# Empirical Industrial Organization (ECO 310) <br> Winter 2021. Victor Aguirregabiria 

Problem Set \#2
Due on Friday, March 26, 2021 [before 11:59pm]

INSTRUCTIONS. Please, follow the following instructions for the submission of your completed problem set.

1. Write your answers electronically in a word processor.
2. For the answers that involve coding in STATA, include in the document the code in STATA that you have used to obtain your empirical results.
3. Convert the document to PDF format.
4. Submit your problem set (in PDF) online via Quercus.
5. You should submit your completed problem set before Friday, March 26, 2021 [before 11:59pm].
6. Problem sets should be written individually.

## The total number of marks is 200.

QUESTION 1. [80 points]. Consider an industry with a differentiated product. There are two firms in this industry, firms $A$ and $B$. Each firm produces and sells two brands of the differentiated product: brands $A 1$ and $A 2$ are produced by firm $A$, and brands $B 1$ and $B 2$ by firm $B$. The demand system is a logit demand model, where consumers choose between five different alternatives: $j=0$, represents the consumer decision of no purchasing any product; and $j=A 1, j=A 2, j=B 1$, and $j=B 2$ represent the consumer purchase of product $A 1, A 2, B 1$, and $B 2$, respectively. The utility of no purchase $(j=0)$ is zero. The utility of purchasing product $j \in\{A 1, A 2, B 1, B 2\}$ is $\beta x_{j}-\alpha p_{j}+\varepsilon_{j}$, where the variables and parameters have the interpretation that we have seen in class. Variable $x_{j}$ is a measure of the quality of product $j$, e.g., the number of stars of the product according to consumer ratings. Therefore, we have that $\beta>0$. The random variables $\varepsilon_{1}$ and $\varepsilon_{2}$ are independently and identically distributed over consumers with a type I extreme value distribution, i.e., Logit model of demand. Let $H$ be the number of consumers in the market. Let $s_{0}, s_{A 1}, s_{A 2}$, $s_{B 1}$, and $s_{B 2}$ be the market shares of the five choice alternatives, such that $s_{j}$ represents the proportion of consumers choosing alternative $j$ and $s_{0}+s_{A 1}+s_{A 2}+s_{B 1}+s_{B 2}=1$.

Q1.1. (5 points) Based on this model, write the equation for the market share $s_{A 1}$ as a function of the prices and the qualities $x$ 's of all the products.
Q1.2. (15 points) Obtain the expression for the derivatives: (a) $\frac{\partial s_{j}}{\partial p_{j}}$; and (b) $\frac{\partial s_{j}}{\partial p_{k}}$ for $j \neq k$. Write the expression for these derivatives in terms only of the market shares $s_{j}$ and $s_{k}$ and the parameters of the model.

The profit function of firm $A$ is $\pi_{A}=p_{A 1} q_{A 1}+p_{A 2} q_{A 2}-c_{A 1} q_{A 1}-c_{A 2} q_{A 2}-F C\left(x_{A 1}\right)-$ $F C\left(x_{A 2}\right)$, where: $q_{j}$ is the quantity sold by firm $j$ (i.e., $q_{j}=H s_{j}$ ); $c_{j}$ is the marginal cost of producing good $j$, that is assumed constant, i.e., linear cost function; and $F C\left(x_{j}\right)$ is the fixed cost of producing a good with quality $x_{j}$.

Q1.3. (20 points) Suppose that firms take the qualities $x$ of their products as given and compete in prices ala Bertrand.
(a) Show that the marginal conditions of profit maximization of firm $A$ in this Bertrand game, $\frac{d \pi_{A}}{d p_{1 A}}=0$ and $\frac{d \pi_{A}}{d p_{2 A}}=0$, have the following form:

$$
\begin{aligned}
& \frac{d \pi_{A}}{d p_{1 A}}=0 \text { implies } \frac{1}{\alpha}-\left(p_{A 1}-c_{A 1}\right)\left(1-s_{A 1}\right)+\left(p_{A 2}-c_{A 2}\right) s_{A 2}=0 \\
& \frac{d \pi_{A}}{d p_{2 A}}=0 \quad \text { implies } \frac{1}{\alpha}+\left(p_{A 1}-c_{A 1}\right) s_{A 1}-\left(p_{A 2}-c_{A 2}\right)\left(1-s_{A 2}\right)=0
\end{aligned}
$$

(b) Define the Price-Cost-Margins of products $A 1$ and $A 2$ as $P C M_{A 1} \equiv p_{A 1}-c_{A 1}$ and $P C M_{A 2} \equiv p_{A 2}-c_{A 2}$, respectively. In the First-Order-Conditions of profit maximization in Q1.3(a), replace $p_{A 1}-c_{A 1}$ with $P C M_{A 1}$, and $p_{A 2}-c_{A 2}$ with $P C M_{A 2}$. Then, taking $s_{A 1}$ and $s_{A 2}$ as given, these First-Order-Conditions can be seen as a system of linear equation where the unknowns are $P C M_{A 1}$ and $P C M_{A 2}$. Solve this system to obtain the following solution for $P C M_{A 1}$ and $P C M_{A 2}$ :

$$
P C M_{A 1}=P C M_{A 2}=\frac{1}{\alpha\left(1-s_{A 1}-s_{A 2}\right)}
$$

Q1.4. (20 points) Suppose that products $A 1$ and $A 2$ were produced by two different firms that operate separately and maximize their respective profits. Answer the same Questions as in Q1.3(a) and Q1.3(B) but for these independent firms.

Q1.5. (20 points) Compare the expressions for the equilibrium price cost margins in Questions Q1.3(b) and Q1.4(b).
(a) Does the multi-product firm change higher or lower price-cost margins than the single product firm?
(b) Based on this result, explain in words the implications of multiproduct firms on prices, firms' profits, and consumer surplus.

QUESTION 2. [ $\mathbf{1 2 0}$ points]. To answer the questions in this part of the problem set you need to use the dataset cars1.dta. Use this dataset to implement the estimations describe below. ${ }^{1}$ Please, provide the STATA code that you use to obtain the results. For all the models that you estimate below, impose the following conditions.

- For market size (number of consumers), use Population/4, i.e., pop/4
- Use prices measured in euros (price).
- For the product characteristics in the demand system, include the characteristics: horsepower, fuel, width, height, weight, domestic.
- Include also as explanatory variables the market characteristics: $\log (p o p)$ and $\log (g d p)$.
- In all the OLS estimations include fixed effects for market (country), year (year), and brand (brand).
- Include the price in levels (not in logarithms).


## Q2.1. (70 points)

(a) [10 points] Obtain the OLS-Fixed effects estimator of the Standard logit model. Interpret the results.
(b) [20 points] Obtain the Instrumental Variables (IV) estimator of the Standard logit model. Use the following set of instruments. For each product characteristic $X_{1 j}=$ horsepower $X_{2 j}=$ fuel, $X_{3 j}=$ width, $X_{4 j}=$ height, $X_{5 j}=$ weight, and $X_{6 j}=$ domestic, construct the following three instruments:

$$
\begin{aligned}
& I V_{k j, m t}^{(1)}=\sum_{i \neq j}\left(X_{k i}-X_{k j}\right) \text { for those products } i \text { in country } m \text { and year } t \\
& I V_{k j, m t}^{(2)}=\sum_{i \neq j}\left|X_{k i}-X_{k j}\right| \text { for those products } i \text { in country } m \text { and year } t
\end{aligned}
$$

This is a total of 12 instrumental variables. Interpret the results from this IV estimation.
(c) [10 points] Using the IV estimation, obtain how much the average consumer is willing to pay (in Euros) for a reduction of 1 unit in the characteristic fuel (that is, for an improvement in fuel efficiency of 1 liter per km).
(d) [10 points] Test the over-identifying restrictions in the IV estimation.
(e) [10 points] Using the IV estimation, test the null hypothesis that all countries have the same price coefficient.

[^0](f) [10 points] Based on the IV estimation, obtain the average price elasticity of demand evaluated at the mean values of prices and market shares.

Q2.2. (50 points) Consider the equilibrium condition (first order conditions of profit maximization) under the assumption that each product is produced by only one firm.
(a) (5 points) Write the equation for this equilibrium condition. Using this condition, obtain an estimate of the (realized) marginal cost for every car-market-year observation in the data.
(b) (15 points) Run an OLS-Fixed effects regression where the dependent variable is the estimated value of the marginal cost, and the explanatory variables (regressors) are the product characteristics horsepower, fuel, width, height, weight, domestic. Interpret the estimated coefficients.
(c) (20 points) Now consider that the marginal cost may also depend on the amount produced and sold of the product (variable qu). Include this variable in the regression for the marginal cost function. Estimate this marginal cost function by Instrumental variables using the same instruments as for the demand estimation, i.e., the characteristics of competing products. Interpret the estimated coefficients.
(d) (10 points) Based on the IV estimation of the marginal cost function in Q2.2(c), obtain the change in the marginal cost in Euros of a change in output of when a firm increases in 10,000 units (cars per market and year) the variable qu. Is this estimate plausible? Discuss this result.


[^0]:    ${ }^{1}$ This is the same dataset that Frank Verboven provides for his Stata command mergesim. Note that this is a different sample that the one we have used in the Tutorials.

