_1(\theta), P_2(\theta), ..., P_L(\theta)\right)$$

- Therefore, the likelihood function is:

$$\mathcal{L}(\theta) = \prod_{\ell=0}^{L} P_\ell(\theta)^{n_\ell}$$

- or

$$\ln \mathcal{L}(\theta) = \sum_{\ell=0}^{L} n_\ell \;\ln P_\ell(\theta)$$

One City; Small N; and epsi = 0 [4]

- We can estimate θ by MLE using the **Nested Fixed Point algorithm**.
- We maximize $\ln \mathcal{L}(\theta)$ using a Newton's or **BHHH** iterative method:

$$\widehat{\theta}_{k+1} = \widehat{\theta}_k - \left[\sum_{\ell=0}^{L} \frac{\partial \ln P_\ell(\widehat{\theta}_k)}{\partial \theta} \frac{\partial \ln P_\ell(\widehat{\theta}_k)}{\partial \theta'} \right]^{-1} \left[\sum_{\ell=0}^{L} n_\ell \frac{\partial \ln P_\ell(\widehat{\theta}_k)}{\partial \theta} \right]$$

- At each iteration k, given $\widehat{\theta}_k$ we compute the equilibrium $\mathbf{P}(\widehat{\theta}_k)$.
- When L is large, the computation of an equilibrium can be computational demanding.
- To deal with this computational cost **Haiqing Xu (IER, 2018)** proposes approximating the equilibrium by using L local equilibria, on for each location. The local equilibrium at location ℓ is obtained using only its nearest neighbors.

One City; Small N and Unobserved Location Heter.

- Now, the equilibrium vector depends on the vector of unobservables $\boldsymbol{\xi} = (\xi_1, \xi_2, ..., \xi_L)$. We have $\mathbf{P}(\theta; \boldsymbol{\xi})$.
- The log-likelihood function is integrated over the distribution of $\boldsymbol{\xi}$:

$$\begin{aligned} \ln \mathcal{L}(\theta) &= \sum_{\ell=0}^{L} n_\ell \ \ln \Pr(n_\ell | \theta) \\ &= \sum_{\ell=0}^{L} n_\ell \ \ln \left[\int P_\ell(\theta; \boldsymbol{\xi}) \ f(\boldsymbol{\xi}) \ d\boldsymbol{\xi} \right] \end{aligned}$$

- Since L is large, the dimension of $\boldsymbol{\xi}$ and the integral is large. Very demanding computational problem.
- **Monte Carlo simulation** is a common approach to compute an approximation to $I = \int P_\ell(\theta; \boldsymbol{\xi}) \ f(\boldsymbol{\xi}) \ d\boldsymbol{\xi}$:

$$I \simeq \frac{1}{R} \sum_{r=1}^{R} P_\ell(\theta; \boldsymbol{\xi}^{(r)})$$

M Cities; Large M; Small N

- Now, we have M cities and for each city m we observe $\{x_{m\ell}, n_{m\ell} : \ell = 1, 2, ..., L_m\}$.
- The log-likelihood function is: **without** $\boldsymbol{\xi}'s$:

$$\ln \mathcal{L}(\theta) = \sum_{m=0}^{M} \sum_{\ell=0}^{L_m} n_{m\ell} \ \ln P_{m\ell}(\theta)$$

- The estimation is the same as before, but for each trial value of θ we need to compute M equilibria, one for each city.
- **With** $\boldsymbol{\xi}'s$, the estimation is similar as described above for one single network, but again with as many equilibria as cities and values of $\boldsymbol{\xi}$ per city.

Seim (2006) application: Main Results

- Seim (2006) finds very significant results of spatial differentiation (γ parameters decline very significantly with distance)
- Market structure and spatial structure of stores under two different scenarios of city growth.
  - Growth in population but keeping city boundaries.
  - Growth in population and in city boundaries
- The model can be used to study how changes in the exogenous characteristics x_ℓ of a single location (e.g., new amenities, schools, new local regulations, transportation, developments) can affect the landscape of firms in a city.

3. Models of Firms' Spatial Location: Multi-product (store) firms

Model with Multi-Store Firms

- Consider the same spatial configuration as before, but now the N potential entrants can open as many stores as possible locations L.
- Now, the number of players N is very small (a few retail chains). For instance, two firms indexed by $i \in \{1, 2\}$.
- The decision variable for firm i:

$$\mathbf{a}_i = (a_{i1},\ a_{i2},\ ...,\ a_{iL})$$

where $a_{i\ell} = 1\{\text{Firm } i \text{ opens a store in location } \ell\} \in \{0, 1\}$.

Multi-Store Firms: Profit

- Now, the profit function should incorporate not only the competition effects from the stores of other firms but also the competition or/and spillover effects from the own stores.

- For instance (we can extend it to allow for B bands):

$$\Pi_i = \sum_{\ell=1}^{L} a_{i\ell} \left[x_\ell \beta_i + \xi_\ell + \theta_i^{COM} \left(\sum_{\ell'=1}^{L} \frac{a_{j\ell'}}{d_{\ell\ell'}} \right) + \theta_i^{CAN} \left(\sum_{\ell' \neq \ell'} \frac{a_{i\ell'}}{d_{\ell\ell'}} \right) + \varepsilon_{i\ell} \right]$$

  where $d_{\ell\ell'}$ = distance between ℓ and ℓ'.

- θ_i^{COM} captures competition (if $\theta_i^{COM} < 0$) or spillovers (if $\theta_i^{COM} > 0$) from rivals.

- θ_i^{CAN} captures cannibalization (if $\theta_i^{CAN} < 0$) or economies of scope/density (if $\theta_i^{CAN} > 0$) from own stores.

Best responses

- The space of the vector $\mathbf{a}_i = (a_{i1}, a_{i2}, ..., a_{iL})$ has 2^L possible points.
- For instance, Jia (2008) studies competition between in entry/location between Walmart and Kmart in $L = 2,065$ locations (US counties). This implies $2^L = 2^{2065} \simeq 10^{621}$.
- The computation of an equilibrium in this model is computationally very costly.
- Researchers have consider different approaches to deal with this issue.
    (a) Moment inequalities based on restrictions on the unobservables: Ellickson, Houghton, and Timmins (RAND, 2013)
    (b) Lattice theory approach: Jia (Econometrica, 2008); Nishida (Marketing Science, 2014)

Ellickson, Houghton, and Timmins (2013)

- Consider a game between N multi-store firms but ignore for the moment cannibalization and economies of scope/density such that:

$$\Pi_i = \sum_{\ell=1}^{L} a_{i\ell} \left[x_\ell \beta_i + \sum_{j \neq i} \gamma_{ij} \; a_{j\ell} + \varepsilon_{i\ell} \right]$$

- They assume that: $\varepsilon_{i\ell} = \alpha_i + \xi_\ell$. They also assume complete information.

- By revealed preference, the profit of the observed action of firm i, $\mathbf{a}_i$, should be larger than the profit of any alternative action, $\mathbf{a}_i'$:

$$\Pi_i \left(\mathbf{a}_i \right) - \Pi_i \left(\mathbf{a}_i' \right) \geq 0 \quad \text{for any } \mathbf{a}_i' \neq \mathbf{a}_i$$

- EHT (2013) consider hypothetical choices $\mathbf{a}_i'$ that difference out the error term such that we do not need to integrate over a space of 2^L unobservables.

Ellickson, Houghton, and Timmins [2]

- Suppose that the observe choice of firm i, $\mathbf{a}_i$, is such that $a_{i\ell} = 1$ and $a_{i\ell'} = 0$.
- Consider the hypothetical choice $\mathbf{a}_i^*$ that consists in the relocation of a store from ℓ into ℓ', such that $a_{i\ell}^* = 0$ and $a_{i\ell'}^* = 1$. Then:

$$\Pi_i\left(\mathbf{a}_i\right) - \Pi_i\left(\mathbf{a}_i^*\right) =$$

$$\left[x_\ell - x_{\ell'}\right]\beta_i + \sum_{j \neq i} \gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + \left[\xi_\ell - \xi_{\ell'}\right] \geq 0$$

Ellickson, Houghton, and Timmins [3]

- Now, suppose that for a different firm, $k \neq i$, the observe choice, $\mathbf{a}_k$, is such that $a_{k\ell} = 0$ and $a_{k\ell'} = 1$.
- Consider the hypothetical choice $\mathbf{a}_k^*$ that consists in the relocation of a store from ℓ' into ℓ, such that $a_{k\ell}^* = 0$ and $a_{k\ell'}^* = 1$.
- Then, for firm k we have:

$$\Pi_k(\mathbf{a}_k) - \Pi_k(\mathbf{a}_k^*) =$$

$$[x_{\ell'} - x_\ell]\,\beta_k + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] + [\xi_{\ell'} - \xi_\ell] \geq 0$$

Ellickson, Houghton, and Timmins [4]

- Adding the inequalities:

$$[x_\ell - x_{\ell'}]\,\beta_i + \sum_{j \neq i} \gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + [\xi_\ell - \xi_{\ell'}] \geq 0$$

$$[x_{\ell'} - x_\ell]\,\beta_k + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] + [\xi_{\ell'} - \xi_\ell] \geq 0$$

- We have:

$$[x_\ell - x_{\ell'}]\;\;[\beta_i - \beta_k] + \sum_{j \neq i} \gamma_{ij}\left[a_{j\ell} - a_{j\ell'}\right] + \sum_{j \neq k} \gamma_{kj}\left[a_{j\ell'} - a_{j\ell}\right] \geq 0$$

, Houghton, and Timmins [4]

- Using different pairs of locations and/or firms, we can construct many different inequalities like

$$[x_\ell - x_{\ell'}] \ \ [\beta_i - \beta_k] + \sum_{j \neq i} \gamma_{ij} \left[a_{j\ell} - a_{j\ell'}\right] + \sum_{j \neq k} \gamma_{kj} \left[a_{j\ell'} - a_{j\ell}\right] \geq 0$$

- Using these inequalities, we can estimate the parameters β and γ using a **Maximum Score estimator (MSE)** (Manski, 1975; Horowitz, 1992; Fox, 2010).

- If we describe these inequalities as $z_{ik\ell\ell'}\theta \geq 0$, the **score function** is

$$S(\theta) = \sum_{i,k,\ell,\ell'} 1\{z_{ik\ell\ell'}\theta \geq 0\}$$

  and the MSE is the value of θ that maximizes $S(\theta)$.

- EHT (RAND, 2013) apply this approach to study competition in entry/location between department store chains in US.