ECO 310: Empirical Industrial Organization

Lecture 9: Models of Competition in Prices or Quantities: Conjectural Variations

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Outline on today's lecture

1. **Introduction**
2. **Estimating the form of competition when MCs are observed**
3. **Estimating the form of competition without data on MCs**

1. Introduction

Introduction

- In the previous lecture we saw how given a (estimated) demand system and an assumption about competition, **we can obtain (estimate) firms' marginal costs**.
- In today's lecture we will see how given a demand system and firms' marginal costs, **we can identify the form of competition in a market.**
- More specifically, we can identify firms' beliefs about how the other firms in the market respond strategically.
- This approach is called the **conjectural variation approach** or **conjectural variation model**.

2. Conjectural variation model: Homogeneous product markets

Conjectural Variation Model: Homogeneous product markets

- Consider an industry where, at period t, the inverse demand curve is $p_t = P\left(Q_t, X_t^D\right)$, and firms, indexed by i, have cost functions $C_i(q_{it})$.
- Every firm i, chooses its amount of output, q_{it}, to maximize its profit, $\Pi_{it} = p_t \; q_{it} - C_i(q_{it})$.
- Without further assumptions, the marginal condition for the profit maximization of a firm is **marginal revenue = marginal cost**, where the marginal revenue of firm i is:

$$MR_{it} = p_t + P_Q'\left(Q_t \; X_t^D\right) \; \left[1 + \frac{dQ_{(-i)t}}{dq_{it}}\right] \; q_{it}$$

- $\dfrac{dQ_{(-i)t}}{dq_{it}}$ represents the **belief** that firm i has about how the other firms will respond if she changes its own amount. We denote this **belief** as the **conjectural variation of firm i**, CV_i.

Conjectural Variations and Beliefs

- As researchers, we can consider different assumptions about firms' beliefs or conjectural variations, CV_{it}.
- An assumption on CVs implies a particular model of competition.
- Different assumptions imply different equilibrium outcomes, q_{it}, Q_t, and p_t.
- However, not all the assumptions are consistent with an **equilibrium**.
- In fact, most assumptions about CVs imply an equilibrium where firms are not rational in the sense that they have beliefs that do not hold in equilibrium.

Conjectural Variations: Nash-Cournot equilibrium

- In our model of firm competition, Nash conjecture implies that:

$$CV_{it} \equiv \frac{\partial Q_{(-i)t}}{\partial q_{it}} = 0$$

- This conjecture implies the Cournot equilibrium (or Nash-Cournot equilibrium).

- For every firm i, the "perceived" marginal revenue is:

$$MR_{it} = p_t + P'_Q \left(Q_t \ X_t^D \right) \ q_{it}$$

and the condition $p_t + P'_Q \left(Q_t \ X_t^D \right) \ q_{it} = MC_i(q_{it})$ implies the Cournot equilibrium.

Conjectural Variations: Perfect Competition

- Are other assumptions on firms' CVs that are consistent with a rational equilibrium?
- Yes, there are CVs that generate **perfect competition equilibrium** and the **collusive or monopoly equilibrium** which are consistent (rational) with the equilibrium outcome that they generate.
- **Perfect competition**. For every firm i, $CV_{it} = -1$.
- Note that this conjecture implies that:

$$MR_{it} = p_t + P'_Q\left(Q_t\ X_t^D\right)\ [1-1]\ q_{it} = p_t$$

and the conditions $p_t = MC_i(q_{it})$ imply the perfect competition equilibrium.

Conjectural Variations: Collusion

- There are also beliefs that can generate the collusive outcome (monopoly outcome) as a rational equilibrium.
- **Collusion (Monopoly)**. For every firm i, $CV_{it} = N_t - 1$. This conjecture implies:

$$MR_{it} = p_t + P'_Q \left(Q_t \ X_t^D \right) \ N_t \ q_{it}$$

- This conjecture implies the equilibrium conditions:

$$p_t + P'_Q \left(Q_t \ X_t^D \right) \ N_t \ q_{it} = MC_i(q_{it})$$

- When firms have constant and homogeneous MCs, these conditions imply:

$$p_t + P'_Q \left(Q_t \ X_t^D \right) \ Q_t = MC$$

which is the equilibrium condition for the Monopoly (collusive or cartel) outcome.

Conjectural Variations: Nature of Competition

- The value of the beliefs CV are related to the "nature of competition", i.e., Cournot, Perfect Competition, Cartel (Monopoly).

$$\begin{array}{rll} \text{Perfect competition:} & CV_{it} = -1; & MR_{it} = p_t \\ \text{Nash-Cournot:} & CV_{it} = 0; & MR_{it} = p_t + P'_Q(Q_t)\ q_{it} \\ \text{Cartel all firms:} & CV_{it} = N_t - 1; & MR_{it} = p_t + P'_Q(Q_t)\ Q_t \end{array}$$

- Given this result, one can argue that CV is closely related to the **nature of competition**, and therefore with equilibrium price and quantities.
- If CV is negative, the degree of competition is stronger than Cournot. The closer to -1, the more competitive.
- If CV is positive, the degree of competition is weaker than Cournot. The closer to $N_t - 1$, the less competitive.

Conjectural Variations: Nature of Competition [2]

- Interpreting the beliefs CV as an **index of competition** is correct.
- However, it is important to take into account that for values of CV different to -1, or 0, or $N_t - 1$, the "Conjectural Variation" equilibrium that we obtain is not a rational equilibrium.
- We can think in the CV as firms' beliefs that are determined over time as the result of firms interactions and learning (a dynamic game).

Conjectural Variation: Estimation

- Consider an homogeneous product industry and a researcher with data on firms' quantities and marginal costs, and market prices over T periods of time:

$$\text{Data} = \{p_t, \ MC_{it}, \ q_{it}\} \text{ for } i = 1, 2, ..., N_t \ \& \ t = 1, 2, ..., T$$

- Under the assumption that every firm chooses the amount of output that maximizes its profit given its belief CV_{it}, we have that the following condition holds:

$$p_t + P'_Q \left(Q_t \ X_t^D \right) \ [1 + CV_{it}] \ q_{it} = MC_{it}$$

- And solving for the conjectural variation,

$$CV_{it} = \frac{p_t - MC_{it}}{-P'_Q \left(Q_t \ X_t^D \right) \ q_{it}} - 1 = \left[\frac{\left(p_t - MC_{it} \right) / p_t}{q_{it} / Q_t} \right] |\eta_t| - 1$$

where η_t is the demand elasticity.

Conjectural Variation: Estimation [2]

$$CV_{it} = \left[\frac{(p_t - MC_{it}) \, / \, p_t}{q_{it} \, / \, Q_t} \right] |\eta_t| - 1$$

- This equation shows that, given data on quantities, prices, demand and marginal costs, we can identify the firms' beliefs that are consistent with these data and with profit maximization.
- Let us denote $\left[\frac{(p_t - MC_{it}) \, / \, p_t}{q_{it} \, / \, Q_t} \right]$ as the **Lerner-index-to-market-share ratio** of a firm.
- If the Lerner-index-to-market-share ratios are close zero, then the estimated values of CV will be close to -1, unless the absolute demand elasticity is large.
- If the Lerner-index-to-market-share ratios are large (i.e., larger than the inverse demand elasticity), then estimated CV values will be greater than zero, and can reject the hypothesis of Cournot competition.

Conjectural Variation: Estimation [3]

$$CV_{it} = \left[\frac{(p_t - MC_{it}) \, / \, p_t}{q_{it} \, / \, Q_t} \right] |\eta_t| - 1$$

- Part of the sample variation of CV_{it} can be due to estimation error in demand and marginal costs.
- To implement a formal statistical test of the value of CV_{it} we need to take into account this error.
- For instance, let $\overline{CV}$ be the sample mean of the values CV_{it}. Under the null hypothesis of Cournot competition, $CV_{it} = 0$ for every (i, t) and $\overline{CV}$ has a Normal distribution $(0, , s^2)$. We can estimate s and implement a t-test based on the statistic $\overline{CV} / \widehat{s}$.

3. Estimating CV parameters without data on MCs

Estimating CV parameters without data on MCs

- So far, we have considered the estimation of CV parameters when the researcher knows both demand and firms' marginal costs.
- We now consider the case where the **researcher knows the demand, but it does not know firms' marginal costs**.
- Identification of CVs requires also de identification of MCs.
- Under some conditions, we can **jointly identify CVs and MCs** using the marginal conditions of optimality and the demand.

Data

- Researcher observes data:

$$\text{Data} = \left\{ P_t, \; q_{it}, X_t^D, X_t^{MC} : \; i = 1, ...N_t; \, t = 1, ..., T \right\}$$

- X_t^D are variables affecting consumer demand, e.g., average income, population.

- X_t^{MC} are variables affecting marginal costs, e.g., some input prices.

Model: Demand and MCs

- Consider the linear (inverse) demand equation:

$$P_t = \alpha_0 + \alpha_1 \ X_t^D - \alpha_2 \ Q_t + \varepsilon_t^D$$

with $\alpha_2 \geq 0$, and ε_t^D is unobservable to the researcher.

- Consider the marginal cost function:

$$MC_{it} = \beta_0 + \beta_1 \ X_t^{MC} + \beta_2 \ q_{it} + \varepsilon_{it}^{MC}$$

with $\beta_2 \geq 0$, and ε_{it}^{MC} is unobservable to the researcher.

Model: Profit maximization

- Profit maximization implies $MR_{it} = MC_{it}$, or equivalently:

$$P_t + \frac{dP_t}{dQ_t} \left[1 + CV_{it}\right] \; q_{it} = MC_{it}$$

- In the model above, $\dfrac{dP_t}{dQ_t} = -\alpha_2$. Therefore,

$$P_t - \alpha_2 \left[1 + CV_{it}\right] \; q_{it} = \beta_0 + \beta_1 \; X_t^{MC} + \beta_2 \; q_{it} + \varepsilon_{it}^{MC}$$

- Or equivalently,

$$P_t = \beta_0 + \beta_1 \; X_t^{MC} + \left[\beta_2 + \alpha_2 (1 + CV_{it})\right] \; q_{it} + \varepsilon_{it}^{MC}$$

- This equation describes the marginal condition for profit maximization. We assume now that $CV_{it} = CV$ for every observation i, t in the data.

Complete structural model

- The structural equations of the model are:

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \, X_t^D - \alpha_2 \, Q_t + \varepsilon_t^D$$

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \, X_t^{MC} + [\beta_2 + \alpha_2 (1 + CV)] \, q_{it} + \varepsilon_{it}^{MC}$$

- Using this model and data, **can we identify (estimate consistently, without asymptotic bias) the CV parameter?**

- First, we will see that NO. In this model we cannot separately identify CV and MC.

- Second, we will see that a simple modification of this model implies separate identification of CV and MC.

Identification of demand parameters

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t + \varepsilon_t^D$$

- Endogeneity problem: in equilibrium, $cov(Q_t, \varepsilon_t^D) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, Q_t depends on X_t^{MC}. Note that X_t^{MC} does not enter in demand. If X_t^{MC} is not correlated with ε_t^D, then X_t^{MC} satisfies all the conditions for being a valid instrument.
- Parameters α_0, α_1, and α_2 are identified using this IV estimator.

Identification of CV and MCs

$$\text{F.O.C.:} \quad P_t = \quad \beta_0 + \beta_1\ X_t^{MC} + [\beta_2 + \alpha_2(1 + CV)]\ q_{it} + \varepsilon_{it}^{MC}$$

- Endogeneity problem: in equilibrium, $cov(q_{it}, \varepsilon_{it}^{MC}) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, q_{it} depends on X_t^D. Note that X_t^D does not enter enter in the F.O.C. If X_t^D is not correlated with ε_{it}^{MC}, then X_t^D satisfies all the conditions for being a valid instrument.
- Parameters β_0, β_1, and $\gamma \equiv \beta_2 + \alpha_2(1 + CV)$ are identified using this IV estimator.

The identification problem

$$\text{F.O.C.:} \quad P_t = \quad \beta_0 + \beta_1 \ X_t^{MC} + \gamma \ q_{it} + \varepsilon_{it}^{MC}$$

- Note that we can identify the parameter γ, where $\gamma \equiv \beta_2 + \alpha_2(1 + CV)$, and the slope of inverse demand function, α_2.
- However, knowledge of γ and α_2 is not sufficient to identify separately CV and the slope of the MC, β_2.
- Suppose that $\gamma = 1$ and $\alpha_2 = 0.4$, such that we have the constraint:

$$1 = \beta_2 + 0.4 \ (1 + CV)$$

- This equation is satisfied by any of the following:
  [Perfect competition] $CV = -1$ and $\beta_2 = 1.0$
  [Cournot] $CV = 0$ and $\beta_2 = 0.6$
  [Cartel, with $N = 3$] $CV = N - 1 = 2$ and $\beta_2 = 0.2$

The identification problem [2]

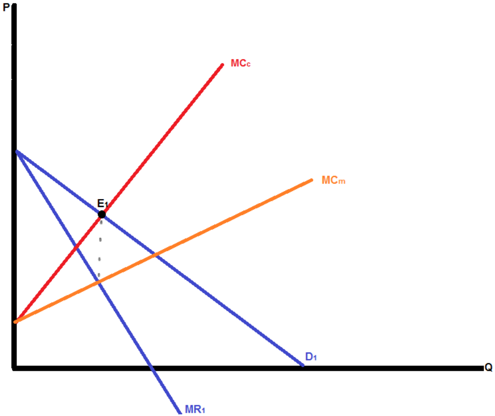
- The IV estimator identifies the MC by using the instrument X_t^D that shifts the demand.

- When we make an assumption about the form of competition, shifts in the demand curve are able to trace out the marginal cost curve, i.e., to identify the MC parameters.

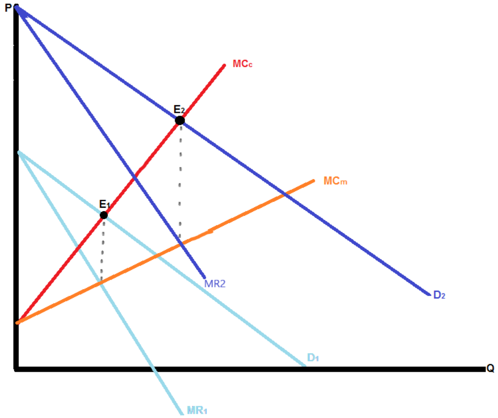
- However, without specifying the form of competition, shifts in the demand alone are not sufficient to separately identify MC and CV.

- Let $\widehat{q}_{it}(X_t^D)$ be the part of q_{it} explained X_t^D. When X_t^D varies, we see a positive correlation between P_t and $\widehat{q}_{it}(X_t^D)$. But the magnitude of this correlation can be explained by the combination of:
  - either zero/negative CV and positive and large β_2;
  - or positive CV and small or zero β_2.

The identification problem [3]



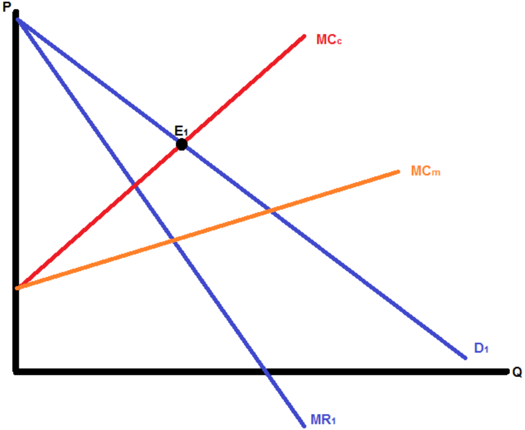
The identification problem [4]



Solving the identification problem

- Solving the identification problem involves generalizing demand so that changes in exogenous variables do **more than just parallel shift** the demand curve and MR.
- In particular, we need to allow for additional exogenous variables that are capable of **rotating** the demand curve as well.
- "Demand Rotators" are exogenous variables affecting the slope of the demand curve:

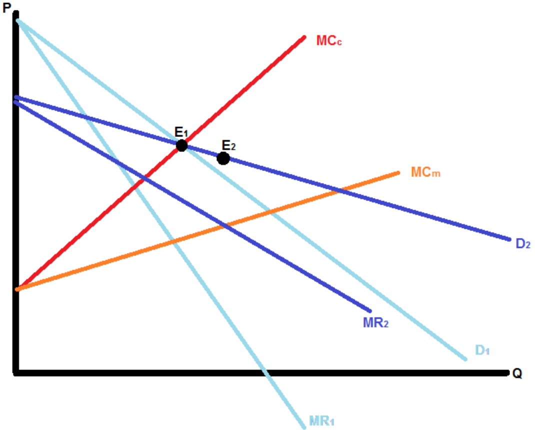
Solving the identification problem [2]



- Note that E_1 could be an equilibrium either for a perfectly competitive industry with cost MC_c or for a monopolist with cost MC_m.

- There is no observable distinction between the hypotheses of competition and

Solving the identification problem [3]



- Now, rotate the demand curve to D_2, with MR_2
- Competitive equilibrium stays at E_1. But monopoly equilibrium moves to E_2

Solving the identification problem [4]

- Consider now the following demand equation:

$$P_t = \alpha_0 + \alpha_1 \ X_t^D - \alpha_2 \ Q_t - \alpha_3 \ [R_t \ Q_t] + \varepsilon_t^D$$

- R_t is an observable variable that affects the slope of the demand, i.e., the price of a substitute or complement product.
- Key condition: $\alpha_3 \neq 0$.
- That is, when R_t varies, there should be rotation (i.e., change in the slope of the demand curve).

Solving the identification problem [5]

- Given this demand model, we have that:

$$\frac{dP_t}{dQ_t} = -\alpha_2 - \alpha_3 \ R_t$$

- And the F.O.C. for profit maximization

$$P_t + \frac{dP_t}{dQ_t} [1 + CV] \ q_{it} = MC_{it}$$

become:

$$P_t + (-\alpha_2 - \alpha_3 \ R_t) \ [1 + CV] \ q_{it} = MC_{it}$$

or equivalently:

$$P_t = MC_{it} + (\alpha_2 + \alpha_3 \ R_t) \ [1 + CV] \ q_{it}$$

Solving the identification problem [6]

- Combining this F.O.C. with the MC function, $MC_{it} = \beta_0 + \beta_1 X_t^{MC} + \beta_2 \; q_{it} + \varepsilon_{it}^{MC}$, we have:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \beta_2 \; q_{it} + (\alpha_2 + \alpha_3 \; R_t) \; [1 + CV] \; q_{it} + \varepsilon_{it}^{MC}$$

- That we can represent using the following regression model:

$$P_t = \beta_0 + \beta_1 X_t^{MC} + \gamma_1 \; q_{it} + \gamma_2 \; (R_t \; q_{it}) + \varepsilon_{it}^{MC}$$

with $\gamma_1 \equiv \beta_2 + \alpha_2 \; [1 + CV]$ and $\gamma_2 \equiv \alpha_3 \; [1 + CV]$.

Solving the identification problem [7]

- The structural equations of the model are:

$$\text{Demand:} \quad P_t = \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t - \alpha_3 \; [R_t \; Q_t] + \varepsilon_t^D$$

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \; X_t^{MC} + \gamma_1 \; q_{it} + \gamma_2 \; (R_t \; q_{it}) + \varepsilon_{it}^{MC}$$

- Using this model and data, **we can identify separately CV and MC parameters.**

Identification of demand parameters

$$\text{Demand:} \quad P_t = \quad \alpha_0 + \alpha_1 \; X_t^D - \alpha_2 \; Q_t - \alpha_3 \; [R_t \; Q_t] + \varepsilon_t^D$$

- Endogeneity problem: in equilibrium, $cov(Q_t, \varepsilon_t^D) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, Q_t depends on X_t^{MC}. Note that X_t^{MC} does not enter in demand. If X_t^{MC} is not correlated with ε_t^D, then X_t^{MC} satisfies all the conditions for being a valid instrument.
- Parameters α_0, α_1, α_2, and α_3 are identified using this IV estimator.

Identification of CV and MCs

$$\text{F.O.C.:} \quad P_t = \quad \beta_0 + \beta_1 \ X_t^{MC} + \gamma_1 \ q_{it} + \gamma_2 \ \left(R_t \ q_{it}\right) + \varepsilon_{it}^{MC}$$

- Endogeneity problem: in equilibrium, $cov(q_{it}, \varepsilon_{it}^{MC}) \neq 0$.
- The model implies a valid instrument to estimate demand.
- In equilibrium, q_{it} depends on X_t^D. Note that X_t^D does not enter enter in the F.O.C. If X_t^D is not correlated with ε_{it}^{MC}, then X_t^D satisfies all the conditions for being a valid instrument.
- Parameters β_0, β_1, γ_1, and γ_2 are identified.

Identification of CV and MCs [2]

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \, X_t^{MC} + \gamma_1 \, q_{it} + \gamma_2 \, (R_t \, q_{it}) + \varepsilon_{it}^{MC}$$

- Note that:

$$\gamma_1 = \beta_2 + \alpha_2 \left[1 + CV\right]$$

$$\gamma_2 = \alpha_3 \left[1 + CV\right]$$

- It is clear that given γ_2 and α_3, we identify CV.
- And given γ_1, α_2, and CV we identify β_2.

Identification of CV and MCs [3]

$$\text{F.O.C.:} \quad P_t = \beta_0 + \beta_1 \, X_t^{MC} + \gamma_1 \, q_{it} + \gamma_2 \, (R_t \, q_{it}) + \varepsilon_{it}^{MC}$$

- with
$$\gamma_2 = \alpha_3 \left[1 + CV\right]$$
- The identification of CV is very intuitive: $1 + CV = \gamma_2 / \alpha_3$. It measures the ratio between the sensitivity of P_t with respect to $(R_t \; q_{it})$ in the F.O.C. and the sensitivity of P_t with respect to $(R_t \; Q_t)$ in the demand.
- Example: $\alpha_3 = 0.5$ and $N = 3$.
  [Perfect competition] $CV = -1$ such that $\gamma_2 / \alpha_3 = 0$
  [Cournot] $CV = 0$ such that $\gamma_2 / \alpha_3 = 1/0.5 = 2$
  [Cartel, with $N = 3$] $CV = N - 1 = 2$ such that $\gamma_2 / \alpha_3 = 2/0.5 = 4$