ECO 2901
EMPIRICAL INDUSTRIAL ORGANIZATION

Lecture 12:
Dynamic games with firms' non-equilibrium beliefs and learning

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1. Introduction

Introduction

- Firms are **heterogeneous** in their ability to **acquire and process information**.
- The implications of firms' heterogeneous expectations on firms' performance and market outcomes have been long recognized in economics, at least since the work of **Herbert Simon** (1958, 1959).
- However, the assumption of rational expectations has been the status quo to represent agents' beliefs in many areas in economics, and in particular in IO.
- It has not been until recently that **firms' biased beliefs** has received substantial attention in structural models in empirical IO.

Aguirregabiria & Magesan (REStud, 2020)

- [1] Present a **dynamic game of oligopoly competition** that allows for **biased beliefs** and learning, but it is agnostic about the source of biased beliefs and the form of learning (if any).
- [2] Study **nonparametric identification of firms' belief functions** and structural parameters in the profit function.
- [3] Application to **market entry and geographic expansion** of McDonalds and Burger King during the early years of this industry in Britain.

2. Model

Model: Dynamic Game

- N firms indexed by i. Every period t, each firm takes an action $a_{it} \in \{0, 1, ..., J\}$.

- One-period **profit function** is:

$$\Pi_{it} = \pi_{it}(a_{it}, \mathbf{a}_{-it}, \mathbf{x}_t) + \varepsilon_{it}(x_{it})$$

- $\mathbf{x}_t$ = vector of common knowledge state vars. with **transition prob.**

$$f_t\left(\mathbf{x}_{t+1} \mid a_{it},\ \mathbf{a}_{-it},\ \mathbf{x}_t\right)$$

- $\varepsilon'_{it}s$ are private info of player i and unobservable to researcher. It is i.i.d. over time and players.

Maintain some assumptions from MPE

- **ASSUMPTION 1 (Payoff relevant state variables):** *Players' strategy functions depend only on payoff relevant state variables:* $\mathbf{x}_t$ and ε_{it}.

- **ASSUMPTION 2 (Maximization of intertemporal payoffs):** *Players are forward looking and maximize expected intertemporal payoffs.*

- **ASSUMPTION 3 (Rational beliefs on own future behavior):** *Players have rational expectations on their own behavior in the future.*

- We **relax** the assumption that firms have **unbiased or equilibrium beliefs on other players' behaviour**,

Strategies, Choice Probabilities, and Beliefs

- Let $\sigma_{it}(\mathbf{x}_t, \varepsilon_{it})$ be the strategy function for player i at period t.
- $P_{it}(a_i|\mathbf{x}_t) \equiv \Pr(\sigma_{it}(\mathbf{x}_t, \varepsilon_{it}) = a_i|\mathbf{x}_t)$ choice probability of player i.
- $B_{it+s}^{(t)}(\mathbf{a}_{-i}|\mathbf{x}_{t+s})$ **beliefs** of player i at period t about the behavior of other players at period $t+s$.
- The model allows the belief functions $B_{it+s}^{(t)}$ to vary freely both over t (i.e., over the period when these beliefs are formed) and over $t+s$ (i.e., over the period of the other players' behavior).
- In particular, the model allows players to update their beliefs and learn (or not) over time t.

Sequence of Beliefs $B_{it+s}^{(t)}$

Beliefs formed (t)	Period of the opponents' behavior $(t+s)$				
	$t+s=1$	$t+s=2$	$t+s=3$	...	$t+s=T$
$t=1$	$B_{i1}^{(1)}$	$B_{i2}^{(1)}$	$B_{i3}^{(1)}$	...	$B_{iT}^{(1)}$
$t=2$	-	$B_{i2}^{(2)}$	$B_{i3}^{(2)}$	...	$B_{iT}^{(2)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$t=T$	-	-	-	...	$B_{iT}^{(T)}$

Best Response Functions

- Given her beliefs at period t, $\mathbf{B}_i(t) = \left\{ B^{(t)}_{i,t+s} : s \geq 0 \right\}$, a player best response at period t is the solution of a single-agent Dynamic Programming problem.

- At period t, the DP problem can be described in terms of: **(1)** a sequence of **expected one-period payoff functions**:

$$\pi^{\mathbf{B}(t)}_{it+s}(a_{it+s}, \mathbf{x}_{t+s}) \equiv \sum_{\mathbf{a}_{-i}} B^{(t)}_{it+s}(\mathbf{a}_{-i}|\mathbf{x}_{t+s}) \;\; \pi_{it+s}(a_{it+s}, \mathbf{a}_{-i}, \mathbf{x}_{t+s})$$

- And **(2)** a sequence of **transition probability functions**:

$$f^{\mathbf{B}(t)}_{it+s}(\mathbf{x}_{t+s+1}|a_{it+s}, \mathbf{x}_{t+s}) \equiv \sum_{\mathbf{a}_{-i}} B^{(t)}_{it+s}(\mathbf{a}_{-i}|\mathbf{x}_{t+s}) \;\; f_{t+s}(\mathbf{x}_{t+s+1}|a_{it+s}, \mathbf{a}_{-i}, \mathbf{x}_{t}$$

Best Response Functions (2)

- The solution of this DP problem implies the vector of **conditional choice value functions** at period t:

$$v_{it}^{\mathbf{B}(t)}(\mathbf{x}_t) = \left\{ v_{it}^{\mathbf{B}(t)}(a_i, \mathbf{x}_t) : a_i = 0, 1, ..., J \right\}$$

- And the **best response choice probabilities**:

$$P_{it}(a_i|\mathbf{x}_t) = \Pr\left(v_{it}^{\mathbf{B}(t)}(a_i, \mathbf{x}_t) + \varepsilon_{it}(a_i) \geq v_{it}^{\mathbf{B}(t)}(a_i', \mathbf{x}_t) + \varepsilon_{it}(a_i') \ \forall a_i' \right)$$

- For instance, in a logit model:

$$P_{it}(a_i|\mathbf{x}_t) = \frac{\exp\left\{ v_{it}^{\mathbf{B}(t)}(a_i, \mathbf{x}_t) \right\}}{\sum_{j=0}^{J} \exp\left\{ v_{it}^{\mathbf{B}(t)}(j, \mathbf{x}_t) \right\}}$$

3. Identification

Data

- We have a random sample of M markets, indexed by m, where we observe

$$\{a_{imt},\ \mathbf{x}_{mt} : i = 1, 2, ..., N;\ t = 1, 2, ..., T^{data}\}$$

- N and T^{data} are small and M is large.

- The payoff functions $\pi_{it}(a_{it}, \mathbf{a}_{-it}, \mathbf{x}_t)$ and the beliefs functions $B^{(t)}_{it+s}(\mathbf{a}_{-i}|\mathbf{x}_{t+s})$ are **nonparametrically specified**.

- The distribution of the unobservables Λ is assumed known.

- I focus here in a model with two players, i and j, but the paper results can be extended to N players.

Inversion of CCPs

- The model is described by the conditions:

$$P_{it}(a_i|\mathbf{x}_t) \quad = \quad \Lambda\left(a_i \ ; v_{it}^{\mathbf{B}(t)}(\mathbf{x}_t)\right)$$

- The CCPs $P_{it}(a_i|\mathbf{x}_t)$ are identified using data from M markets.

- Hotz-Miller inversion theorem implies that we can invert the best response mapping to obtain value differences $\widetilde{v}_{it}^{\mathbf{B}(t)}(a_i, \mathbf{x}_t) \equiv v_{it}^{\mathbf{B}(t)}(a_i, \mathbf{x}_t) - v_{it}^{\mathbf{B}(t)}(0, \mathbf{x}_t)$ as functions of CCPs:

$$\widetilde{v}_{it}^{\mathbf{B}(t)}(a_i, \mathbf{x}_t) \quad = \quad \Lambda^{-1}\left(a_i \ ; \mathbf{P}_{it}(\mathbf{x}_t)\right)$$

- The identification problem is to obtain beliefs and payoff functions given that $\Lambda^{-1}\left(a_i \ ; \mathbf{P}_{it}(\mathbf{x}_t)\right)$ are known.

Structure of the restrictions

- By definition the value differences $\widetilde{v}_{it}^{\mathbf{B}(t)}(a_i, \mathbf{x}_t)$ have the following structure:

$$\widetilde{v}_{it}^{\mathbf{B}(t)}(a_i, \mathbf{x}_t) \quad = \quad \mathbf{B}_{it}^{(t)}(\mathbf{x}_t)' \left[\widetilde{\boldsymbol{\pi}}_{it}(a_{it}, \mathbf{x}_t) + \widetilde{\mathbf{c}}_{it}^{\mathbf{B}(t)}(a_{it}, \mathbf{x}_t) \right]$$

  where $\mathbf{B}_{it}^{(t)}(\mathbf{x}_t)$ is the vector of beliefs $\left[B_{it}^{(t)}(\mathbf{a}_{-i} | \mathbf{x}_t) \right.$ for any value $\left. \mathbf{a}_{-i} \right]$.

- $\widetilde{\boldsymbol{\pi}}_{it}(a_{it}, \mathbf{x}_t)$ is the vector of payoff differences $\left[\pi_{it}(a_{it}, \mathbf{a}_{-i}, \mathbf{x}_t) - \pi_{it}(0, \mathbf{a}_{-i}, \mathbf{x}_t) \right.$ for any value $\left. \mathbf{a}_{-i} \right]$.

- $\widetilde{\mathbf{c}}_{it}^{\mathbf{B}(t)}(a_{it}, \mathbf{x}_t)$ is the vector of **differences of continuation values** $\left[c_{it}^{\mathbf{B}(t)}(a_{it}, \mathbf{a}_{-i}, \mathbf{x}_t) - c_{it}^{\mathbf{B}(t)}(0, \mathbf{a}_{-i}, \mathbf{x}_t) \right.$ for any value $\left. \mathbf{a}_{-i} \right]$ where:

$$c_{it}^{\mathbf{B}(t)}(a_{it}, \mathbf{a}_{-i}, \mathbf{x}_t) = \beta \sum V_{it+1}^{\mathbf{B}(t)}(\mathbf{x}_{t+1}) \; f_t(\mathbf{x}_{t+1} | a_{it}, \mathbf{a}_{-i}, \mathbf{x}_t)$$

Identification Assumptions

- **ASSUMPTION ID-1.** A player has the same beliefs in markets with the same **x** variables.

$$B_{imt+s}^{(t)}(.|\mathbf{x}) = B_{it+s}^{(t)}(.|\mathbf{x}) \quad \text{for any market } m$$

- **ASSUMPTION ID-2 (Exclusion Restriction 1)**: $\mathbf{x}_t = (s_{it}, s_{jt}, \mathbf{w}_t)$ such that s_{it} enters in the payoff function of player i but not in the payoff of the other player.

$$\pi_{it}\left(a_{it}, a_{jt}, s_{it}, s_{jt}, \mathbf{w}_t\right) = \pi_{it}\left(a_{it}, a_{jt}, s_{it}, \mathbf{w}_t\right)$$

- **ASSUMPTION ID-3 (Exclusion Restriction 2)**: The transition probability of the state variable s_{it} is such that the value of s_{it+1} does not depend on (s_{it}, s_{jt}):

$$f_t\left(s_{it+1} \mid a_{it}, s_{it}, s_{jt}, \mathbf{w}_t\right) = f_t\left(s_{it+1} \mid a_{it}, \mathbf{w}_t\right)$$

Exclusion restriction in the payoff function (ID-2)

- The exclusion restriction ID-2 appears naturally in many applications of dynamic games of oligopoly competition.
- Incumbent status, capacity, capital stock, or product quality of a firm at period $t-1$ are state variables that enter in a firm's payoff function at period t because there are investment and adjustment costs that depend on these lagged variables.
- A firm's payoff π_{it} depends also on the competitors' values of these variables at period t, but it does not depend on the competitors' values of these variables at $t-1$.
- Importantly, the assumption does not mean that player i does not condition her behavior on those excluded variables. Each player conditions his behavior on all the (common knowledge) state variables that affect the payoff of a player in the game, even if these variables are excluded from his own payoff.

Exclusion restriction in the transition probability (ID-3)

$$f_t\left(s_{it+1} \mid a_{it}, s_{it}, s_{jt}, \mathbf{w}_t\right) = f_t\left(s_{it+1} \mid a_{it}, \mathbf{w}_t\right)$$

- An important class of models that satisfies this condition is when $s_{it} = a_{i,t-1}$, such that the transition rule is simply:

$$s_{it+1} = a_{it}$$

- Many dynamic games of oligopoly competition belong to this class, e.g., market entry/exit, technology adoption, and some dynamic games of quality or capacity competition, among others.

Example: Quality competition

- Quality ladder dynamic game (Pakes and McGuire, 1994).

- s_{it} is the firm's quality at $t-1$.

- The decision variable a_{it} is the firm's quality at period t, such that:

$$s_{it+1} = a_{it}$$

- The model is dynamic because the payoff function includes a cost of adjusting quality that depends on $a_{it} - s_{it}$:

$$AC_i \left(a_{it} - s_{it} \right)$$

- Given competitors quality at period t, a_{jt}, firm i's profit does not depend on competitors' qualities at $t-1$.

Role of the Exclusion restrictions

$$\ln\left(\frac{P_{it}\left(a_i \mid s_{it}, \mathbf{s}_{-it}\right)}{P_{it}\left(0 \mid s_{it}, \mathbf{s}_{-it}\right)}\right) = \mathbf{B}_{it}^{(t)}(s_{it}, \mathbf{s}_{-it})' \left[\widetilde{\boldsymbol{\pi}}_{it}(a_{it}, s_{it}) + \widetilde{\mathbf{c}}_{it}^{\mathbf{B}(t)}(a_{it}, s_{it})\right]$$

- Under the two exclusion restrictions, the state variables $\mathbf{s}_{-it}$ (the competitors s_j) do not enter in the payoffs $\widetilde{\boldsymbol{\pi}}_{it}(a_{it}, s_{it})$ and on the continuation values $\widetilde{\mathbf{c}}_{it}^{\mathbf{B}(t)}(a_{it}, s_{it})$.
- Note: Though $\widetilde{\mathbf{c}}_{it}^{\mathbf{B}(t)}(a_{it}, s_{it})$ depends on beliefs, these are beliefs at periods $t+s>t$ and therefore depend on $(s_{it+s}, \mathbf{s}_{-it+s})$ for $t+s>t$.
- Therefore, the dependence of $\ln\left(\frac{P_{it}\left(a_i \mid s_{it}, \mathbf{s}_{-it}\right)}{P_{it}\left(0 \mid s_{it}, \mathbf{s}_{-it}\right)}\right)$ with respect to $\mathbf{s}_{-it}$ captures the dependence of beliefs $\mathbf{B}_{it}^{(t)}(s_{it}, \mathbf{s}_{-it})$ with respect to $\mathbf{s}_{-it}$.

Identification of Beliefs

- For any player i, any period t in the data, any value of $(\mathbf{a}_{-i}, s_{it})$, and any combination of three values $\mathbf{s}_{-it}$, say $(\mathbf{s}_{-i}^{(a)}, \mathbf{s}_{-i}^{(b)}, \mathbf{s}_{-i}^{(c)})$, the following function of beliefs is identified:

$$\frac{B_{it}^{(t)}(\mathbf{a}_{-i} \mid s_{it}, \mathbf{s}_{-i}^{(c)}) - B_{it}^{(t)}(\mathbf{a}_{-i} \mid s_{it}, \mathbf{s}_{-i}^{(a)})}{B_{it}^{(t)}(\mathbf{a}_{-i} \mid s_{it}, \mathbf{s}_{-i}^{(b)}) - B_{it}^{(t)}(\mathbf{a}_{-i} \mid s_{it}, \mathbf{s}_{-i}^{(a)})}$$

Identification of Beliefs [2]

- For instance, in a binary choice logit with two-players:

$$\frac{B_{it}^{(t)}(1 \mid s_{it}, \mathbf{s}_{-i}^{(c)}) - B_{it}^{(t)}(1 \mid s_{it}, \mathbf{s}_{-i}^{(a)})}{B_{it}^{(t)}(1 \mid s_{it}, \mathbf{s}_{-i}^{(b)}) - B_{it}^{(t)}(1 \mid s_{it}, \mathbf{s}_{-i}^{(a)})} =$$

$$\frac{\ln\left(\dfrac{P_{it}(1 \mid s_{it}, \mathbf{s}_{-i}^{(c)})}{P_{it}(0 \mid s_{it}, \mathbf{s}_{-i}^{(c)})}\right) - \ln\left(\dfrac{P_{it}(1 \mid s_{it}, \mathbf{s}_{-i}^{(a)})}{P_{it}(0 \mid s_{it}, \mathbf{s}_{-i}^{(a)})}\right)}{\ln\left(\dfrac{P_{it}(1 \mid s_{it}, \mathbf{s}_{-i}^{(b)})}{P_{it}(0 \mid s_{it}, \mathbf{s}_{-i}^{(b)})}\right) - \ln\left(\dfrac{P_{it}(1 \mid s_{it}, \mathbf{s}_{-i}^{(a)})}{P_{it}(0 \mid s_{it}, \mathbf{s}_{-i}^{(a)})}\right)}$$

- Note that we cannot identify beliefs about competitors' behavior at future periods: $B_{it+s}^{(t)}$ for $s > 0$. However, $B_{it}^{(t)}$ can provide substantial information about learning.

Monte Carlo Experiments

- Main purposes of these experiments:

  [1] To assess the power of our identification assumptions in small samples. What is the price, in terms of precision of our estimates, of relaxing the assumption of unbiased beliefs?

  [2] Evaluate the bias induced by imposing the assumption of equilibrium beliefs when this assumption does not hold in the DGP.

Monte Carlo Experiments: Model

- *Dynamic game of market entry and exit.*

$$\pi_{1mt}(1, a_{2mt}, \mathbf{x}_{mt}) = \alpha_1 - \delta_1 \ a_{2mt} - (1 - a_{1mt-1}) \ \theta_1^{EC}$$

$$\pi_{2mt}(1, a_{1mt}, \mathbf{x}_{mt}) = \alpha_2 - \delta_2 \ a_{1mt} - \theta_S \ S_{2m} - (1 - a_{2mt-1}) \ \theta_2^{EC}$$

- S_{2m} has a discrete uniform distribution with support $\{-2, -1, 0, 1, 2\}$.

Monte Carlo Experiments: Design

Table 3
Summary of DGPs in the Monte Carlo Experiments

For all the experiments:		$\alpha = 2.4$; $\delta = 3.0$; $\theta^{EC} = 0.5$; $\beta = 0.95$
		$S_{2m} \sim Uniform\ \{-2, -1, 0, +1, +2\}$
		$M = 2,000$; $T = 5$; $MC\ rep = 10,000$
Experiment	1U:	$\theta_S = -0.5$; Unbiased beliefs
Experiment	1B:	$\theta_S = -0.5$; Biased beliefs
Experiment	2U:	$\theta_S = -1.0$; Unbiased beliefs
Experiment	2B:	$\theta_S = -1.0$; Biased beliefs

MCs: Cost of Relaxing Equil Beliefs

Monte Carlo Experiment 1U

Parameter (True value)	Estimation WITH equil. rest. Bias (%)	Std (%)	Estimation WITHOUT equil. rest. Bias (%)	Std (%)
Payoffs				
α (2.4)	-0.0992 (4.13)	0.2208 (9.20)	0.1412 (5.88)	0.3702 (15.42)
δ (3.0)	-0.1004 (3.35)	0.2349 (7.83)	0.1448 (4.83)	0.3763 (12.54)
θ^{EC} (0.5)	-0.0021 (0.42)	0.0665 (13.30)	-0.0760 (15.20)	0.1118 (22.35)

MCs: Benefits of Relaxing Equil Beliefs

Monte Carlo Experiment 1B

Parameter (True value)	Estimation WITH equil. rest. Bias (%)	Std (%)	Estimation WITHOUT equil. rest. Bias (%)	Std (%)
Payoffs				
α (2.4)	-0.3332 (13.88)	0.2666 (11.11)	-0.0081 (0.34)	0.2829 (11.79)
δ (3.0)	0.2979 (9.93)	0.2746 (9.15)	0.1543 (5.14)	0.3071 (10.24)
θ^{EC} (0.5)	-0.3277 (65.55)	0.0778 (15.56)	0.0134 (2.68)	0.1482 (29.63)

EMPIRICAL APPLICATION

- Dynamic game of store location by McDonalds (MD) and Burger King (BK) using data for United Kingdom during the period 1990-1995.
- Panel of 422 local markets (districts) and six years, 1990-1995.
- Information on the number of stores of McDonalds (MD) and Burger King (BK) in United Kingdom.
- Information on local market characteristics such as population, density, income per capita, age distribution, average rent, local retail taxes, and distance to the headquarters of each firm in UK.

Table 1
Descriptive Statistics for the Evolution of the Number of Stores
Data: 422 markets, 2 firms, 5 years = 4,220 observations

	Burger King					McDonalds			
	1991	1992	1993	1994	1995	1991	1992	1993	1994
# Markets	98	104	118	131	150	213	220	237	248
Δ # Markets	17	6	14	13	19	7	7	17	11
# of stores	115	128	153	181	222	316	344	382	421
Δ # of stores	36	13	25	28	41	35	28	38	39
stores per mark	1.17	1.23	1.30	1.38	1.48	1.49	1.56	1.61	1.70

Model

- $k_{imt} \in \{0, 1, ..., |\mathcal{K}|\}$ number of stores of firm i in market m at period $t-1$.
- $a_{imt} \in \{0, 1\}$ decision of firm i to open a new store.
- $a_{imt} + k_{imt} = \#$ stores of firm i at period t.
- Firm i's total profit function is equal to:

$$\Pi_{imt} = VP_{imt} - EC_{imt} - FC_{imt}$$

Model (2)

- Variable profit function:

$$VP_{imt} = (\mathbf{W}_m \ \gamma) \ (a_{imt} + k_{imt}) \left[\begin{array}{c} \theta_{0i}^{VP} + \theta_{can,i}^{VP}(k_{imt} + a_{imt}) \\ +\theta_{com,i}^{VP}(a_{jmt} + k_{jmt}) \end{array} \right]$$

- Entry cost:

$$EC_{imt} \ = \ 1\{a_{imt} > 0\} \ \left[\theta_{0i}^{EC} + \theta_{K,i}^{EC} \ 1\{k_{imt} > 0\} + \theta_{S,i}^{EC} \ S_{imt} + \varepsilon_{it}\right]$$

- Fixed cost:

$$FC_{imt} \ = \ 1\{(k_{imt} + a_{imt}) > 0\} \left[\begin{array}{c} \theta_{0i}^{FC} + \theta_{lin,i}^{FC}(k_{imt} + a_{imt}) \\ +\theta_{qua,i}^{FC}(k_{imt} + a_{imt})^2 \end{array} \right]$$

Tests of Unbiased Beliefs

Data: 422 markets, 5 years = 2,110 observations

BK: $\widehat{D}$ (p-value)	66.841	(0.00029)
MD: $\widehat{D}$ (p-value)	42.838	(0.09549)

- We can reject hypothesis that BK beliefs are unbiased (p-value 0.00029).
- Restriction is more clearly rejected for large values of the state variable (distance to chain network) S_{MD}.

Where to impose unbiased beliefs?

- We propose three different criteria:
  [1] Minimize distance $\|B_i - P_j\|$
  [2] Test for monotonicity of beliefs: if not rejected, impose unbiased beliefs in extreme values of S_j.
  [3] Most visited values of S_j.

- In this empirical application, the three criteria have the same implication: impose unbiased beliefs at the lowest values for the distance S_j.

Estimation of Dynamic Game

Data: 422 markets, 2 firms, 5 years = 4,220 observations

	$\beta = 0.95$ (not estimated)			
	Equilibrium Beliefs		Biased Beliefs	
	BK	MD	BK	MD
Var Profits:				
θ_0^{VP}	0.5413	0.8632	0.4017	0.8271
	$(0.1265)^*$	$(0.2284)^*$	$(0.2515)^*$	$(0.4278)^*$
θ_{can}^{VP} cannibalization	-0.2246	0.0705	-0.2062	0.0646
	$(0.0576)^*$	$(0.0304)^*$	$(0.1014)^*$	(0.0710)
θ_{com}^{VP} competition	**-0.0541**	-0.0876	**-0.1133**	-0.0856
	(0.0226)*	(0.0272)	**(0.0540)***	(0.0570)
Log-Likelihood	-848.4		-840.4	

Estimation of Dynamic Game				
Data: 422 markets, 2 firms, 5 years = 4,220 observations				
	$\beta = 0.95$ (not estimated)			
	Equilibrium Beliefs		Biased Beliefs	
	BK	MD	BK	MD
Fixed Costs:				
θ_0^{FC} fixed	0.0350	0.0374	0.0423	0.0307
	(0.0220)	(0.0265)	(0.0478)	(0.0489)
θ_{lin}^{FC} linear	0.0687	0.0377	0.0829	0.0467
	$(0.0259)^*$	$(0.0181)^*$	$(0.0526)^*$	(0.0291)
θ_{qua}^{FC} quadratic	-0.0057	0.0001	-0.0007	0.0002
	(0.0061)	(0.0163)	(0.0186)	(0.0198)

Estimation of Dynamic Game				
Data: 422 markets, 2 firms, 5 years = 4,220 observations				
	$\beta = 0.95$ (not estimated)			
	Equilibrium Beliefs		Biased Beliefs	
	BK	MD	BK	MD
Entry Cost:				
θ_0^{EC} fixed	0.2378	0.1887	0.2586	0.1739
	$(0.0709)^*$	$(0.0679)^*$	$(0.1282)^*$	$(0.0989)^*$
θ_K^{EC} (K)	-0.0609	-0.107	-0.0415	-0.1190
	(0.043)	$(0.0395)^*$	(0.096)	$(0.0628)^*$
θ_S^{EC} (S)	0.0881	0.0952	0.1030	0.1180
	$(0.0368)^*$	$(0.0340)^*$	$(0.0541)^*$	$(0.0654)^*$

Implications of biased beliefs on BK's profits

- We compare the value of BK's profits during years 1991 to 1994 given its actual entry decisions with this firm's profits if its entry decisions were based on unbiased beliefs on MD's behaviour.

- Having unbiased would increase BK's total profits in these markets by: 2.78% in 1991; 2.11% in 1992; 1.20% in 1993; and 0.87% in 1994.

- Biased beliefs occur in markets which are relatively far away from the firm's network of stores. These markets are relatively smaller, and biased beliefs decline over time in the sample period as the result of geographic expansion.

Summary and Conclusions

- Strategic uncertainty can be important for competition in oligopoly markets. Under these conditions, the assumption of equilibrium beliefs can be too restrictive.
- We present sufficient conditions for the NP identification of preferences and beliefs.
- We apply these ideas to actual data and find that bias beliefs can be useful to explain a puzzle in the data.