ECO 310: Empirical Industrial Organization

Lecture 2: Production Functions: Introduction

Victor Aguirregabiria (University of Toronto)

January 18, 2021

Outline

1. Model
2. Data
3. What determines productivity?
4. Estimation: The simultaneity problem

1. Model

What is a Production Function?

- It is a function that relates the amount of physical output of a production process (Y) to the amount of physical inputs or factors of production (X).
- Estimation of PFs plays a key role in empirical questions such as:
  - Productivity growth: measurement, heterogeneity (dispersion).
  - Misallocation of inputs. How allocation of capital and labor relates to TFP.
  - Estimation Firms' Costs.
  - Technological change over time or across industries. Capital intensity. Skill labor intensity.
  - Evaluating the effects of adopting new technologies
  - Measuring learning-by-doing.

Production functions

- A general representation is:

$$Y = A \times f(X_1, X_2, ..., X_J)$$

Y is a measure of firm output;
X_1, X_2, .., and X_J are measures of J firm inputs;
A represents the firm's **Total Factor Productivity**.

- The marginal productivity of input j is: $MP_j = \dfrac{\partial Y}{\partial X_j} = A \, \dfrac{\partial f}{\partial X_j}$.

- Note that TFP increases proportionally the MP of all the inputs. We say that TFP is (Hicks) neutral.

Cobb-Douglas PF

- A common specification is the Cobb-Douglas PF:

$$Y = A \ X_1^{\alpha_1} \ X_2^{\alpha_2} \ ... \ X_J^{\alpha_J}$$

α_1, α_2, ..., α_J are technological parameters (all positive).

- For the Cobb-Douglas PF the marginal productivity of input j is:

$$MP_j = \alpha_j \frac{Y}{X_j}$$

- All the inputs are complements in production. MP_j increases with the amount of any other input k:

$$\frac{\partial MP_j}{\partial X_k} = \frac{\alpha_j}{X_j} \frac{\alpha_k}{X_k} Y > 0$$

Production function and Cost Function

- Given the production function and input prices, the cost function $C(Y)$ is defined as the minimum cost of producing the amount of output Y:

$$C(Y) = \left[\begin{array}{c} \min_{\{X_1, X_2, ..., X_J\}} W_1 X_1 + W_2 X_2 + ... + W_J X_J \\ \\ \text{subject to: } Y = A\ f(X_1, X_2, ..., X_J) \end{array} \right]$$

- Or using a Lagrange representation:

$$C(Y) = \min_{\{\lambda, X_1, ..., X_J\}} W_1 X_1 + ... + W_J X_J + \lambda \left[Y - A\ f(X_1, ..., X_J) \right]$$

where λ is the Lagrange multiplier of the restriction.

- The marginal conditions of optimality imply that for every input j,

$$W_j - \lambda\ MP_j = 0$$

Cost Function: Cobb-Douglas

- For the Cobb-Douglas PF $Y = A\ X_1^{\alpha_1} ...\ X_1^{\alpha_J}$ the marginal condition of optimality for input j implies:

$$W_j\ X_j = \lambda \alpha_j\ Y$$

- Therefore, the cost is equal to:

$$\sum_{j=1}^{J} W_j X_j = \lambda \alpha Y$$

where $\alpha \equiv \sum_{j=1}^{J} \alpha_j$ and measures returns to scale: constant if $\alpha = 1$, decreasing if $\alpha < 1$, and increasing if $\alpha > 1$.

Cost Function: Cobb-Douglas [2]

- We need to obtain the value of the Lagrange multiplier λ. For this, we solve the marginal conditions $X_j = \lambda \alpha_j \; Y / W_j$ into the PF:

$$Y = A \left(\frac{\lambda \alpha_1 Y}{W_1} \right)^{\alpha_1} \left(\frac{\lambda \alpha_2 Y}{W_2} \right)^{\alpha_2} \ldots \left(\frac{\lambda \alpha_J Y}{W_J} \right)^{\alpha_J}$$

- Solving in this expression for the Lagrange multiplier:

$$\lambda = \left(\frac{W_1}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha}} \left(\frac{W_2}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha}} \ldots \left(\frac{W_J}{\alpha_J} \right)^{\frac{\alpha_J}{\alpha}} \left(\frac{Y}{A} \right)^{\frac{1}{\alpha}} \frac{1}{Y}$$

- Plugging this expression of the multiplier into the equation $C(Y) = \lambda \alpha \; Y$ for the cost, we obtain the cost function:

$$C(Y) = \alpha \left(\frac{Y}{A} \right)^{\frac{1}{\alpha}} \left(\frac{W_1}{\alpha_1} \right)^{\frac{\alpha_1}{\alpha}} \left(\frac{W_2}{\alpha_2} \right)^{\frac{\alpha_2}{\alpha}} \ldots \left(\frac{W_J}{\alpha_J} \right)^{\frac{\alpha_J}{\alpha}}$$

Cost Function: Cobb-Douglas (3)

$$C(Y) = \alpha \left(\frac{Y}{A}\right)^{\frac{1}{\alpha}} \left(\frac{W_1}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha}} \left(\frac{W_2}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha}} \dots \left(\frac{W_J}{\alpha_J}\right)^{\frac{\alpha_J}{\alpha}}$$

- The marginal cost is:

$$C'(Y) = Y^{\left(\frac{1}{\alpha}-1\right)} \left(\frac{1}{A}\right)^{\frac{1}{\alpha}} \left(\frac{W_1}{\alpha_1}\right)^{\frac{\alpha_1}{\alpha}} \left(\frac{W_2}{\alpha_2}\right)^{\frac{\alpha_2}{\alpha}} \dots \left(\frac{W_J}{\alpha_J}\right)^{\frac{\alpha_J}{\alpha}}$$

- The sign of $C''(Y)$ is equal to the sign of $\frac{1}{\alpha} - 1$.
  $\alpha = 1$ (*constant returns*): $C''(Y) = 0$ (Constant MC)
  $\alpha < 1$ (*decreasing returns*): $C''(Y) > 0$ (Increasing MC)
  $\alpha > 1$ (*increasing returns*): $C''(Y) < 0$ (Decreasing MC).

More on the Cobb-Douglas

- A nice property (for estimation) of the Cobb-Douglas is that its logarithm transformation is linear in parameters:

$$\ln(Y) = \ln(A) + \alpha_1 \ \ln(X_1) + \alpha_2 \ \ln(X_2) + \ ... + \alpha_J \ \ln(X_J)$$

- We will represent $\log(Y)$ and $\log(X)$ using the lower letters y and x, resp., and the log-TFP using ω, such that:

$$y = \omega + \alpha_1 \ x_1 + \alpha_2 \ x_2 + \ ... + \alpha_J \ x_J$$

- Differences in log-TFP (ω) are in percentage terms:
  - Consider two firms, 1 and 2, using the same amount of inputs X but with $\omega_1 = 1.1$ and $\omega_2 = 1.5$ such that $\omega_2 - \omega_1 = 0.4$. Therefore, firm 2 is 40% more productive than firm 1.

More on the Cobb-Douglas (2)

- Most empirical applications that we will see in the course consider two inputs: labor (L) and capital (K):

$$y = \alpha_L \; \ell + \alpha_K \; k + \omega$$

with $\ell \equiv \ln(L)$ and $k \equiv \ln(K)$.

- Sometimes the specification also includes materials (M):

$$y = \alpha_L \; \ell + \alpha_K \; k + \alpha_M \; m + \omega$$

with $m \equiv \ln(M)$.

2. Data

Data

- Panel data of N firms over T periods with information on output, labor, and capital (in logs):

$$\{\ y_{it},\ \ell_{it},\ k_{it} : i = 1, 2, ..., N\ ;\quad t = 1, 2, ...T\ \}$$

- We are interested in the estimation of the Cobb-Douglas PF (in logs):

$$y_{it} = \alpha_L\ \ell_{it} + \alpha_K\ k_{it} + \omega_{it} + e_{it}$$

ω_{it} = *log-TFP. Unobserved inputs (for the researcher) which are known to the firm when it decides K and L (e.g., managerial ability, quality of land, different technologies).*

e_{it} = *measurement error in output or shock affecting output that is unknown to the firm when it decides K and L.*

Measurement: Observing revenue instead of physical output

- $R_{it} = P_{it}\ Y_{it}$ such that $\ln(R_{it}) = \ln(P_{it}) + \ln(Y_{it}) = p_{it} + y_{it}$, but the researcher only observes $\ln(R_{it})$.

- **Possible "solution" 1**

  * Try to measure $\ln(P_{it})$ as good as possible using industry level price indexes.

  * In general, $\ln(P_{it}) = \ln(P_{Industry,t}) + u_{it}$, where u_{it} is measurement error.

  * This measurement error can interpreted as part of the TFP ω_{it}.

Observing revenue instead of physical output [2]

- **Possible "solution" 2**
- Assume isoelastic demand & monopolisitc competition:

$$y_{it} = b_{it} - \beta \ p_{it}$$

where b_{it} is unobservable to the researcher and β is the elasticity of demand.

- Then, $p_{it} = \dfrac{b_{it} - y_{it}}{\beta}$ and $\ln(R_{it}) = p_{it} + y_{it} = \dfrac{b_{it}}{\beta} + \left(1 - \dfrac{1}{\beta}\right) y_{it}$ such that:

$$\ln(R_{it}) = \alpha_L^* \ \ell_{it} + \alpha_K^* \ k_{it} + \omega_{it}^* + e_{it}$$

with $\alpha_L^* = \left(1 - \frac{1}{\beta}\right) \alpha_L$, $\alpha_K^* = \left(1 - \frac{1}{\beta}\right) \alpha_K$, and $\omega_{it}^* = \omega_{it} + \frac{b_{it}}{\beta}$.

- This is very relevant for interpretation of results and of "log-TFP".
- For instance, $\alpha_L^* + \alpha_K^* = \left(1 - \frac{1}{\beta}\right) (\alpha_L + \alpha_K) < \alpha_L + \alpha_K$.

Measurement: Capital

- We typically observe firms' investments in physical capital but not the capital stock K_{it}.
- Instead we observe the "book value" (accounting value) of capital and of amortization.
- The most common approach to construct the economic stock of capital is the **perpetual inventory method**.

$$K_{it} = (1-\delta)\ K_{it-1} + I_{it}$$

such that:

$$K_{it} = I_{it} + (1-\delta) I_{it-1} + ... + (1-\delta)^{t-1} I_1 + (1-\delta)^t K_0$$

- We know investments. We need to know the depreciation rate (δ) and the initial capital stock (K_0).

3. What determines productivity?

Total Factor Productivity (TFP)

- Production function:

$$Y_{it} = A_{it} \; F(K_{it}, L_{it}, M_{it})$$

- A_{it} is denoted Total Factor Productivity (TFP).
- It is a factor-neutral shifter that captures variations in output not explained by observable inputs.
- TFP is a residual.

Large and persistent differences in TFP across firms

- **Ubiquitous**, even within narrowly defined industries and products.
- **Large:** 90th to 10th percentile TFP ratios: $\dfrac{A_{90th}}{A_{10th}}$
  - U.S. manufacturing, **average** within 4-digit SIC industries $=$ **1.92** (Syverson, 2004)
  - Denmark: average $=$ **3.75** (Fox and Smeets, 2011)
  - China or India, **average** $>$ **5** (Hsieh & Klenow, 2009).
- **Persistent:**
  - AR(1) of log-TFP with annual frequency: autoregressive coefficients between 0.6 to 0.8.
- **It matters:** Higher productivity producers are more likely to survive, innovate, invest, ...

Why firms differ in their productivity levels?

- What supports such large productivity differences in equilibrium?
- Can producers control the factors that influence productivity or are they purely external effects of the environment?
- If firms can partly control their TFP, what type of choices increase it?

Why dispersion is possible in equilibrium?

- Because the profit function is concave in output and the optimal amount of profit for a monopolist (or duopolist, ...) is smaller than total demand.

- Let the profit of a firm be:

$$\pi_i = P_i(Y_i)\ Y_i - C(Y_i, A_i)$$

$P_i(Y_i)$ = Inverse demand function; $C(Y_i, A_i)$ = Cost function.

- **Key condition:** either $P_i(Y_i)\ Y_i$ is strictly concave in Y_i, or $C(.)$ is strictly convex in Y_i. [The profit function is strictly concave].

- **Example: DRS even with perfect competition**. $P\ Y_i$ is linear in Y_i but $C(.)$ is strictly convex because DRS.

- **Example: Oligopoly competition even with CRS**. $C(.)$ is linear but $P_i(Y_i)\ Y_i$ is strictly concave in Y_i if demand is downward sloping.

Why dispersion is possible in equilibrium? [2]

- Equilibrium implies the marginal condition for optimal output:

$$MR_i \equiv \frac{\partial\ [P(Y_i)\ \ Y_i]}{\partial Y_i} = \frac{\partial C(Y_i, A_i)}{\partial Y_i} \equiv MC_i$$

- If variable profit is strictly concave, this equilibrium can support firms with different TFPs, A_i.

- It is not optimal for the firm with highest TFP to provide all the output in the industry.

- Firms with different TFPs (above a certain threshold value) operate in the same market.

How can a firm affect its TFP?

- (HR) Managerial Practices. (Bloom & Van Reenen, 2007; Ichniowski and Shaw, 2003)
- Learning-by-Doing (Benkard, 2000).
- Organizational structure (vertical integration vs outsourcing).
- Higher-Quality (Labor and Capital) inputs.
- Adoption of new (IT) technologies.(Brynjolfsson et al., 2008).
- Investment in R&D. Long literature linking R&D investment and productivity.
- Innovation. Many firms undertake both process and product innovation without formally reporting R&D spending.

4. Estimation: Endogeneity / Simultaneity Problem

Endogeneity / Simultaneity problem

- Consider the PF:

$$y_{it} = \alpha_L \ \ell_{it} + \alpha_K \ k_{it} + \omega_{it} + e_{it}$$

- We are interested in the estimation of α_L and α_K. These parameters represent "ceteris paribus" causal effects of labor and capital on output, respectively.
- When the manager decides the optimal (k_{it}, ℓ_{it}) she has some information about log-TFP ω_{it} (that we do not observe).
- This means that there is a correlation between the observable inputs (k_{it}, ℓ_{it}) are correlated with the unobservable ω_{it}.
- This correlation implies that the OLS estimators of α_L and α_K are biased and inconsistent.