ECO 310: Empirical Industrial Organization

Lecture 7: Demand Systems: Discrete Choice Models [2]

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Outline on today's lecture

1. **Estimation of the Standard Logit Model**
    - **1.1. Endogeneity problem & bias of OLS estimator**
    - **1.2. Instrumental Variables estimation**
2. **Logit model with heterogeneous coefficients**

1. Estimation of the Standard Logit Model

Estimation Standard Logit Model: Data

- Suppose that we have data on quantities (sold), prices, and characteristics of all the J products in a market:

$$\text{Data} \;=\{q_j,\; p_j,\, X_{1j},\, ...,X_{Kj}\text{: for } j=1,2,...,J\}$$

- Suppose that we also observe the consumers who have not purchased any of the J products, q_0.

- For instance, the Stata dataset `verboven_cars.dta` contains the following variables for $J = 356$ car models in the markets of five different European countries.
 price; quantity; brand; displacement (in cc); horsepower (in kW); weight (in kg); seats; doors; length; (in cm); width (in cm); height (in cm); fuel efficiency (liter per km); maximum speed (km/hour); time to acceleration (secs from 0 to 100 km/h).

Estimation Logit Model

- Given quantities, we can construct market shares. Market size (number of consumers) is: $H = q_0 + q_1 + ... + q_J$. And the market share of product j is $s_j = q_j / H$.

- The logit model implies the regression model:

$$y_j = \beta_p \ p_j + \beta_1 \ X_{1j} + ... + \beta_K \ X_{Kj} + \xi_j$$

where $y_j \equiv \ln(s_j) - \ln(s_0)$, $\beta_p = -\alpha$.

- The error term ξ_j represents characteristics of product j valuable to the consumers but unobservable to us as researchers.

- Given these data, we can estimate parameters $(\beta_p, \beta_1, ..., \beta_K)$.

OLS Estimation: Endogeneity problem

- Unfortunately, the OLS estimator does not provide unbiased (consistent) estimates of the parameters of the model.
- Products with higher unobserved quality ξ_j tend to have higher prices [See next slide]:

$$cov(p_j, \xi_j) > 0$$

- This endogeneity problem implies that the OLS estimate $\widehat{\beta}_p^{OLS}$ estimates the combination of two effects:
  (a) the causal effect of price on y_j: i.e., $\beta_p < 0$;
  (b) an indirect positive effect (not causal) that comes from the correlation between price and unobserved product quality.

$$\widehat{\beta}_p^{OLS} \to \frac{cov(y_j, p_j)}{var(p_j)} = \beta_p + \frac{cov(p_j, \xi_j)}{var(p_j)}$$

- We could even get $\widehat{\beta}_p^{OLS} > 0$.

Endogeneity problem: Example

- Suppose that the profit maximization condition, Marginal Revenue = Marginal Cost, implies the following optimal price for the firm selling product j:

$$p_j = \gamma_1 \; X_{1j} + ... + \gamma_K \; X_{Kj} + \gamma_\xi \; \xi_j$$

  where $\gamma' s$ are parameters.

- Product characteristics affect price because: (1) they affect MCs, i.e., higher quality products are more costly to produce; and (2) they enter in demand and affect marginal revenue.

- The model consist of the logit demand equation and the pricing equation. For simplicity, let's omit the X variables:

$$\begin{aligned} y_j &= \beta_p \; p_j + \xi_j \\ p_j &= \gamma_\xi \; \xi_j \end{aligned}$$

Endogeneity problem: Example [2]

- These are the structural equations of the model (I have omitted the constant terms; the variables are in deviations with respect to their respective means).

$$y_j = \beta_p \, p_j + \xi_j$$

$$p_j = \gamma_\xi \, \xi_j$$

- Solving the price equation into the demand equation, we have that:

$$y_j = (\beta_p \gamma_\xi + 1) \, \xi_j$$

- Therefore:

$$cov(y_j, p_j) = (\beta_p \gamma_\xi + 1) \, \gamma_\xi \; var(\xi_j)$$

$$var(p_j) = (\gamma_\xi)^2 \; var(\xi_j)$$

Endogeneity problem: Example [3]

- Then, in this model, the OLS estimator is such that:

$$\begin{aligned}\widehat{\beta}_p^{OLS} &\rightarrow \frac{cov(y_j, p_j)}{var(p_j)} = \frac{(\beta_p \gamma_\xi + 1)\gamma_\xi \ var(\xi_j)}{(\gamma_\xi)^2 \ var(\xi_j)} \\ &= \beta_p + \frac{1}{\gamma_\xi}\end{aligned}$$

- Since $\frac{1}{\gamma_\xi} > 0$, the OLS estimator is an upward biased estimate of the true β_p.
- Since $\beta_p < 0$, we have that the estimate is biased towards zero, or it could be even positive.

Instrumental Variables (IV) Estimation

- To deal with this endogeneity problem, we can use IV estimation.
- We need a variable (or multiple variables), Z_j, that satisfies the following conditions.
- **[1] Exclusion.** Z_j is NOT an explanatory variable in the demand equation of product j, i.e., Z_j is not part of vector $\mathbf{X}_j$.
- **[2] No correlation with error.** Z_j is NOT correlated with product j unobserved quality ξ_j.
- **[3] Relevance.** In the regression of price, p_j, on the vector $\mathbf{X}_j$ and on Z_j, variable Z_j has a significant (partial) correlation with p_j.

IV Estimation in Two stages (2SLS)

- To implement the IV estimator we can use a two stage least squares (2SLS) method.

- **[Stage 1]** We run an OLS regression for price on the exogenous variables of the model (vector $\mathbf{X}_j$) and the instrument (Z_j):

$$p_j = \gamma_z \; Z_j + \gamma_1 \; X_{1j} + ... + \gamma_K \; X_{Kj} + e_j$$

  And obtain the fitted values: $\widehat{p}_j = \widehat{\gamma}_z \; Z_j + \widehat{\gamma}_1 \; X_{1j} + ... + \widehat{\gamma}_K \; X_{Kj}$.

- **[Stage 2]** We run an OLS regression of the demand equation but using the fitted values from stage 1 ($\widehat{p}_j$) instead of price (p_j) as explanatory variable:

$$y_j = \beta_p \; \widehat{p}_j + \beta_1 \; X_{1j} + ... + \beta_K \; X_{Kj} + \xi_j^*$$

- The estimator in this second stage is the IV estimator. Standard errors should be corrected.

How to get instruments?

- Under the assumption that the observable characteristics (other than price) X_{kj} are not correlated with the unobserved quality ξ_j, the model of demand and price competition of differentiated products provides IVs.

- This model implies that the profit-maximizing price for product j depends not only on its own characteristics ($\mathbf{X}_j$ and ξ_j) but also on the characteristics of other products competing with product j ($\mathbf{X}_i$ and ξ_i).

- Intuitively, if the values of $\mathbf{X}$ are such that there are other products with similar characteristics as product j, price competition is intense and price p_j is low:

$$p_j \text{ depends positively on distance}(\mathbf{X}_j, \mathbf{X}_i)$$

How to get instruments? [2]

- Under this argument, we can use the characteristics of products other than j (i.e., X_{ki} for $i \neq j$) as an instrument for product p_j.
- These instruments are called the **Berry-Levinsohn-Pakes (BLP) instruments** in the demand of differentiated products.
- For instance, we can use:

$$Z_j = \min_{i \neq j} \|\mathbf{X}_j - \mathbf{X}_i\|$$

where $\|\mathbf{a} - \mathbf{b}\|$ is the Euclidean distance between vectors $\mathbf{a}$ and $\mathbf{b}$:

$$\|\mathbf{a} - \mathbf{b}\| = \sqrt{(a_1 - b_1)^2 + ... + (a_K - b_K)^2}$$

- Or we can use as instrument Z_j other functions of $\mathbf{X}_j$ and $\mathbf{X}_i$:

$$Z_j = \frac{\sum_{i \neq j} \|\mathbf{X}_j - \mathbf{X}_i\|}{J-1} \quad \text{or} \quad Z_j = \frac{\sum_{i \neq j} \mathbf{X}_i}{J-1}$$

2. Logit Model with Heterogeneous Coefficients

Deriving Elasticities for Standard Logit Demand

- In the logit model, $s_j = \exp\{\delta_j\} / [1 + \sum_{i=1}^{J} \exp\{\delta_i\}]$, implies that:

$$\frac{ds_j}{d\delta_j} = s_j \ (1 - s_j) \quad \text{and} \quad \frac{ds_j}{d\delta_i} = -s_j \ s_i \quad \text{for } i \neq j$$

- Let's show that $\dfrac{ds_j}{d\delta_j} = s_j \ (1 - s_j)$. We can write:

$$s_j = \frac{\exp\{\delta_j\}}{\exp\{\delta_j\} + C}$$

where $C = 1 + \sum_{i \neq j} \exp\{\delta_i\}$.

- Taking the derivative with respect to δ_j:

$$\begin{aligned} \frac{ds_j}{d\delta_j} &= \frac{\exp\{\delta_j\} [\exp\{\delta_j\} + C]}{[\exp\{\delta_j\} + C]^2} - \frac{\exp\{\delta_j\} \exp\{\delta_j\}}{[\exp\{\delta_j\} + C]^2} \\ &= \frac{\exp\{\delta_j\}}{\exp\{\delta_j\} + C} - \left[\frac{\exp\{\delta_j\}}{\exp\{\delta_j\} + C}\right]^2 = s_j(1 - s_j) \end{aligned}$$

Deriving Elasticities Standard Logit Demand [2]

- Similarly, let's show that $\dfrac{ds_j}{d\delta_i} = -s_j \ s_i$ for $i \neq j$. We can write:

$$s_j = \frac{\exp\left\{\delta_j\right\}}{\exp\left\{\delta_i\right\} + A}$$

where $A = 1 + \sum_{k \neq i} \exp\left\{\delta_k\right\}$.

- Taking the derivative with respect to δ_i:

$$\begin{aligned} \frac{ds_j}{d\delta_i} &= -\frac{\exp\left\{\delta_j\right\} \exp\left\{\delta_i\right\}}{\left[\exp\left\{\delta_i\right\} + A\right]^2} \\ &= -\left[\frac{\exp\left\{\delta_j\right\}}{\exp\left\{\delta_i\right\} + A} \frac{\exp\left\{\delta_i\right\}}{\exp\left\{\delta_i\right\} + A}\right] = s_j \ s_i \end{aligned}$$

Deriving Elasticities Standard Logit Demand [3]

- Taking into account that

$$\frac{ds_j}{d\delta_j} = s_j \ (1 - s_j) \quad \text{and} \quad \frac{ds_j}{d\delta_i} = -s_j \ s_i \quad \text{for } i \neq j$$

- The own price elasticity is (chain rule):

$$\frac{ds_j}{dp_j} \frac{p_j}{s_j} = \left[\frac{ds_j}{d\delta_j} \frac{d\delta_j}{dp_j} \right] \frac{p_j}{s_j} = s_j \ (1 - s_j) \ [-\alpha] \ \frac{p_j}{s_j} = -\alpha \ (1 - s_j) \ p_j$$

- And the cross price elasticities are:

$$\frac{ds_j}{dp_i} \frac{p_i}{s_j} = \left[\frac{ds_j}{d\delta_i} \frac{d\delta_i}{dp_i} \right] \frac{p_i}{s_j} = -s_j \ s_i \ [-\alpha] \ \frac{p_i}{s_j} = \alpha \ s_i \ p_i$$

Why are these elasticities very restrictive?

- The standard Logit model imposes some strong restrictions on price elasticities.
- Consider the car models: ECON1, ECON2, and LUX, with:

$$\begin{array}{ccc} s_{ECON1} = 0.20 & s_{ECON2} = 0.20 & s_{LUX} = 0.01 \\ s_{ECON1} = 1 & s_{ECON2} = 1 & s_{LUX} = 20 \end{array}$$

- Consider the effect of a change in p_{ECON1} on s_{ECON2} and s_{LUX}:

$$\frac{ds_{ECON2}}{dp_{ECON1}} \frac{p_{ECON1}}{s_{ECON2}} = \alpha \ s_{ECON1} \ p_{ECON1} = 0.20 * \alpha$$

$$\frac{ds_{LUX}}{dp_{ECON1}} \frac{p_{ECON1}}{s_{LUX}} = \alpha \ s_{ECON1} \ p_{ECON1} = 0.20 * \alpha$$

- This is very unrealistic. An increase in the price of product $ECON1$ implies the same proportional increase in the demand of ECON2 as in the demand of a luxury car.

Dealing with Limitations of Standard Logit Model

- We can introduce two alternative extensions in the model that relax this restriction.

  [A] Consumer heterogeneous coefficients β_h

  [B] Nested Logit model for ε_h

Logit with heterogeneous coefficients: Micro data

- Suppose that our dataset is such that we observe consumer purchasing decisions for G groups of consumers, indexed by $g = 1, 2, ..., G$.
- These G groups are based on consumer demographic characteristics such as age, gender, income, geographic location, etc.
- For instance, group 1 could be defined as: "Consumers in age group 20-to-30; Female; income group [$70K-$80K]; in city A".
- For each group g, we observe quantities q_{gj} and the number of consumers H_g, such that we can construct the market shares $s_{gj} = q_{gj} / H_g$.

Logit with heterogeneous coefficients

- Suppose that consumer groups are heterogeneous in the preferences: in the utility parameters α and β.

- The logit model for group g is:

$$\ln\left(s_{gj}\right) - \ln\left(s_{g0}\right) = -\alpha_g \ p_j + \beta_{1g} \ X_{1j} + ... + \beta_{Kg} \ X_{Kj} + \xi_{gj}$$

- Note that the explanatory variables (p_j and X_j) are the same for each group, but the dependent variable and the parameters are different.

- We have G different regression equations, one for each group. We can estimate the model parameters separately for each group using the IV method described above.

Heter. coeff overcome limitations standard Logit

- For each group g, the model has the same structure as the standard logit. However, now the aggregate demand of product j has a different structure.

- The aggregate demand of product j is:

$$q_j = \sum_{g=1}^{G} q_{gj} = \sum_{g=1}^{G} H_g \ s_{gj} = \sum_{g=1}^{G} H_g \left[\frac{\exp\left\{\delta_{gj}\right\}}{\sum_{i=0}^{J} \exp\left\{\delta_{gi}\right\}} \right]$$

with $\delta_{gj} = -\alpha_g \ p_j + \beta_{1g} \ X_{1j} + ... + \beta_{Kg} \ X_{Kj} + \xi_{gj}$.

- Now, we have:

$$\frac{dq_j}{dp_i} \frac{p_i}{q_j} = \left[\sum_{g=1}^{G} H_g \ \frac{ds_{gj}}{dp_i} \right] \frac{p_i}{q_j} = \left[\sum_{g=1}^{G} H_g \ \alpha_g \ s_{gj} \ s_{gi} \right] \frac{p_i}{q_j}$$

Heterogeneous coeff. Logit: Price elasticities

$$\frac{\partial q_j}{\partial p_i}\frac{p_i}{q_j} = \left[\sum_{g=1}^{G} H_g \;\alpha_g \;s_{gj} \;s_{gi}\right]\frac{p_i}{q_j}$$

- Note that $\sum_{g=1}^{G} H_g \;\alpha_g \;s_{gj} \;s_{gi}$ is no longer equal to $\alpha \;s_j \;s_i$.
- The term $\sum_{g=1}^{G} H_g \;\alpha_g \;s_{gj} \;s_{gi}$ measures the **Covariance of the market shares of products** j and i across groups.
- This covariation depends on the characteristics of these products.
- If the two products have similar characteristics, Covariance is large.
- If the two products have very different characteristics, Covariance is small.

Heterogeneous coeff. Logit: Price elasticities

- For simplicity, suppose that $H_g = H$ and $\alpha_g = \alpha$ for all groups.
- It is simple to varifu that:

$$\frac{1}{G}\sum_{g=1}^{G} s_{gj}\ s_{gi} = \bar{s}_j\ \bar{s}_i + Cov(s_{gj}, s_{gi})$$

where $\bar{s}_j = \frac{1}{G}\sum_{g=1}^{G} s_{gj}$; $\bar{s}_i = \frac{1}{G}\sum_{g=1}^{G} s_{gi}$; and $Cov(s_{gj}, s_{gi}) = \frac{1}{G}\sum_{g=1}^{G}(s_{gj} - \bar{s}_j)(s_{gi} - \bar{s}_i)$.

- Therefore, elasticity between products j and i:

$$\text{Elasticity } j, i = H\ \alpha\ G\left[\bar{s}_j\ \bar{s}_i + Cov(s_{gj}, s_{gi})\right]\frac{p_i}{q_j}$$

- Product with similar characteristics have high $Cov(s_{gj}, s_{gi})$ and therefore high cross elasticity.