Empirical Industrial Organization (ECO 310)
Winter 2021 – Sections 0101, 9101, 0201, 9201
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Problem Set #1

Due on Friday, February 5th, 2021, before 11:59pm

INSTRUCTIONS. Please, follow the following instructions for the submission of your completed problem set.

1. Write your answers electronically in a word processor.
2. For the answers that involve coding in STATA, include in the document the code in STATA that you have used to obtain your empirical results.
3. Convert the document to PDF format.
4. Submit your problem set (in PDF) online via Quercus.
5. You should submit your completed problem set before Friday, February 5th, at 11:59pm.
6. Problem sets should be written individually.

The total number of marks is 200.

QUESTION 1. [60 points]. Consider an industry for an homogeneous product with the following characteristics.

- The production function uses labor, capital, and TFP, and it has a Cobb-Douglas form.
- All the firms use the same amount of capital (the same fixed equipment).
- Firms sell their output in the same product market at the same price.
- Firms are in different geographic locations with different labor markets.
- Firms are price takers in labor and product markets and maximize profits.

Q1.1. (10 points) (a) Write the expression for the Cobb-Douglas production function, and for this PF in logarithms. (b) Write the expression for a firm's profit function. (c) Obtain the first order conditions for the maximization of profits with respect to the amount of labor. (d) Obtain the equation that describes a firm's labor demand. (e) Write the labor demand equation in logarithms: in terms of the logarithms of output and labor.

Q1.2. (10 points) (a) Write the simultaneous equations model that consists of the production function and the demand equation in logarithms. (b) Solve this system of two equations and two unknowns to obtain the equilibrium amounts of output and labor as functions of the exogenous variables.

Q1.3. (10 points) Suppose that the industry is such that firms are heterogeneous in their log-TFP (represented as ω_i) and in their log-real wage (represented as r_i). Let σ_ω^2, σ_r^2, and $\sigma_{\omega r}$ be the variance of ω_i, the variance of r_i, and the covariance between ω_i and r_i, respectively. Use the equilibrium equations in Q1.2 to obtain the expression for: (a) the covariance between log-output and log-labor as a function of σ_ω^2, σ_r^2, and $\sigma_{\omega r}$; and (b) the variance of log-labor as a function of σ_ω^2, σ_r^2, and $\sigma_{\omega r}$.

Q1.4. (10 points) Consider the OLS estimation of the labor intensity parameter α_L in the production function. (a) Write the expression for this OLS estimator as a function of the data on log-output and log-labor. By the Law of Large Numbers, when the number of observations (firms) N is large, the sample variances and covariances in the expression for the OLS estimator converge to their population counterparts. Given the results in Q1.3, obtain the expression for the OLS estimator (when the sample size is large) as a function of the parameters α_L, σ_ω^2, σ_r^2, and $\sigma_{\omega r}$.

Q1.5. (20 points) Using the expression that you have derived in Q1.4 obtain the expression for the bias of the OLS estimator under the following different scenarios for the industry. (a) No heterogeneity in real wages: $\sigma_r^2 = 0$. (b) No heterogeneity in TFP: $\sigma_\omega^2 = 0$. (c) Heterogeneity in real wages and TFP but not correlation between them: $\sigma_r^2 > 0$, $\sigma_\omega^2 > 0$, and $\sigma_{\omega r} = 0$. Try to obtain the sign of the bias in each of these three cases. Explain the results.

QUESTION 2. [40 points]. Consider an industry for an homogeneous product. Firms use capital and labor to produce output according to a Cobb-Douglas technology with parameters α_L and α_K and Total Factor Productivity (TFP) A. Suppose that firms are price takers in the input markets for labor and capital. Let W_L and W_K be the price of labor and capital, respectively.

Q2.1. (5 points) (a) Derive the expression for the cost function of a firm $C(Y)$ as a function of output Y, the technological parameters α_L and α_K, TFP, and input prices. Explain in detail the different steps in your derivation.

Q2.2. (5 points) For this question Q2.2, suppose that $\alpha_L = 0.6$ and $\alpha_K = 0.3$. Obtain the values of the following elasticities. Explain your answer. (a) Elasticity of cost with respect to output. (b) Elasticity of cost with respect to TFP. (c) Elasticity of cost with respect to the price of labor. (d) Elasticity of cost with respect to the price of capital. (e) Elasticity of output with respect to labor. (f) Elasticity of output with respect to TFP.

Suppose that the output market in this industry is competitive: firms are price takers. The demand function is linear with the following form: $P = 100 - Q$, where P and Q are the industry price and total output, respectively. **Suppose that $\alpha_L = \alpha_K = 1/4$, and the value of input prices are $W_L = W_K = 1/4$. Suppose also that each firm has a fixed cost (the cost of fixed land) that is exogenous and equal to 2.**

Q2.3. (5 points) Using these primitives, write the expression for the profit function of a firm (revenue minus cost) as a function of the market price, P, the firm's output, Y_i, and its TFP, A_i.

Q2.4. (5 points) Using the condition "price equal to marginal cost", obtain the optimal amount of output of a firm as a function of the market price, P, and the firm's TFP, A_i. Explain your derivation.

Q2.5. (5 points) A firm is active in the market (i.e., it finds optimal to produce a positive amount of output) only if its profit is greater or equal than zero. Using this condition show that a firm is active in this industry only if its TFP satisfies the condition $A_i \geq 2/P$. Explain your derivation.

Let (P^*, Q^*, Y_1^*, Y_2^*, ..., Y_N^*) the equilibrium price, total output, and individual firms' outputs. Based on the previous results, the market equilibrium can be characterized by the following conditions:

(i) The demand equation holds: $P^* = 100 - Q^*$.

(ii) Total output is equal to the sum of firms' individual outputs: $Q^* = Y_1^* + ... + Y_N^*$.

(iii) Firm i is active ($Y_i^* > 0$) if and only if its total profit is greater than zero: $Y_i^* > 0$ if and only if $A_i \geq 2/P^*$.

(iv) For firms with $Y_i^* > 0$, the optimal amount of output is given by the condition $P^* = MC_i(Y_i^*)$, where $MC_i(.)$ represents the marginal cost function for firm i.

Q2.6. (5 points) Combine conditions (i) to (iv) to show that the equilibrium price can be written as the solution to this equation:

$$P^* = 100 - P^* \left[\sum_{i=1}^{N} A_i^2 \ 1\{A_i \geq 2/P^*\} \right]$$

where $1\{x\}$ is the indicator function that is defined as $1\{x\} = 1$ if condition x is true, and $1\{x\} = 0$ if condition x is false. Explain your derivation.

Suppose that the subindex i sorts firms by their TFP such that firm 1 is the most efficient, then firm 2, etc. That is, $A_1 > A_2 > A_3 > .....$

Q2.7. (5 points) Suppose that $A_1 = 7$, $A_2 = 5$, and $A_3 = 1$. Obtain the equilibrium price, total output, and output of each individual firm in this industry. [Hint: Start with the conjecture that only firms 1 and 2 produce in equilibrium. Then, confirm this conjecture. Note that we do not need to know the values of A_4, A_5, etc].

Q2.8. (5 points) Explain why the most efficient firm, with the largest TFP, does not produce all the output of the industry.

QUESTION 3. [100 points]. The datafile *blundell_ bond_ 2000_ production_ function.dta* contains annual information on sales, labor, and capital for 509 firms for the period 1982-1989 (8 years). Consider a Cobb-Douglas production function in terms of labor and capital. Use this dataset to implement the following estimators and hypothesis tests. Provide the code in STATA and the table of estimation results.

Q3.1. (10 points) (a) OLS with time dummies. (b) Test the null hypothesis $\alpha_L + \alpha_K = 1$. Comment the results.

Q3.2. (10 points) (a) Fixed Effects estimator with time dummies. (b) Test the null hypothesis of no time-invariant unobserved heterogeneity: $\eta_i = \eta$ for every firm i. Comment the results.

Q3.3. (10 points) (a) Fixed Effects - Cochrane Orcutt estimator with time dummies. (b) Test the two over-identifying restrictions of the model. Comment the results.

Q3.4. (10 points) Arellano-Bond estimator with time dummies and non-serially correlated transitory shock. Comment the results.

Q3.5. (10 points) Arellano-Bond estimator with time dummies and AR(1) transitory shock. Comment the results.

Q3.6. (10 points) Blundell-Bond system estimator with time dummies and non-serially correlated transitory shock. Comment the results.

Q3.7. (10 points) Blundell-Bond system estimator with time dummies and AR(1) transitory shock. Comment the results.

Q3.8. (10 points) Based on the previous results, select your preferred estimates of the production function. Explain your choice.

Q3.9. (20 points) For this question, your favorite estimates according to your answer to Q3.8, and log-TFPs for year 1989. (a) Obtain the median, the percentile 5, and the percentile 95 in the distribution of log-TFP. Suppose that all the firms operate in the same input markets and $W_L = W_K = 1$. (b) Present a figure with three marginal cost functions (i.e., output Y in the horizontal axis and marginal cost MC in the vertical axis) for the firms with median, percentile 5, and percentile 95 TFPs, respectively. (c) Comment the results.