

Industrial Organization II (ECO 2901)
Winter 2020. Victor Aguirregabiria

Problem Set

Due on Sunday, March 8th, 2020 [before 11:59pm]

INSTRUCTIONS.

1. Write your answers electronically in a word processor.
2. For the answers that involve coding, include in the document the code that you have used to obtain your empirical results, indicating the language of the code (e.g., R, Matlab, Julia, Python) as well as the version of the software/compiler you have used.
3. Convert the document with your answers and code to PDF format and submit the PDF online via Quercus.
4. You should submit your completed problem set before 11:59pm on Sunday, March 8th, 2020.
5. Problem sets should be written individually.

The total number of marks is 100.

This problem set deals with the **solution, simulation, and estimation of dynamic games of oligopoly competition.**

MODEL AND DATA GENERATING PROCESS (DGP)

Consider an industry with $M = 1000$ geographic (local) markets, indexed by m , and $N = 5$ firms, indexed by i . Time is discrete and indexed by t . Each firm is a potential entrant in every local market. Competition takes place at the level of local markets and there are not spillover effects between these markets. Here we focus on the entry-exit decisions of firms in local markets. Let $a_{imt} \in \{0, 1\}$ be the binary variable that indicates whether firm i is active in market m at period t .

The profit of firm i in market m is:

$$\Pi_{imt} = \begin{cases} \theta_S s_{mt} - \theta_N \ln \left(1 + \sum_{j \neq i} a_{jmt} \right) - \theta_{EC} (1 - a_{i,t-1}) - \theta_{FC,i} + \varepsilon_{imt}(1) & \text{if } a_{imt} = 1 \\ \varepsilon_{imt}(0) & \text{if } a_{imt} = 0 \end{cases} \quad (1)$$

where θ_S , θ_N , θ_{EC} , $\theta_{FC,1}$, ..., and $\theta_{FC,5}$ are structural parameters. Variable s_{mt} is exogenous and it represents market size. Parameter θ_N measures competition effects. Parameters θ_{EC}

and $\theta_{FC,i}$ represent entry cost and fixed cost, respectively. Variables $\varepsilon_{imt}(0)$ and $\varepsilon_{imt}(1)$ are unobservable to the researcher, private information of firm i , and *i.i.d.* over (i, m, t) with type 1 extreme value distribution.

Suppose that, in the population under study, the structural parameters take the following values:

$$\begin{aligned} \theta_S = 1.0; \quad \theta_N = 1.0; \quad \theta_{EC} = 1.0; \\ \theta_{FC,1} = 1.9; \quad \theta_{FC,2} = 1.8; \quad \theta_{FC,3} = 1.7; \quad \theta_{FC,4} = 1.6; \quad \theta_{FC,5} = 1.5; \end{aligned} \tag{2}$$

The market size variable, s_{mt} , has a discrete support set $\mathcal{S} = \{1, 2, 3, 4, 5\}$ and it follows an exogenous Markov chain with the following transition probability matrix:

$$\mathbf{F}_s = \begin{bmatrix} 0.8 & 0.2 & 0.0 & 0.0 & 0.0 \\ 0.2 & 0.6 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.2 & 0.6 & 0.2 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.6 & 0.2 \\ 0.0 & 0.0 & 0.0 & 0.2 & 0.8 \end{bmatrix} \tag{3}$$

The discount factor is $\delta = 0.95$.

QUESTION 1 [40 POINTS]. SOLUTION

1.1. [10 points]

(a) What is the vector of observable state variables \mathbf{x}_{mt} in this model? What is the dimension of the space of \mathbf{x}_{mt} ? Define the conditional choice probability (CCP), $P_i(\mathbf{x}_{mt})$, of a firm in this model.

(b) Write the equations, in vector form, that describe a MPE in this dynamic game as a fixed point of a mapping in the space of firms' CCPs. Describe each vector and matrix and its dimensions.

1.2. [20 points]

(a) Write the code to compute a MPE in this dynamic game. Use the value function iterations algorithm to compute the solution to the dynamic programming problem in a firm's best response.

(b) Implement your code to obtain a MPE in this model. Present a table with the following values for the equilibrium CCPs.

State (x)	CCP Firm 1	CCP Firm 2	CCP Firm 3	CCP Firm 4	CCP Firm 5
$s = 1; a_{1t-1} = 1; a_{jt-1} = 0 \text{ for } j \neq 1$					
$s = 5; a_{1t-1} = 1; a_{jt-1} = 0 \text{ for } j \neq 1$					
$s = 1; a_{5t-1} = 1; a_{jt-1} = 0 \text{ for } j \neq 5$					
$s = 5; a_{5t-1} = 1; a_{jt-1} = 0 \text{ for } j \neq 5$					

1.3. [10 points]

(a) Let $\mathbf{F}^{\mathbf{P}}$ be the transition probability of the \mathbf{x}_{mt} induced by the equilibrium CCPs \mathbf{P} . Write the expression for the element $\Pr(s_{mt+1} = s', \mathbf{a}_{mt} = \mathbf{a}' | s_{mt} = s, \mathbf{a}_{mt-1} = \mathbf{a})$ in this matrix.

(b) Write the code to compute this matrix.

(c) Let $q(\mathbf{x})$ be the ergodic or steady-state distribution of \mathbf{x}_{mt} induced by the transition probability $\mathbf{F}^{\mathbf{P}}$. Let \mathbf{q} be the vector with the ergodic distribution $\{q(\mathbf{x}) : \mathbf{x} \in \mathcal{X}\}$. By definition of \mathbf{q} , it satisfies the following condition:

$$\mathbf{q} = \mathbf{F}^{\mathbf{P}} \mathbf{q} \quad (4)$$

Write the code to compute \mathbf{q} . [Hint: You can use fixed point iterations in the previous expression].

(d) Implement your code to obtain the ergodic distribution. Present a table with the following values of $q(\mathbf{x})$.

State (x)	Ergodic distribution $q(\mathbf{x})$
$s = 1; a_{1t-1} = 1; a_{jt-1} = 0 \text{ for } j \neq 1$	
$s = 5; a_{1t-1} = 1; a_{jt-1} = 0 \text{ for } j \neq 1$	
$s = 1; a_{5t-1} = 1; a_{jt-1} = 0 \text{ for } j \neq 5$	
$s = 5; a_{5t-1} = 1; a_{jt-1} = 0 \text{ for } j \neq 5$	

QUESTION 2 [20 POINTS]. SIMULATION

2.1. [10 points] Using the MPE computed in Question 1, simulate a dataset with $M = 400$ markets and $T = 5$ periods.

(a) Write the code to simulate this dataset.

[Note] Remember: Let Y be a discrete random variable with support $\{y^1, y^2, \dots, y^J\}$ and let $p_j \equiv \Pr(Y = y^j)$. Let u be a random draw from a $U(0, 1)$ random variable. Then, the following value of y is a random draw from the distribution of Y :

$$\begin{aligned} & \text{if } u \leq p_1, \quad \text{then } y = y^1; \\ & \text{else if } p_1 < u \leq p_1 + p_2, \quad \text{then } y = y^2; \\ & \quad \quad \quad \vdots \\ & \text{else if } p_1 + \dots + p_{J-2} < u \leq p_1 + \dots + p_{J-1}, \quad \text{then } y = y^{J-1}; \\ & \quad \quad \quad \text{else if } u > p_1 + \dots + p_{J-1}, \quad \text{then } y = y^J. \end{aligned}$$

(b) For every market m , the value of the state variables at period $t = 1$, $\mathbf{x}_1 = (s_{m1}, a_{10}, \dots, a_{50})$, should be drawn from the ergodic distribution $q(\cdot)$.

(c) Then, for $t \geq 1$, use the equilibrium CCPs, $\mathbf{P}(\mathbf{x}_t)$, to simulate firms' actions $\{a_{imt}\}$, and the transition probabilities $F_s(\cdot | s_{mt})$ to simulate $s_{m,t+1}$.

2.2. [10 points] Use your simulated dataset to present the following table of statistics.

Statistic	Mean (Standard error)
Number of active firms in the market	
Number of new entrants	
Number of exits	
AR(1) parameter in autoregression for # active firms	
Frequency of Firm 1 active in the market	
Frequency of Firm 2 active in the market	
Frequency of Firm 3 active in the market	
Frequency of Firm 4 active in the market	
Frequency of Firm 5 active in the market	

QUESTION 3 [40 POINTS]. ESTIMATION

Use the your simulated dataset in Question 2 to obtain the following estimations of the structural parameters.

3.1. [10 points] Obtain nonparametric estimates of \mathbf{P} and \mathbf{F}_s . Consider the following Kernel estimator of the parameters $P_i(0, \mathbf{x}_{-i})$ and $P_i(1, \mathbf{x}_{-i})$, where $\mathbf{x}_{-it} = (s_t, a_{jt-1} : j \neq i)$, and $F_s(s'|s)$:

$$\begin{aligned}\hat{P}_i(0, \mathbf{x}_{-i}) &= \frac{\sum_{m=1}^M \sum_{t=2}^T a_{imt} 1\{a_{im,t-1} = 0\} K_N(\mathbf{x}_{-i} - \mathbf{x}_{-imt})}{\sum_{m=1}^M \sum_{t=2}^T 1\{a_{im,t-1} = 0\} K_N(\mathbf{x}_{-i} - \mathbf{x}_{-imt})} \\ \hat{P}_i(1, \mathbf{x}_{-i}) &= \frac{\sum_{m=1}^M \sum_{t=2}^T a_{imt} 1\{a_{im,t-1} = 1\} K_N(\mathbf{x}_{-i} - \mathbf{x}_{-imt})}{\sum_{m=1}^M \sum_{t=2}^T 1\{a_{im,t-1} = 1\} K_N(\mathbf{x}_{-i} - \mathbf{x}_{-imt})} \\ \hat{F}_s(s'|s) &= \frac{\sum_{m=1}^M \sum_{t=2}^T 1\{s_{mt} = s'\} K_N(s_{m,t-1} - s)}{\sum_{m=1}^M \sum_{t=2}^T K_N(s_{m,t-1} - s)}\end{aligned}$$

where $K_N(\mathbf{d})$ is a Kernel function, for instance:

$$K_N(\mathbf{d}) = \frac{1}{1 + \sqrt{N} \|\mathbf{d}\|}$$

and $\|\cdot\|$ is the Euclidean distance.

(a) Write the code to obtain this Kernel estimators.

(b) Implement your code and report the following tables of estimates and standard errors.

State (x)	CCP-Firm 1 Est. (s.e)	CCP-Firm 2 Est. (s.e)	CCP-Firm 3 Est. (s.e)	CCP-Firm 4 Est. (s.e)	CCP-Firm 5 Est. (s.e)
$s = 1; a_1 = 1; a_j = 0 \text{ for } j \neq 1$					
$s = 5; a_1 = 1; a_j = 0 \text{ for } j \neq 1$					
$s = 1; a_5 = 1; a_j = 0 \text{ for } j \neq 5$					
$s = 5; a_5 = 1; a_j = 0 \text{ for } j \neq 5$					

3.2. [20 points] Obtain the two-step Pseudo maximum likelihood estimator of the structural parameters θ .

(a) Present the equation that describes the expected profit of firm i if this firm is active: $\Pi_i^{\mathbf{P}}(1, \mathbf{x})$. Show that this function is linear in the structural parameters θ_i .

- (b) Write the code to implement (a).
- (c) Present the expression of the valuation operator $v_i(\mathbf{P})$ that provides the vector of values of firm i given the vector of CCPs, \mathbf{P} . Show that this function is linear in the structural parameters θ_i .
- (d) Write the code to implement (c). Use the true value of $\delta = 0.95$.
- (e) Write the expression for the best response probability of firm i as a function of all the CCPs in \mathbf{P} , i.e., $\Psi_i(\mathbf{x}_{mt}, \mathbf{P})$. Show that this function has the structure of a standard binary Logit model where the index is linear in the structural parameters θ_i .
- (f) Write the code to implement (e).
- (g) Write the expression for the log pseudo likelihood function that uses the best response probabilities $\Psi_i(\mathbf{x}_{mt}, \mathbf{P})$.
- (h) Write the code to implement the pseudo likelihood function in (g). Use the true value of $\delta = 0.95$.
- (i) Write the code to the two-step pseudo maximum likelihood estimator of the structural parameters.
- (j) Present a table with the two-step estimates of the structural parameters.

Parameter	True Value	Two-step Estimates Estimate (Standard error)
θ_S	1.0	
θ_N	1.0	
θ_{EC}	1.0	
$\theta_{FC,1}$	1.9	
$\theta_{FC,2}$	1.8	
$\theta_{FC,3}$	1.7	
$\theta_{FC,4}$	1.6	
$\theta_{FC,5}$	1.5	

3.3. [10 points] Apply recursively the idea of the two-step estimator to obtain the 10-steps Pseudo maximum likelihood estimator of the structural parameters.

- (a) Present the equations that describe the updating the structural parameters and the CCPs in this recursive method.
- (b) Write the code to implement (a).
- (c) Present the following table with 2-step and 10-step estimates of the structural parameters.

Parameter	True Value	2-step Estimates Estimate (Standard error)	10-step Estimates Estimate (Standard error)
θ_S	1.0		
θ_N	1.0		
θ_{EC}	1.0		
$\theta_{FC,1}$	1.9		
$\theta_{FC,2}$	1.8		
$\theta_{FC,3}$	1.7		
$\theta_{FC,4}$	1.6		
$\theta_{FC,5}$	1.5		

END OF THE PROBLEM SET
