ECO 2901
EMPIRICAL INDUSTRIAL ORGANIZATION

Lectures 5:
Static games of incomplete information with non-equilibrium beliefs: Empirical Applications

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Empirical Applications

[1] **Goldfarb and Xiao (2011)**

Entry decisions into local US telecommunication markets

[2] **Aguirregabiria & Xie (2020)**

Biased beliefs in lab experiments: Coordination game

1. Goldfarb and Xiao (2011)
Entry decisions into
local US telecommunication markets

US Telecommunication industry after deregulation

- **Goldfarb and Xiao (AER, 2011)** study entry decisions into local US telecommunication markets following the deregulatory Telecommunications Act of 1996.

- This act:
  (1) eliminated the possbility that State regulators deny entry to potential entrants;
  (2) Forced incumbents to facilitate interconnections and share other infrastructure with new entrants.

- A new entrant is denoted **"Competitive Local Exchange Carriers" (CLEC)**.

- New entry started to be effective in 1998.

- **Shakeout of the industry in 2002**: Excess entry followed by exit.

Data

- **CLEC annual reports** from years 1998 and 2002: year of entry; firm characteristics; CEO names
- **CEO characteristics, CV** from multiple sources: public companies annual reports, "Who is Who", company websites, archives.
  - Experience; education achievements.
- **Local market characteristics.** Local market is a "census place".
  - Population census: Population; median income; racial composition; median age; HH size; Poverty rate.
  - Business census: # establishments; # employees per establishment; sectorial composition.
- **Information from incumbents**

Data: Potential Entrants

- A nice feature of the local TCOM industry is that it is possible to identify the set of potential entrants in a local market such that we do not need to assume that every firm is a potential entrant everywhere.
- CLEC should apply and being approved by the state regulator to be a potential entrant in every local market of the state.
- Data on potential entrants

Data: Selection of Local Markets

- 234 midsize markets: Population between $100,000$ and $1,000,000$.
- Similar selection approach as other applications of market entry following Bresnahan & Reiss (1990).

Descriptive Statistics: Market level

TABLE 1B—DESCRIPTIVE STATISTICS BY MARKET

Variable	Mean	SD	Min	Max
Population (in thousands)	224.1	160.8	100.3	951.3
Median household income (in $1,000)	41.7	11.7	23.5	88.8
Median age	32.8	3.1	22.9	41.8
Household size	2.6	0.418	2.03	4.55
Percent foreign born	15.6	12.5	1.1	72.1
Percent African American	17.8	18.0	0.3	84.0
Percent below poverty line	14.5	6.3	2.2	35.6
GTE	0.107	0.310	0	1
RBOC	0.808	0.395	0	1
Number of establishments in thousands	4.7	3.8	0.661	24.5
Average number of employees/establishment	16.9	5.0	8.18	58.0
Percent establishments in manufacturing	18.1	10.3	0.001	60.36
Number of operating CLECs	2.02	2.9	0	18
Number of potential entrants	25.2	7.2	8	35
Observations			234	

Descriptive Statistics: CLEC level

TABLE 1A—DESCRIPTIVE STATISTICS BY CLEC

Variable	1998 Mean	1998 SD	2002 Mean	2002 SD
Number of markets to enter	61.5	66.9	90.3	70.6
Number of markets entered	4.9	9.4	15.7	16.8
Firm age	7.9	17.9	10.3	14.9
Subsidiary	0.312	0.465	0.218	0.416
Privately owned	0.645	0.480	0.625	0.487
Financed by venture capital	0.177	0.383	0.296	0.460
Employees (in thousands) 1998 ($N = 81$)	3.517	16.71	N/A	
Survive to 2002	0.427	0.497	N/A	
Alternate definition of survive to 2002	0.667	0.474	N/A	
Revenue 2002 (million \$, $N = 48$)	535	1550	N/A	
Local phone revenue 2002 (million \$, $N = 46$)	150	362	N/A	
Manager characteristics (with imputations)				
Experience	17.7	9.3	20.3	11.3
Undergraduate school average SAT $\geq$ 1400	0.094	0.293	0.096	0.297
Any graduate degree	0.554	0.475	0.501	0.469
Any economics or business degree	0.733	0.445	0.682	0.433
Any engineering or science degree	0.364	0.463	0.339	0.443
Observations (CLECs)	96		83	

"Motivating Analysis": Ex-post Monopoly markets

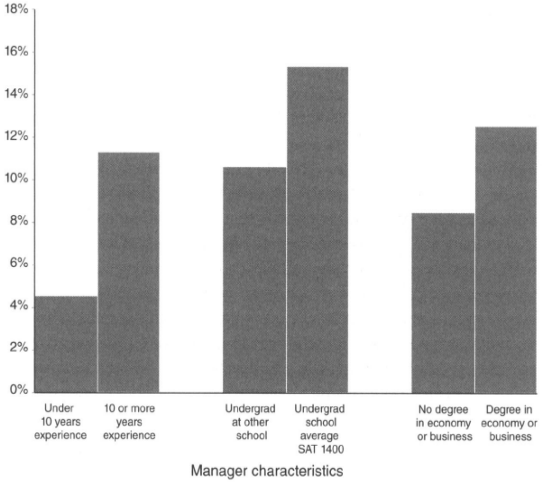


Figure 1. Percent Markets where the Firm Is the Only Operating CLEC

"Motivating Analysis"

$$(1) \quad Entry_{jm} = \alpha_0 + \alpha_1(\#competitors)_m + \mathbf{Z}_j\alpha_2 + (\#competitors)_m\mathbf{Z}_j\alpha_3 + \mathbf{X}_m\alpha_4 + \varepsilon_{jm},$$

- The main interest is the sign of the parameters α_3.
- The interaction between managers characteristics and number of competitors.

"Motivating Analysis" [2]

TABLE 2A—OLS REGRESSIONS OF 1998 ENTRY ON MANAGER CHARACTERISTICS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
(1) No. of competitors × log(experience)	−0.007 (0.002)***			−0.007 (0.002)***	−0.022 (0.009)**	−0.007 (0.002)***	−0.022 (0.009)**
(2) No. of competitors × manager attended school with SAT score above 1400		−0.018 (0.007)***		−0.016 (0.007)**	−0.016 (0.007)**	−0.017 (0.007)**	−0.016 (0.007)**
(3) No. of competitors × manager has degree in economics or business			−0.010 (0.003)***	−0.008 (0.004)**	−0.061 (0.031)*	−0.008 (0.004)**	−0.059 (0.030)*

Structural Model: Profit function

$$(2) \qquad \Pi_{jm} = \beta_0 + \mathbf{X}_m \beta + \psi(\#competitors)_m + \xi_m + \varepsilon_{jm}.$$

- Private information entry game.
- Note strong assumption: Profit does not depend on manager characteristics (ability) $\mathbf{Z}_j$.
- Strong exclusion restriction: $\mathbf{Z}_j$ only affects (the probability distribution of) a manager's level of strategic sophistication.

Structural Model: Cognitive Hierarchy (CH)

- Level $k = 0$: Believe they are monopolist in the market.
- Firm j's type is a random draw from a Poisson distribution with parameter τ_j, with:
$$\tau_j = \exp\{\gamma_0 + \mathbf{Z}_j \, \gamma\}$$
- A type k firm beliefs that the type of firm j comes from a:
$$Poisson\left(\tau_j\right) \text{ truncated at } k-1$$

Structural Parameters

- The vector of structural parameters includes:

$$\boldsymbol{\theta} = \left(\beta_0, \boldsymbol{\beta}, \psi, \sigma_\xi, \gamma_0, \boldsymbol{\gamma}\right)$$

- Parameters in the profit function: β_0, $\boldsymbol{\beta}$, ψ, σ_ξ
- Parameters in the Poisson distribution of strategic types: γ_0, $\boldsymbol{\gamma}$
- Note that the model **does not identify** the strategic type k of a firm j.
- But it **does identify** the probability distribution of the type of a firm j: $\text{Poisson}(\tau_j = \exp\{\gamma_0 + \mathbf{Z}_j\ \gamma\})$.
- Maximum Likelihood Estimation

Identification

- Based on the exclusion restriction of the model – $\mathbf{Z}_j$ and $\mathbf{Z}_i$ for $i \neq j$ do not have a direct effect on the profit function – all the parameters are identified.
- As we have seen in previous lecture, this assumption is stronger than necessary.
- $\mathbf{Z}_j$ could be included in the profit function of firm j. We still have that $\mathbf{Z}_i$ for $i \neq j$ does not have a direct effect on the profit function of firm j. This can identify an object that only depends on beliefs.

Empirical Results: Strategic Ability

TABLE 4—STRATEGIC ABILITY AND ENTRY COEFFICIENTS ($N = 5{,}906$)

Variables	Main (1)	No covariates in Z (2)	Only manager characteristics (3)	Alternative treatment of missing variables (4)	No random effects (5)
Coefficients on strategic ability parameter log(τ)					
(1) Log(experience)	0.161		0.180	0.147	0.235
	(0.061)***		(0.053)***	(0.057)***	(0.080)***
(2) Manager attended school with SAT score above 1400	0.069		0.041	0.062	0.117
	(0.039)*		(0.034)	(0.038)	(0.052)**
(3) Manager has degree in economics or business	0.396		0.358	0.375	0.558
	(0.215)*		(0.162)**	(0.193)*	(0.253)**

Empirical Results: Distribution of tau

(28) Mean τ	2.59	2.90	2.83	2.59	2.36
(29) Minimum τ	1.96	2.90	2.23	1.66	1.57
(30) Maximum τ	3.41	2.90	3.38	3.41	3.48

- At Mean $\tau = 2.59$:

$$\text{Type } 0 = 7.5\%; \;\; \text{Type } 1 = 19.4\%$$

$$\text{Type } 2 = 25.2\%; \;\; \text{Type } 3 = 21.7\%;$$

$$\text{Type } 4 \text{ or higher} = 26.2\%$$

2. Aguirregabiria & Xie (2020) Biased Beliefs in Lab Experiments: Coordination Game

Context and Motivation

- Given an easy to implement condition in the experimental design, it is possible to **use experimental data to test for equilibrium beliefs**
- It is also possible to **use experimental data to test the validity of self-reported beliefs**
- **Empirical Applications:**

- **Matching pennies game** – data from Goeree and Holt (AER, 2001)

- **public good / coordination game** – data from Heinemann, Nagel and Ockenfels (REStud, 2009).

Outline

1. Model
2. Experimental Design & Data
3. Identification Results
4. Empirical Applications

Model: Game

- Two players binary choice: row player (R) and column player (C).
- $a_R \in \{0, 1\}$ and $a_C \in \{0, 1\}$ be the actions of the players.
- There is a **monetary payoff matrix** that is common knowledge.

$$\mathbf{m} \equiv \{m_R(a_R, a_C), m_C(a_R, a_C) : (a_R, a_C) \in \{0, 1\}^2\}$$

- This game is played by a population of individuals, or subjects, that we index by $i \in \mathcal{I}$.

Model: Preferences

- Total utility of subject i as player with role in $r \in \{R, C\}$ is:

$$\Pi_{i,r} = \pi_i\left(m_r(a_i, a_j)\right) + \varepsilon_{i,r}(a_i)$$

- $\varepsilon_i's$ and π_i are **private information of individual** i.

- $\widetilde{\varepsilon}_{i,r} \equiv \varepsilon_{i,r}(0) - \varepsilon_{i,r}(1)$ captures the **non-pecuniary preferences of subject** i in role r.

- $\widetilde{\varepsilon}_{i,r}$ is independently distributed across subjects with mean μ_{ir}, variance σ^2_{ir}, and

$$\frac{\widetilde{\varepsilon}_{i,r} - \mu_{ir}}{\sigma_{ir}} \text{ has CDF } F(.)$$

Model: Best responses

- Suppose that individual i has been assigned the role of R player.
- She does not know the utility function and the $\widetilde{\varepsilon}'s$ of her opponent j. She has uncertainty about the choice of player j.
- $B_{i,R}(\mathbf{m})$ represents the belief that, as row player, individual i has about the probability that the other player chooses action 1.
- Player i's expected payoff of taking action a_i is:

$$
\begin{aligned}
\Pi_{i,R}^{e}(a_i) &= [1 - B_{i,R}(\mathbf{m})] \;\pi_i\left(m_R(a_i, 0)\right) \\
&\quad + B_{i,R}(\mathbf{m}) \;\pi_i\left(m_R(a_i, 1)\right) + \varepsilon_{i,R}(a_i)
\end{aligned}
$$

Model: Best responses [2]

- The best response of player i is alternative $a_i = 1$ if

$$[1 - B_{i,R}(\mathbf{m})] \;\; \pi_i \left(m_R(1,0) \right) + B_{i,R}(\mathbf{m}) \;\; \pi_i \left(m_R(1,1) \right) + \varepsilon_{i,R}(1)$$

$$\geq [1 - B_{i,R}(\mathbf{m})] \;\; \pi_i \left(m_R(0,0) \right) + B_{i,R}(\mathbf{m}) \;\; \pi_i \left(m_R(0,1) \right) + \varepsilon_{i,R}(0)$$

- The **best response choice probability** for individual i is:

$$P_{i,R}(\mathbf{m}) = F \left[\alpha_{i.R}(\mathbf{m}) + \beta_{i.R}(\mathbf{m}) \;\; B_{i,R}(\mathbf{m}) \right]$$

$\alpha_{i.R}(\mathbf{m}) \equiv \left[\pi_i \left(m_R(1,0) \right) - \pi_i \left(m_R(0,0) \right) - \mu_{i,R} \right] / \sigma_{i,R}$
$\beta_{i.R}(\mathbf{m}) \equiv \left[\pi_i \left(m_R(1,1) \right) - \pi_i \left(m_R(0,1) \right) - \pi_i \left(m_R(1,0) \right) - \right.$
$\left. \pi_i \left(m_R(0,0) \right) \right] / \sigma_{i,R}$

Model

- This model allows for:
  - General individual heterogeneity in π_i, $B_{i,r}$, $\mu_{i,r}$, $\sigma_{i,r}$
  - Unrestricted (nonparametric) belief function $B_{i,r}(\mathbf{m})$
  - Unrestricted (nonparametric) utility function $\pi_i(m)$

- It includes as particular cases different models that are commonly used in empirical applications of games:
  - Bayesian Nash Equilibrium of incomplete information game
  - Quantal Response Equilibrium
  - Cognitive Hierarchy and Level-K models

Experimental Design & Data

- Researcher has panel data of N individuals over T times/rounds.

$$\{d_{it},\ a_{it} : i = 1, 2, ..., N; \quad t = 1, 2, ..., T\}$$

- d_{it} represents the *treatment* received by subject i in her game t, and a_{it} is her action in that game.

- The researcher chooses M **different matrices of monetary payoffs**. Let $\mathcal{M}$ be the set of monetary payoff matrices in the experiment.

- A **treatment** is pair $(\mathbf{m}, r)$, where $\mathbf{m} \in \mathcal{M}$ is a payoff matrix and $r \in \{R, C\}$ is a player role.

- At each round t, the N subjects are **randomly assigned to one of the $2M$ treatments**.

Assumptions on the DGP

ASSUMPTIONS.

(1) $(\pi_i, \mu_{i,r}, \sigma_{i,r}, F)$ *are invariant over the* T *rounds that an individual plays the game.*

(2) Beliefs depend on treatment $(\mathbf{m}, r)$ *but conditional on the same treatment, it is invariant over rounds:* $B_{it,r}(\mathbf{m}) = B_{ir}(\mathbf{m})$.

(3) $F(.)$ *– the CDF of* $\dfrac{\widetilde{\varepsilon}_{i,r} - \mu_{ir}}{\sigma_{ir}}$ *– is known to the researcher.*

(4) Treatment variable d_{it} *and the non-pecuniary utility components* $(\widetilde{\varepsilon}_{it,R}, \widetilde{\varepsilon}_{it,C})$ *are independently distributed.*

A Key Property of the Experimental Design

- The set of monetary payoff matrices in the randomized experiment, $\mathcal{M}$, is such that there are at least three treatments, say $(\mathbf{m}_1, R)$, $(\mathbf{m}_2, R)$, $(\mathbf{m}_3, R)$ such that:

- **(A)** Player in role R has the same monetary payoff matrix in the three treatments but the payoff matrix for the C player is different:

$$\mathbf{m}_{R,1} = \mathbf{m}_{R,2} = \mathbf{m}_{R,3} \quad \textit{and} \quad \mathbf{m}_{C,1}, \mathbf{m}_{C,2}, \mathbf{m}_{C,3} \text{ different}$$

- **(B)** For individual i, her choice probability in role R varies across at least two of these treatments:

$$P_{i,R}(\mathbf{m}_1) \neq P_{i,R}(\mathbf{m}_2)$$

Panel Data with Large T

- We consider a panel data set where N is small and T is large.
- This sampling framework corresponds exactly to most empirical applications of discrete choice games of oligopoly competition in empirical IO (Berry and Tamer, 2007).
- This sampling framework is also common in empirical applications in experimental economics.
- Both N and T are not very large in most applications in experimental economics. However, the main point here is that the researcher observed each subject playing the game a number of times T. In many experimental papers T can be larger than 20 or even 50.

Extension: G players symmetric game

- G players binary choice game. Public good game.
- Utility function: $\pi_i\,(m) + \varepsilon_i(a_i)$.
- Let $q_{-i} \equiv \dfrac{1}{G-1} \sum_{j \neq i}^{G} a_j$The monetary payoff of player i is:

$$m_i\,(a_i, q_{-i}) = \begin{cases} m_s & \text{if} \quad a_i = 0 \\ \\ m_s - c & \text{if} \quad a_i = 1 \text{ and } q_{-i} < \phi \\ \\ m_s + r & \text{if} \quad a_i = 1 \text{ and } q_{-i} \geq \phi \end{cases}$$

G players symmetric game (Cont.)

- In this game, the exogenous characteristics that define players' monetary payoffs are $\mathbf{m} \equiv (m_s,\ c,\ r,\ \phi,\ G)$.

- Player i needs to form beliefs:

$$B_i(\mathbf{m}) = \text{Subjective Prob}\left[q_{-i} \geq \phi\right]$$

- The best response of individual i is to choose alternative $a_i = 1$ if

$$\left[1 - B_i(\mathbf{m})\right] \pi_i\left(m_s - c\right) + B_i(\mathbf{m})\pi_i\left(m_s + r\right) + \varepsilon_i(1) \geq \pi_i\left(m_s\right) + \varepsilon_i(0$$

- And the best response choice probability is:

$$P_i(\mathbf{m}) = F\left[\alpha_i(m_s, c, r) + \beta_i(m_s, c, r)\ B_i(\mathbf{m})\right]$$

with: $\alpha_i(m_s, c, r) \equiv \left[\pi_i\left(m_s - c\right) - \pi_i\left(m_s\right) - \mu_i\right]/\sigma_i$; and $\beta_i(m_s, c, r) \equiv \left[\pi_i\left(m_s + r\right) - \pi_i\left(m_s - c\right)\right]/\sigma_i$.

Extension: Preferences with Altruism / Envy

- Our identification result is based on variation of other players' payoff matrix.

- A potential issue is that the identified object might capture not beliefs but a subject's concern for the utility of other players. For instance:

$$\Pi_{i,R} = \pi_i(m_R) + \psi_i(m_C) + \varepsilon_i(a_{i,R})$$

- We show that – given additional conditions matrix m_C, i.e., $m_C(1, a_j) = m_C(0, a_j)$ – the identified object depends on subject $i's$ beliefs and not on his altruism/envy preferences.

Identification of Choice Probabilities

- For every individual i and treatment $(\mathbf{m},r)$:

$$P_{i,r}(\mathbf{m}) = \mathbb{E}\left[a_{it} \mid i,\, d_{it} = (\mathbf{m},r)\right]$$

- Given the data on $\{a_{it},d_{it}\}$, the choice probabilities $P_{i,r}(\mathbf{m})$ are identified for every individual i and treatment $(\mathbf{m},r)$.

- Let $F^{-1}(.)$ be the inverse function of the CDF F, that is known.

- The model implies that:

$$F^{-1}(P_{i,R}(\mathbf{m})) = \alpha_{i.R}(\mathbf{m}) + \beta_{i.R}(\mathbf{m})\ B_{i,R}(\mathbf{m})$$

Identification of Beliefs

- Let $\mathbf{m}_1$, $\mathbf{m}_2$, and $\mathbf{m}_3$ be treatments with $\mathbf{m}_{R,1} = \mathbf{m}_{R,2} = \mathbf{m}_{R,3}$ and different $\mathbf{m}_{C,1}$, $\mathbf{m}_{C,2}$, $\mathbf{m}_{C,3}$.

- Then, the model implies that:

$$\frac{F^{-1}(P_{i,R}(\mathbf{m}_3)) - F^{-1}(P_{i,R}(\mathbf{m}_1))}{F^{-1}(P_{i,R}(\mathbf{m}_2)) - F^{-1}(P_{i,R}(\mathbf{m}_1))} = \frac{B_{i,R}(\mathbf{m}_3) - B_{i,R}(\mathbf{m}_1)}{B_{i,R}(\mathbf{m}_2) - B_{i,R}(\mathbf{m}_1)}$$

- The behavior of individual i reveals an object that depends only on her beliefs.

- We can use this identified object to test the restrictions imposed by different equilibrium models, e.g., BNE, QRE, Level-K, CH.

Test of Equilibrium Beliefs under BNE

- Under this model we have that, for any individual i:

$$B_{i,R}(\mathbf{m}) = \mathbb{E}\left[P_{j,C}(\mathbf{m})\right]$$

where the expectation is over the population of individuals j.

- For every individual i, we can test, the restriction:

$$\frac{B_{i,R}(\mathbf{m}_3) - B_{i,R}(\mathbf{m}_1)}{B_{i,R}(\mathbf{m}_2) - B_{i,R}(\mathbf{m}_1)} = \frac{\mathbb{E}\left[P_{j,C}(\mathbf{m}_3)\right] - \mathbb{E}\left[P_{j,C}(\mathbf{m}_1)\right]}{\mathbb{E}\left[P_{j,C}(\mathbf{m}_2)\right] - \mathbb{E}\left[P_{j,C}(\mathbf{m}_1)\right]}$$

- We can test (nonparametrically) these equilibrium restrictions.

Full identification of the model

- If the null hypothesis of unbiased believes is not rejected in one triple $(\mathbf{m}_1, \mathbf{m}_2, \mathbf{m}_3)$ we can **impose this restriction in two of these treatments**.

- Under this restriction:
  - The beliefs function $B_{i,R}(\mathbf{m})$ is identified at every value $\mathbf{m}$ in the experiment.
  - The utility function of money $\pi_i(m_R)$ is identified at every value m_R in the experiment.

Matching pennies game

Payoff Matrices in the 3 Treatments

Treatment 1		$a_C = 0$	$a_C = 1$
	$a_R = 0$	40 , 44	80 , 40
	$a_R = 1$	80 , 40	40 , 80

Treatment 2		$a_C = 0$	$a_C = 1$
	$a_R = 0$	40 , 80	80 , 40
	$a_R = 1$	80 , 40	40 , 80

Treatment 3		$a_C = 0$	$a_C = 1$
	$a_R = 0$	40 , 320	80 , 40
	$a_R = 1$	80 , 40	40 , 80

Empirical Choice Probabilities

Empirical Choice Probabilities

	Player R $(a_R = 1)$	**Player C** $(a_C = 1)$
Treatment 1	0.20	0.92
	(0.080)	(0.054)
Treatment 2	0.52	0.52
	(0.100)	(0.100)
Treatment 3	0.84	0.04
	(0.073)	(0.039)

Standard errors in parentheses

Unbiased Belief Test

Model for CDF $\widetilde{\varepsilon}$	**Estimate [s.e.] (p-value)**
Probit	0.0503 [0.3753] **(0.8818)**
Logit	0.0726 [0.6479] **(0.9032)**
Exponential	-0.1942 [0.3683] **(0.5426)**
Double Exponential	-0.0965 [0.4738] **(0.8226)**

- We cannot reject the null.
- Players in R-role are able to correctly predict the change of player C's behavior as player C's monetary payoff varies.

Coordination Game

Other players

$q =$ fraction of other players choosing $a = 1$

		$q < \phi$	$q \geq \phi$
Player R	$a_R = 0$	m_S	m_S
	$a_R = 1$	0	15 Euros

Treatments

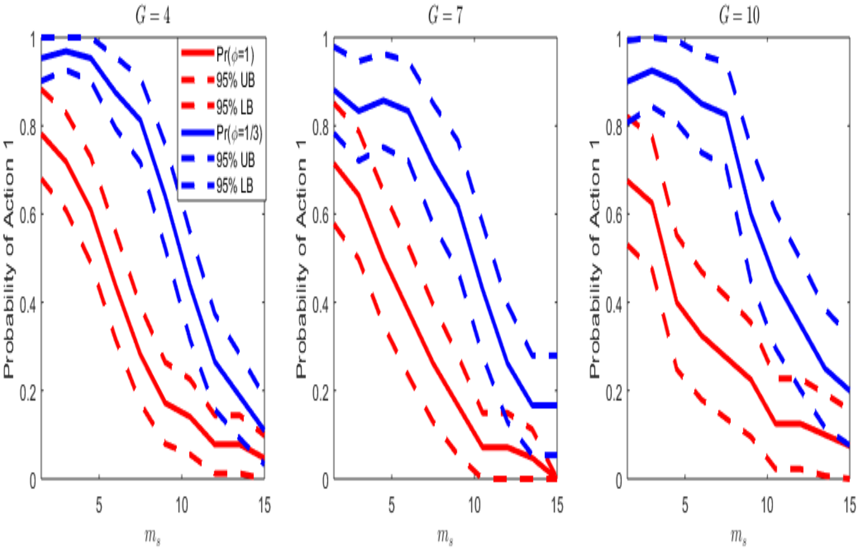
$T = 81$ treatments. Set of treatment consists of every combination (G, m_S, ϕ) with:

G (# players) $\in \{4, 7, 10\}$

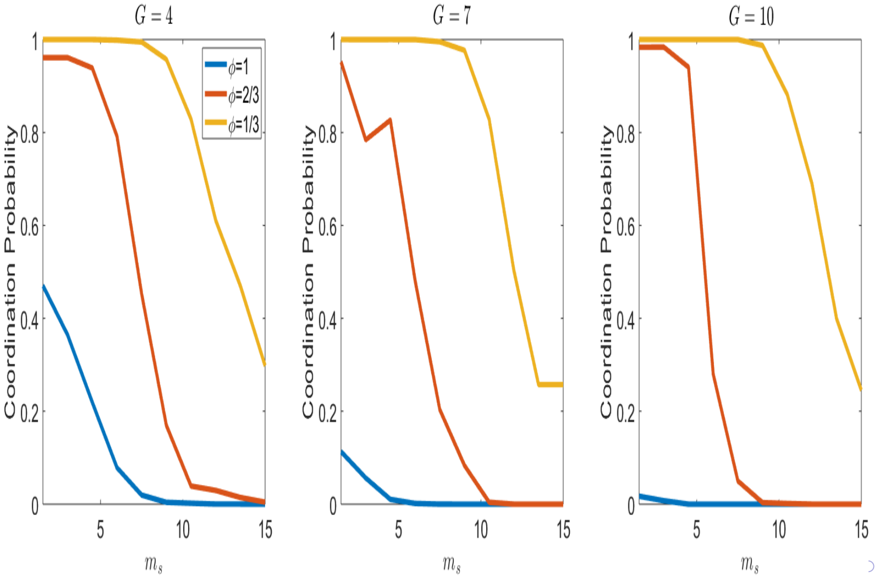
$m_S \in \{j * 1.5 : j = 1, 2, ..., 9\}$ Euros

$\phi \in \{1/3,\ 2/3,\ 1\}$

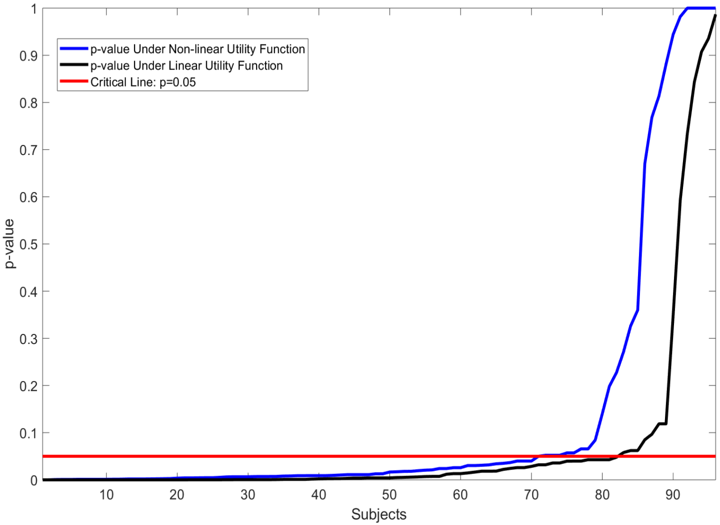
Empirical Choice Probabilities



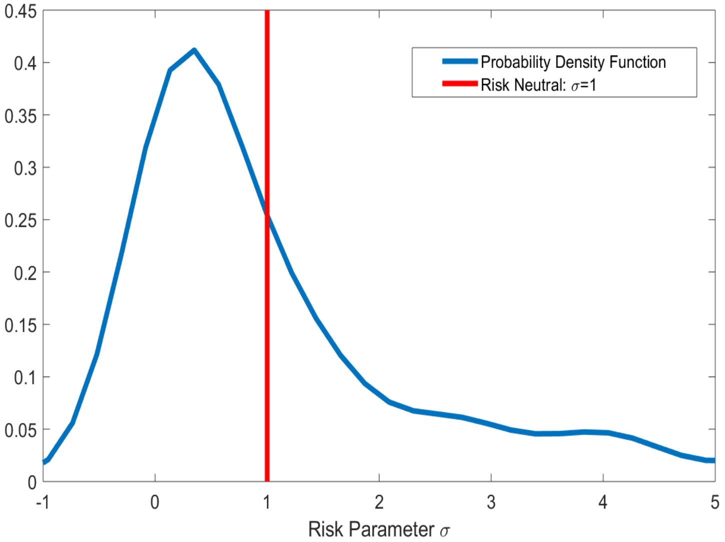
Actual Coordination Probability



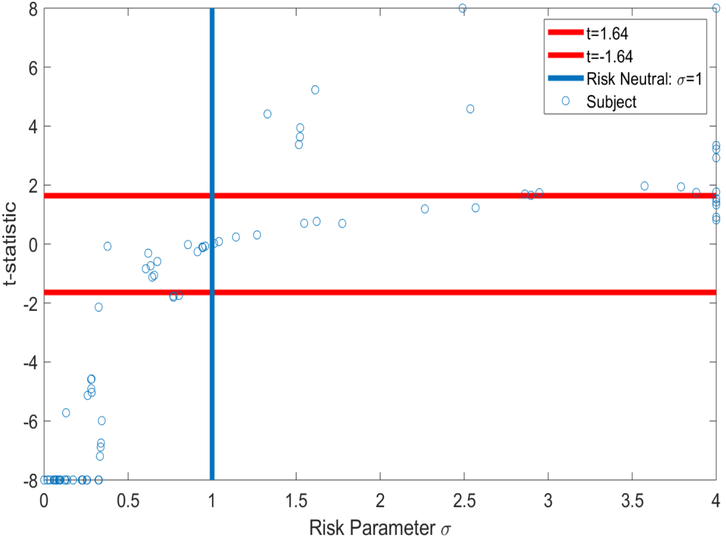
Test of unbiased beliefs: p-values



Estimation of Risk Aversion: Distribution



Estimation of Risk Aversion: Test of Risk Neutrality



Testing Validity of Elicited Beliefs