ECO 2901
EMPIRICAL INDUSTRIAL ORGANIZATION

Lecture 6:
Dynamic Games of Oligopoly Competition
Models and Solution Methods

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Models & Solution Methods

1. Introduction

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1. Introduction

Dynamic Games: Introduction

- In oligopoly industries, firms compete in investment decisions that:
  - have returns in the future (forward-looking);
  - involve substantial uncertainty;
  - have important effects on competitors 'profits (competition / game)

- Some examples are:
  - Investment in R&D, innovation.
  - Investment in capacity, physical capital.
  - Product design / quality
  - Market entry / exit ...

Dynamic Games: Introduction [2]

• Measuring and understanding the **dynamic strategic interactions** between firms decisions (e.g., dynamic complementarity or substitutability) is important to understand the forces behind the dynamics of an industry or to evaluate policies.

• Investment costs, uncertainty, and competition effects play an important role in these decisions.

• Structural estimation of these parameters is necessary for some empirical questions.

• Empirical dynamic games provide a framework to estimate these parameters and perform policy analysis.

Examples of Empirical Applications

- Competition in R&D and product innovation between Intel and AMD: Goettler and Gordon (JPE, 2011).

- Product innovation: incumbents & new entrants (hard drive industry): Igami (JPE, 2017).

- Land use regulation and entry-exit in the hotel industry: Suzuki (IER; 2013).

- Environmental regulation, entry-exit and capacity in cement industry: Ryan (ECMA, 2012).

- Subsidies to entry in small markets of the dentist industry: Dunne et al. (RAND, 2013);

Examples of Empirical Applications [2]

- Fees for musical performance & choice of format of radio stations:
Sweeting (ECMA, 2013).

- Hub-and-spoke networks and entry-exit in the airline industry:
Aguirregabiria and Ho (JoE, 2012).

- Dynamic price competition:
Kano (IJIO, 2013); Ellickson, Misra, and Nair (JMR, 2012).

- Cannibalization and preemption strategies in fast-food industry:
Igami and Yang (QE, 2016).

- Demand uncertainty and firm investment in the concrete industry:
Collard-Wexler (ECMA, 2013);

Examples of Empirical Applications [3]

- Release date of a movie: Einav (EI, 2010).

- Time-to-build, investment, and uncertainty in the shipping industry: Kalouptsidi (AER, 2014).

- Endogenous mergers: Jeziorski (RAND, 2014).

- Exploitation of a common natural resource (fishing): Huang and Smith (AER, 2014).

2. Structure of Dynamic Games of Oligopoly Competition

Dynamic Games: Basic Structure

- Time is discrete and indexed by t.

- The game is played by N firms that we index by i.

- Following the standard structure in the **Ericson-Pakes (1995) framework**, firms compete in two different dimensions: a static dimension and a dynamic dimension.

- For instance: given the state of the industry at period t firms compete in prices (static competition), and decide the quality of their products (dynamic investment decision).

Dynamic Games: Basic Structure (2)

• The investment decision can be an entry/exit decision, a choice of capacity, investment in equipment, R&D, product quality, other product characteristics, etc.

• The action is taken to maximize the expected and discounted flow of profits in the market,

$$E_t \left(\sum_{s=0}^{\infty} \delta^s \; \Pi_{it+s} \right)$$

$\delta \in (0, 1)$ is the discount factor, and Π_{it} is firm i's profit at period t.

Decision variable

• $a_{it} \in \{-A, ..., -1, 0, 1, ..., A\}$ = firm $i's$ investment at period t.

• As an example, I use here a model of competition in **product quality** that is similar to Pakes & Mcguire (RAND, 1996).

• k_{it} = Stock of product quality of firm i at the beginning of period t.
$k_{it} = 0$: firm i is not active in the market;
$k_{it} = k > 0$: firm i is active with a product of quality k.

• k_{it} evolves endogenously according to transition rule (more later):

$$k_{i,t+1} = f_k \left(k_{it}, a_{it}, \xi_{i,t+1} \right)$$

• Consumer demand and the firm's costs (variable and fixed) may depend on the quality stock.

State variables

- At every period t the industry can be described in terms of three sets of state variables affecting firms' profits: $\mathbf{k}_t$, $\mathbf{z}_t$, $\boldsymbol{\varepsilon}_t$.

- **Endogenous (common knowledge) state variables:**

$$\mathbf{k}_t = (k_{1t},\ k_{2t}, ..., k_{Nt})$$

- **Exogenous common knowledge state variables:**

$$\mathbf{z}_t \qquad \text{affecting demand and/or costs.}$$

- **Exogenous private information state variables** affecting costs:

$$\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})$$

ε_{it} is private info of firm i.

Profit function

- The profits of firm i at time t are given by

$$\Pi_{it} = VP_{it} - FC_{it} - IC_{it}$$

  VP_{it} represents variable profit;
  FC_{it} is the fixed cost of operating;
  IC_{it} is an investment / adjustment cost

- The specification of VP_{it} can be more or less structural.

- VP_{it} may come from a static Bertrand equilibrium:
  Goettler and Gordon (2011); Aguirregabiria and Ho (2012).
- VP_{it} may come from a static Cournot equilibrium:
  Ryan (2012); Igami (2017);
- VP_{it} may have a reduced form specification:
  Suzuki (2013); Dunne et al. (2013); Igami and Yang (2016).

Timing of the model: Time-to-Build or Not

• Every period t a firm makes two decisions: one **static or not forward-looking** (e.g., price, quantity) and one **dynamic or forward-looking** (investment decision).

• We can distinguish two different timing assumptions of this model depending on whether there is **time-to-build or not in the investment decision**.

• **With time-to-build**, the investment a_{it} does NOT affect quality at period t and therefore variable profits and the the static competition at period t. It only affects the investment cost at t.

• **Without time-to-build**, the investment a_{it} affects quality at period t and therefore variable profits and the static competition at t.

Variable profit function (without time-to-build)

- The variable profit VP_{it} is:

$$VP_{it} = \left(p_{it} - c_i\left(f_k\left(k_{it}, a_{it} \right), \mathbf{z}_t \right) \right) \ q_{it}$$

p_{it} and q_{it} are the price and the quantity sold by firm i.

- The quantity is given by the Logit demand:

$$q_{it} = H_t \ \frac{1\{ f_k\left(k_{it}, a_{it} \right) > 0 \} \ \exp\{ v_i(f_k\left(k_{it}, a_{it} \right), \mathbf{z}_t) - \alpha \ p_{it} \}}{1 + \sum_{j=1}^{N} 1\{ f_k\left(k_{jt}, a_{jt} \right) > 0 \} \ \exp\{ v_j(f_k\left(k_{jt}, a_{jt} \right), \mathbf{z}_t) - \alpha \ p_{jt} \}}$$

- Bertrand equilibrium implies the **"indirect" variable profit function**:

$$\theta_i^{VP}(\mathbf{a}_t, \mathbf{k}_t, \mathbf{z}_t) = \left(p_i^*[\mathbf{a}_t, \mathbf{k}_t, \mathbf{z}_t] - c_i[f_k\left(k_{it}, a_{it} \right), \mathbf{z}_t] \right) \ q_i^*[\mathbf{a}_t, \mathbf{k}_t, \mathbf{z}_t]$$

Fixed cost

- The fixed cost is paid every period that the firm is active in the market:

$$FC_{it} = \theta_i^{FC}\left(f_k\left(k_{it}, a_{it}\right), \mathbf{z}_t\right) + \varepsilon_{it}^{FC}\left(a_{it}\right)$$

- $\theta_i^{FC}\left(f_k\left(k_{it}, a_{it}\right), \mathbf{z}_t\right)$ is "mean value" of the fixed cost of firm i.

- $\varepsilon_{it}^{FC}\left(a_{it}\right)$ are zero-mean shocks that are private information of firm i.

Fixed cost (2)

• There are two main reasons why we incorporate private information shocks in the model.

• [1] As shown in Doraszelski and Satterthwaite (2012), it is a way to guarantee that the dynamic game has at least one equilibrium in pure strategies.

• [2] They are convenient econometric errors. If private information shocks are independent over time and over players, and unobserved to the researcher, they can 'explain' players heterogeneous behavior without generating endogeneity problems.

Investment / Adjustment costs

- There are costs of adjusting the level of quality:

$$IC_{it} = \theta_i^{AC}(a_{it},\ k_{it},\ \mathbf{z}_t) + \varepsilon_{it}^{AC}(a_{it})$$

- $\theta_i^{AC}(a_{it},\ k_{it},\ \mathbf{z}_t)$ is the adjustment cost function, such that:

- $\theta_i^{AC}(a_{it}, k_{it}, \mathbf{z}_t) = 0$
- $\theta_i^{AC}(\Delta, k_{it}, \mathbf{z}_t) > 0$ if $\Delta \neq 0$.
- If $k_{it} = 0$, this adjustment cost is the cost of market entry.
- If $k_{it} > 0$ and $f_k(k_{it}, a_{it}) = 0$, this AC is the cost of market exit.

- $\varepsilon_{it}^{AC}(a_{it})$ is a private information shock in the investment cost

Profit function

- In summary, the profit function has the following structure:

$$\Pi_{it} = \pi_i\left(a_{it}, \mathbf{a}_{-it}, \mathbf{k}_t, \mathbf{z}_t\right) - \varepsilon_{it}(a_{it})$$

where:

$$\begin{aligned}\pi_i\left(a_{it}, \mathbf{a}_{-it}, \mathbf{k}_t, \mathbf{z}_t\right) &= \theta_i^{VP}\left(a_{it}, \mathbf{a}_{-it}, \mathbf{k}_t, \mathbf{z}_t\right) \\ &- \theta_i^{FC}\left(a_{it}, k_{it}, \mathbf{z}_t\right) - \theta_i^{AC}\left(a_{it}, k_{it}, \mathbf{z}_t\right)\end{aligned}$$

and:

$$\varepsilon_{it}(a_{it}) = \varepsilon_{it}^{FC}(a_{it}) + \varepsilon_{it}^{AC}(a_{it})$$

Evolution of the state variables

• (1) **Exogenous common knowledge state variables:** follow an exogenous Markov process with transition probability function $F_z(\mathbf{z}_{t+1}|\mathbf{z}_t)$.

• (2) **Exogenous private information state variables**. ε_{it} is i.i.d. over time and independent across firms with CDF G_i.

• (3) **Endogenous state variables:** The form of the transition rule $k_{i,t+1} = f_k\left(k_{it}, a_{it}, \xi_{i,t+1}\right)$ depends on the application:
• Market entry: $k_{it} = a_{it-1}$, such that $f_k\left(k_{it}, a_{it}, \xi_{i,t+1}\right) = a_{it}$
• Number of stores, or quality choice without uncertainty of depreciation: $k_{i,t+1} = k_{it} + a_{it}$.
• Capital investment with deterministic depreciation: $k_{i,t+1} = \lambda(k_{it} + a_{it})$

3. Markov Perfect Equilibrium

Markov Perfect Equilibrium

• Most dynamic IO model assume Markov Perfect Equilibrium (MPE), (Maskin and Tirole, ECMA 1988).

• A key condition in this solution concept is that **players' strategies are functions of only payoff-relevant state variables**. In this model, the payoff-relevant state variables for firm i are $(\mathbf{k}_t, \mathbf{z}_t, \varepsilon_{it})$.

• **Why this restriction?**

• **Rationality:** if other players have this type of strategies, a player cannot make better by conditioning its behavior on non-payoff relevant information (e.g., lagged values of the state variables)

• **Dimensionality:** It is convenient because it reduces the dimensionality of the state space.

• Later, we will relax this condition to allow for equilibrium concepts where strategy functions can depend on $(\mathbf{k}_{t-1}, \mathbf{z}_{t-1})$, $(\mathbf{k}_{t-2}, \mathbf{z}_{t-2})$, ...

Markov Perfect Equilibrium (2)

- We use $\mathbf{x}_t$ to represent the vector of common knowledge state variables:

$$\mathbf{x}_t \equiv (\mathbf{k}_t, \mathbf{z}_t)$$

- Let $\boldsymbol{\alpha} = \{\alpha_i(\mathbf{x}_t, \varepsilon_{it}) : i \in \{1, 2, ..., N\}\}$ be a set of strategy functions, one for each firm.

- A MPE is an N-tuple of strategy functions $\boldsymbol{\alpha}$ such that every firm is maximizing its value given the strategies of the other players.

- For given strategies of the other firms, the decision problem of a firm is a single-agent dynamic programming (DP) problem.

Markov Perfect Equilibrium (3)

• Let $V_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_t, \varepsilon_{it})$ be the value function of the DP problem that describes the best response of firm i to the strategies $\boldsymbol{\alpha}_{-i}$ of the other firms.

• This value function is the unique solution to the Bellman equation:

$$V_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_t, \varepsilon_{it}) = \max_{a_{it}} \left\{ \begin{array}{l} \Pi_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) - \varepsilon_{it}(a_{it}) \\ \\ +\delta \int V_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_{t+1}, \varepsilon_{it+1}) \ dG_i(\varepsilon_{it+1}) \ F_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_{t+1}|a_{it}, \mathbf{x}_t) \end{array} \right\}$$

• $\Pi_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) =$ One-period profit given other firms' strategies.
• $F_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_{t+1}|a_{it}, \mathbf{x}_t) =$ Transition prob. state variables given other firms' strategies.

Markov Perfect Equilibrium (4)

- The expected one-period profit $\Pi_i^{\boldsymbol{\alpha}}(a_{it}, \mathbf{x}_t)$ is:

$$\Pi_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) = \sum_{\mathbf{a}_{-it}} \left[\prod_{j \neq i} \Pr\left(\alpha_j(\mathbf{x}_t, \varepsilon_{jt}) = a_{jt} \mid \mathbf{x}_t\right) \right] \pi_i\left(a_{it}, \mathbf{a}_{-it}, \mathbf{x}_t\right)$$

- And the expected transition of the state variables is:

$$F_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_{t+1} | a_{it}, \mathbf{x}_t) = F_z(\mathbf{z}_{t+1} | \mathbf{z}_t) \prod_{j \neq i} \Pr\left(\alpha_j(\mathbf{x}_t, \varepsilon_{jt}) = a_{jt} \mid \mathbf{x}_t\right)$$

Markov Perfect Equilibrium (5)

- A Markov perfect equilibrium (MPE) is an N-tuple of strategy functions $\boldsymbol{\alpha}$ such that for any player i and for any $(\mathbf{x}_t, \varepsilon_{it})$ we have that:

$$\alpha_i(\mathbf{x}_t, \varepsilon_{it}) = \arg\max_{a_{it}} \left\{ v_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) - \varepsilon_{it}(a_{it}) \right\}$$

with

$$v_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) \equiv \Pi_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) + \delta \int \widetilde{V}_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_{t+1}) \; F_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_{t+1} | a_{it}, \mathbf{x}_t)$$

and $\widetilde{V}_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_t)$ is the *integrated value function.* This function uniquely solves the (integrated) Bellman equation:

$$\widetilde{V}_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_t) = \int \max_{a_{it}} \left\{ \begin{array}{c} \Pi_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) - \varepsilon_{it}(a_{it}) \\ +\delta \int \widetilde{V}_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_{t+1}) \; F_i^{\boldsymbol{\alpha}_{-i}}(\mathbf{x}_{t+1} | a_{it}, \mathbf{x}_t) \end{array} \right\} dG_i(\varepsilon_{it})$$

Conditional Choice Probabilities

• Given a strategy function $\alpha_i(\mathbf{x}_t, \varepsilon_{it})$, we can define the corresponding *Conditional Choice Probability (CCP)* function as :

$$\begin{aligned} P_i(a|\mathbf{x}) &\equiv \Pr\left(\alpha_i(\mathbf{x}_t, \varepsilon_{it}) = a \mid \mathbf{x}_t = \mathbf{x}\right) \\ &= \int 1\{\alpha_i(\mathbf{x}_t, \varepsilon_{it}) = a\} \; dG_i(\varepsilon_{it}) \end{aligned}$$

• Since choice probabilities are integrated over the continuous variables in ε_{it}, they are lower dimensional objects than the strategies $\boldsymbol{\alpha}$.

• For instance, when both a_{it} and $\mathbf{x}_t$ are discrete, CCPs can be described as vectors in a finite dimensional Euclidean space.

Conditional Choice Probabilities (2)

- There is a **one-to-one relationship between** a best-response strategy functions $\alpha_i(\mathbf{x}_t, \varepsilon_{it})$ and its CCP function $P_i(.|\mathbf{x}_t)$.

- It is obvious that given $\alpha_i(\mathbf{x}_t, \varepsilon_{it})$ there is a unique $P_i(.|\mathbf{x}_t)$.

- The inverse relationship – given $P_i(.|\mathbf{x}_t)$ there is a unique best response function $\alpha_i(\mathbf{x}_t, \varepsilon_{it})$ – is a corollary of **Hotz-Miller inversion Theorem**.

Conditional Choice Probabilities (3)

• **Hotz-Miller inversion Theorem** (Hotz & Miller, REStud, 1993).
"Let $\alpha_i(x_t, \varepsilon_{it})$ be a best response strategy and let $P_i(a|\mathbf{x})$ be its corresponding CCP such that:

$$P_i(a|\mathbf{x}) = \int 1\{\arg\max_{a_{it}}[v_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) - \varepsilon_{it}(a_{it})] = a\}\ dG_i(\varepsilon_{it})$$

This mapping from the vector of conditional-choice values $\{v_i^{\boldsymbol{\alpha}_{-i}}(a, x_t) : a \in A\}$ into the vector of CCPs $\{P_i(a|x_t) : a \in A\}$ is invertible."

• Therefore, given $P_i(.|\mathbf{x}_t)$ we have a unique $v_i^{\boldsymbol{\alpha}_{-i}}(., \mathbf{x}_t)$, and then a unique best response strategy function:

$$\alpha_i(\mathbf{x}_t, \varepsilon_{it}) = \arg\max_{a_{it}}[v_i^{\boldsymbol{\alpha}_{-i}}(a_{it}, \mathbf{x}_t) - \varepsilon_{it}(a_{it})]$$

MPE as Fixed Point Mapping in CCPs

- We can use $\Pi_i^{\mathbf{P}_{-i}}$ and $F_i^{\mathbf{P}_{-i}}$ instead of $\Pi_i^{\boldsymbol{\alpha}_{-i}}$ and $F_i^{\boldsymbol{\alpha}_{-i}}$ to represent expected profit and transition prob.

$$\begin{aligned}
\Pi_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) &= \sum_{a_{-it}} \left[\prod_{j \neq i} P_j \left(a_{jt} \mid \mathbf{x}_t \right) \right] \pi_i \left(a_{it}, \mathbf{a}_{-it}, \mathbf{x}_t \right) \\
F_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1} | a_{it}, \mathbf{x}_t) &= F_z(\mathbf{z}_{t+1} | \mathbf{z}_t) \prod_{j \neq i} P_j \left(a_{jt} \mid \mathbf{x}_t \right)
\end{aligned}$$

- We also define:

$$v_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) \equiv \Pi_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) + \delta \int V_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1}) \; F_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1} | a_{it}, \mathbf{x}_t)$$

- Where

$$\widetilde{V}_i^{\mathbf{P}_{-i}}(\mathbf{x}_t) = \int \max_{a_{it}} \left\{ \begin{array}{c} \Pi_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) - \varepsilon_{it}(a_{it}) \\ +\delta \int \widetilde{V}_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1}) \; F_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1} | a_{it}, \mathbf{x}_t) \end{array} \right\} dG_i(\varepsilon_{it})$$

MPE as Fixed Point Mapping in CCPs [2]

- A MPE is a vector of CCPs, $\mathbf{P} \equiv \{P_i(a|\mathbf{x}) : \text{for any } (i, a, x)\}$, such that:

$$P_i(a|\mathbf{x}) = \Pr\left(a = \arg\max_{a_i} \left\{ v_i^{\mathbf{P}_{-i}}(a_i, \mathbf{x}) - \varepsilon_i(a_i) \right\} \mid \mathbf{x} \right)$$

- where

$$v_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) \equiv \Pi_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) + \delta \int V_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1}) \; F_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1}|a_{it}, \mathbf{x}_t)$$

- and

$$V_i^{\mathbf{P}_{-i}}(\mathbf{x}_t) = \int \max_{a_i}\{ v_i^{\mathbf{P}_{-i}}(a_i, \mathbf{x}_t) - \varepsilon_{it}(a_i)\} \; dG_i(\varepsilon_{it})$$

MPE in terms of CCPs: Example

- Suppose that vector ε_{it}'s are iid Extreme Value Type I.
- Then, a MPE is a vector $\mathbf{P} \equiv \{P_i(a|\mathbf{x}) : \text{for any } (i, a, \mathbf{x})\}$, such that:

$$P_i(a|\mathbf{x}) = \frac{\exp\left\{v_i^{\mathbf{P}_{-i}}(a, \mathbf{x})\right\}}{\sum_{a'} \exp\left\{v_i^{\mathbf{P}_{-i}}(a', \mathbf{x})\right\}}$$

- where

$$v_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) \equiv \Pi_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) + \delta \int V_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1}) \; F_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1}|a_{it}, \mathbf{x}_t)$$

- and $V_i^{\mathbf{P}_{-i}}$ is the unique solution to the Bellman equation:

$$V_i^{\mathbf{P}_{-i}}(\mathbf{x}_t) = \ln\left(\sum_{a_i} \exp\left\{\Pi_i^{\mathbf{P}_{-i}}(a_{it}, \mathbf{x}_t) + \delta \int V_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1}) \; F_i^{\mathbf{P}_{-i}}(\mathbf{x}_{t+1}|a_{it}, \mathbf{x}_t)\right\}\right)$$