Homework 16: ECO220Y – SOLUTIONS

Required Problems:

(1) (a) The Nearly Normal Condition clearly does not hold. Looking at this sample we see clear evidence of strong positive skew. Given the large sample size we can be confident that the population this sample is drawn from is far from symmetric. However, our sample size is nearly 300. Given this large sample size and the Central Limit Theorem we can be sure that the sampling distribution of the sample mean will be Normal even though the population is surely not. Hence we can still use the formulas presented in this chapter. (The Nearly Normal Condition is important when the sample size is small or modest in size.)

(b) To find the 80% confidence interval estimator we use plug into: $\bar{X} \pm t_{\frac{\alpha}{2},\nu} \frac{s}{\sqrt{n}}$. Our degrees of freedom are $\nu = n - 1 = 298 - 1 = 297$. For an 80% CI estimator $\alpha = 0.20$. Looking at the Student t table we see that we cannot get the exact critical value for 297 degrees of freedom. One option is to be conservative and use the critical value for 250 degrees of freedom: this will make our interval a tiny bit wider than necessary and ensure we are at least 80% confident.

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}} = 45.54027 + 1.285 \frac{41.37982}{\sqrt{298}} = 45.54027 + 1.285 * 2.397 = 45.54027 + 3.080$$

This yields a lower confidence limit (LCL) of $42.46 and an upper confidence limit (UCL) of $48.62. A full interpretation: We are 80% confident the average amount that would have been donated had we contacted the entire population with the original letter (no match) would be between $42.46 and $48.62. (This applies the concept that we are making an inference about the unknown population mean and gives the specific context.) This interval is MUCH NARROWER than the difference between the 10th and 90th percentiles in the STATA summary of the sample given with question: $10 and $100, respectively. The reason is that the CI estimator is about a mean. The sample reflects individual donors. We expect a lot more variability across individuals than across averages for large groups of individuals. This is reflected in the formula we used above which tells us that the width of the CI of the mean is inversely related to the sample size. As the sample size goes up the variability in the sample is not expected to change (i.e. s and the percentiles will be roughly the same but for sampling error) but the variability of the sample mean goes down (as this sample statistic would have less and less sampling error). Hence, whenever we have a sample size greater than 1, we expect a discrepancy between the variability of the sample and the width of the CI estimator of the mean: the sample is more variable.

(2) (a) $H_0: \mu = 0.85$ versus $H_1: \mu > 0.85$, where μ is the mean payout rate. (Note: This is NOT a proportion question. The standard is NOT that 85% of plays must be winning plays. It is about the average payout per dollar played.) A Type I error would be inferring that machine meets the minimum standard when, in fact, it does not and should be removed from play. A Type II error would be concluding that there is insufficient evidence to prove the machine is in compliance (i.e. fail to reject the null), when the machine is, in fact, in compliance, which means removing a machine from play even though it does meet the regulation. For $n = 5{,}000$, we use the last row of the Student t table, to obtain a critical value of $t_{\alpha}^{*} = 1.645$, which means the rejection region is a t value above 1.645. Given the formula for the test statistic of $t = \frac{\bar{X}-\mu_0}{\frac{s}{\sqrt{n}}} = \frac{\bar{X}-0.85}{\frac{s}{\sqrt{5{,}000}}}$, this means that we will have to have a sample mean that is sufficiently better than 0.85. How much better will depend on the sample standard deviation.

(b) $H_0: \mu = 0.85$ versus $H_1: \mu < 0.85$. A Type I error would be inferring that a machine does not meet the minimum standard when, in fact, it does and should be allowed to continue in play. In other words, removing a compliant machine from play. A Type II error would be concluding that there is insufficient evidence to prove the machine is out of compliance (i.e. fail to reject the null), when the machine is, in fact, out of compliance, which means allowing a machine that does not meet the regulation to be in play. For $n = 5{,}000$, we use the last row of the Student t table, to obtain a critical value of $t_{\alpha}^{*} = -1.645$, which means the rejection region is a t value below -1.645. Given the formula for the test statistic of $t = \frac{\bar{X}-\mu_0}{\frac{s}{\sqrt{n}}} = \frac{\bar{X}-0.85}{\frac{s}{\sqrt{5{,}000}}}$, this means that we will have to have a sample mean that is sufficiently worse than 0.85. How much worse will depend on the sample standard deviation.

(c) Remember: the burden of proof is on the research hypothesis. If you are most concerned about protecting players, you should set the hypothesis up like (a). In other words, only allow machines proven to meet minimum payout standards to be in play. If you are most concerned about protecting the profits of the casinos, you should set the hypotheses up like (b). In other words, only remove machines from play if they are proven out of compliance with minimum payout standards.

(d) $H_0: \mu = 0.918$ versus $H_1: \mu > 0.918$. A Type I error would claiming your machine is above average in generosity when, in fact, it is not. A Type II error would be having a machine that is, in fact, above average in generosity but you are not able to prove it.

(e) To limit the chance of a Type I error, we can choose a higher burden of proof, which means a *lower* alpha. For example, instead of choosing $\alpha = 0.05$, choose $\alpha = 0.01$. However, the casino is probably not too worried about a Type I error.

(f) To limit the chance of a Type II error, we can plan on collecting a bigger sample, which will lower β and increase power (our ability to prove that a our machine is more generous, provided that it really is more generous). This would involve playing the machine far more times. The casino would likely appreciate this suggestion as it would help ensure that it could advertise its above-average machines.

(g) It is harder to prove that a machine that is only slightly above average is better than average than to prove one that is way above average is. In other words, there will be a substantially larger chance of Type II error for the marginal machine. Note that while we do NOT do power (and probability of Type II error) calculations for means in our course, only for a proportions, the *concepts* you learned in Lecture 15 (and supporting materials) absolutely apply here. Both scenarios would have the same rejection region, but having a more generous machine means you have a higher chance of being in the rejection region.

(3)(a)

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{-0.421 - 0}{\frac{1.244}{\sqrt{19}}} = -1.475$$

Using the Student t table for $\nu = 19 - 1 = 18$, we obtain that the P-value is somewhere between 0.10 and 0.20. (Note: You must remember this is a two-tailed test when computing the P-value.) This result is not statistically significant at any conventional significant level.

(b)

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{0.321 - 0}{\frac{0.501}{\sqrt{30}}} = 3.509$$

Using the Student t table for $\nu = 30 - 1 = 29$, we obtain that the P-value is somewhere between 0.001 and 0.002. (Note: You must remember this is a two-tailed test when computing the P-value.) This result is statistically significant at a 1% significance level (and just misses meeting a 0.1% significance level).

(c)

$$t = \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{0.053 - 0}{\frac{1.351}{\sqrt{1900}}} = 1.71$$

Using the Standard Normal table, which is an outstanding approximation of the Student t for very large degrees of freedom like this case, we obtain a P-value of 0.0872. (Note: You must remember this is a two-tailed test when computing the P-value.) This result is statistically significant at a 10% significance level, but not a 5% significance level.