University of Toronto Mississauga

STA256 L0103 - Fall 2019

Instructor: Dr. Ramya Thinniyam

Test # 1

October 3rd, 2019

SOLUTIONS - Version 2 (Yellow)

Aids Allowed: Non-programmable Calculator (without a text keypad).
Aids Provided: Formula Sheet

INSTRUCTIONS:

- There are 12 questions - answer all questions.
- There are 6 test pages in total, and a separate formula sheet. Make sure you have all pages before starting the test.
- For Multiple Choice questions, circle the answer and put the final answer in the chart as instructed. You can use empty space for rough work, but it will not be marked. **Only final answers in the chart will be marked.**
- For all other questions, complete solutions are required. **Show your work to earn full marks and then circle the final answer**. Clearly name any events you introduce using proper notation. **Answers, even if correct, with no justifications will not receive any marks**.
- Simplify final answers and round to 4 decimal places where appropriate.

Question	1.	2.	3. - 12.	**TOTAL**
Value	25	25	50	100
Mark Earned				

BEST WISHES ! ☺

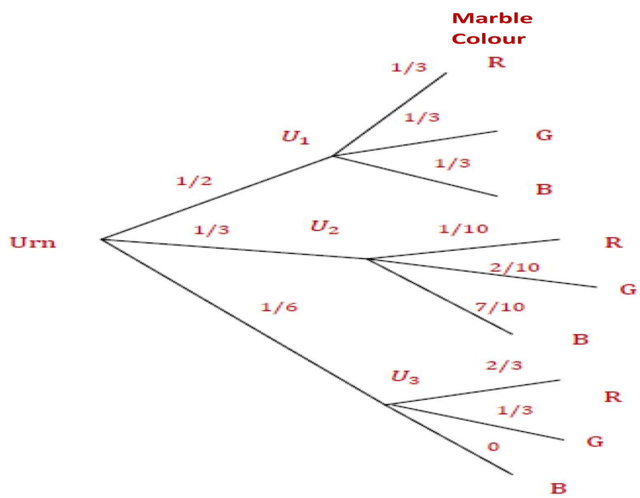
[25 marks]

1. ***There are 3 urns: Urn 1 contains 5 red, 5 green, and 5 blue marbles; Urn 2 has 1 red, 2 green, and 7 blue marbles; Urn 3 has 4 red, 2 green, and no blue marbles. First you roll a fair six-sided die: if it lands on an even number, you choose Urn 1; if it lands on '3' or '5', you choose Urn 2; if it lands on '1', you choose Urn 3. Then you randomly choose one marble from the selected urn.***

You have selected one marble and observe that it is green. What is the probability that it came from an odd numbered urn?

Show your work and then circle the final answer. You must show work (tree diagram properly labelled, define and label any events you introduce, name any theorems/laws that you use, etc.) to earn full marks.

Let U_i represent the event that the ith urn is selected; for $i = 1, 2, 3$.
Let R represent the event that a red marble is selected.
Let G represent the event that a green marble is selected.
Let B represent the event that a blue marble is selected.



$$
\begin{aligned}
P(U_1 \cup U_3|G) &= P(U_1|G) + P(U_3|G) \text{ , since } U_1 \text{ and } U_3 \text{ are disjoint} \\
&= \frac{P(U_1 \cap G)}{P(G)} + \frac{P(U_3 \cap G)}{P(G)} \text{ , using definition of conditional probability} \\
&= \frac{P(G|U_1)P(U_1) + P(G|U_3)P(U_3)}{P(G|U_1)P(U_1) + P(G|U_2)P(U_2) + P(G|U_3)P(U_3)} \text{ , using Bayes Theorem} \\
&= \frac{\frac{1}{3}\left(\frac{1}{2}\right) + \frac{1}{3}\left(\frac{1}{6}\right)}{\frac{1}{3}\left(\frac{1}{2}\right) + \frac{2}{10}\left(\frac{1}{3}\right) + \frac{1}{3}\left(\frac{1}{6}\right)} \\
&= 10/13 \\
&= \boxed{0.7692} \text{ .}
\end{aligned}
$$

[5m for identifying it is a conditional probability and using union of Urn 1 and Urn 3 with proper notation, 5m for tree diagram correctly labelled or explanation, 5m for using Bayes Theorem and quoting it or quoting Multiplication Rule and Law of Total Probability, 5m for showing work and justifying steps, 5m for final answer to 4 decimal places.]

[25 marks]

2. *Proof:* *Let A, B, and C be events in a sample space, $\mathcal{S}$, each with positive probabilty. If $A \subseteq B$, then prove that $P(A \cap C) \leq P(B \cap C)$.*

Prove the statement using the tabular format from lectures and using only the axioms of probability (you may not use other properties without proof).

Case 1: If $A \cap C = \emptyset$ and $B \cap C = \emptyset$

Step	Justification
$P(A \cap C) = P(B \cap C) = P(\emptyset)$	Apply probability function, P
$\Rightarrow P(A \cap C) \leq P(B \cap C)$	Math / definition of inequality

Case 2: If $A \cap C = \emptyset$ and $B \cap C \neq \emptyset$

Step	Justification
$P(A \cap C) = P(\emptyset) = 0$	Apply probability function, P
$\Rightarrow P(A \cap C) = 0 \leq P(B \cap C)$	By Axiom 1

Case 3: If $A \cap C \neq \emptyset$ and $B \cap C \neq \emptyset$

Step	Justification
$B \cap C = (A \cap C) \cup (A^c \cap B \cap C)$ where the sets are disjoint	By Venn diagram
$P(B \cap C) = P((A \cap C) \cup (A^c \cap B \cap C))$	Apply probability function, P
$P(B \cap C) = P(A \cap C) + P(A^c \cap B \cap C)$	By Axiom 3 since the sets are disjoint
$P(B \cap C) \geq P(A \cap C)$	Since $P(A^c \cap B \cap C) \geq 0$ by Axiom 2

[5m for considering all the cases, 5m for proper Venn diagram showing subsets and intersections, 5m for using tabular format, 2m for proof of Case 1, 3m for proof of Case 2 and using Axiom 1, 5m for proof of Case 3 and using Axioms 2 and 3 properly and justifying steps, -5m if tabular format is not used]

Other possible solutions:

- *Consider all the cases. For Case 3, first prove that $(A \cap C) \subset (B \cap C)$ and then prove Property 3. Proofs in tabular format with proper justification*
- *Consider all the cases. For Case 3, first show $B \cap C = (A \cap C) \cup (A^c \cap B \cap C)$ and then prove Distributive Property and use it. Proofs in tabular format with proper justification*

[50 marks - *5m each part; no part marks - answers in chart are marked*]

3.-12. ***Multiple Choice: For each of the following questions, choose only one option. Circle the letter corresponding to the correct answer. You may use empty space for rough work, but it will not be marked. At the end of these questions, there is a chart to put your final answers - write the letter corresponding to the correct answer for each question in the chart. Only final answers in the chart will be marked.***

3. Let $\mathcal{S}$ be a sample space. Which statement is TRUE?

   (a) If $\mathcal{S}$ is finite, it is possible for $P(s) = 0$ for every $s \in \mathcal{S}$

   (b) If $\mathcal{S}$ is uncountable, it is possible for $P(s) = 0$ for every $s \in \mathcal{S}$

   (c) $P(S) = 1$

   (d) (b) and (c)

   (e) All of the above

4. Let A and B be two events in a sample space, $\mathcal{S}$ with $P(A), P(B) > 0$. Which of the following statements is FALSE?

   (a) If $A \supseteq B$ then $P(B) \leq P(A)$

   (b) If $|A| \leq |B|$ then $P(A) \leq P(B)$

   (c) If A and B are disjoint, then A and B are dependent

   (d) All of the above

   (e) None of the above

5. How many arrangements can be made from the letters of the word "MISSISSAUGA" so that all the S's are together but the I's are not together?

   (a) 415,800

   (b) $8! - 7!$

   (c) $\binom{11}{4\ 2\ 2\ 3}$

   (d) 7,560

   (e) None of the above

6. You have not studied and decide to randomly guess on each multiple choice question on this test. (On this test, there are 10 multiple choice questions, each with 5 options, of which only one option is correct.) What is the probability that you get perfect on the multiple choice questions?

   (a) 0.2

   (b) $(0.2)^{10}$

   (c) $1 - (0.8)^{10}$

   (d) $1 - (0.2)^{10}$

   (e) $(0.8)^{10}$

7. There are 4 friends who want to play chess. There are 2 chessboards available so they can all play at the same time. In how many ways can they split up into two pairs, if within each pair, one player plays White (gets to move first) and the other player plays Black (moves second)?

   (a) $\binom{4}{2\ 2}$
   (b) $\frac{1}{2!}\binom{4}{2\ 2}$
   (c) 12
   (d) 6
   (e) None of the above

8. A box contains 6 red balls and 4 yellow balls. If you randomly select 4 balls *without replacement*, what is the probability of getting all yellow balls?

   (a) $\frac{\binom{6}{0}\binom{4}{4}}{\binom{10}{4}}$
   (b) 10!
   (c) $\frac{1}{10^4}$
   (d) $\left(\frac{2}{5}\right)^4$
   (e) None of the above

9. A box contains 6 red balls and 4 yellow balls. If you randomly select 4 balls *with replacement*, what is the probability of getting all yellow balls?

   (a) $\frac{\binom{6}{0}\binom{4}{4}}{\binom{10}{4}}$
   (b) 10!
   (c) $\frac{1}{10^4}$
   (d) $\left(\frac{2}{5}\right)^4$
   (e) None of the above

10. A biased coin with $P(\text{Head}) = p$ is tossed repeatedly until a tail occurs. What is the probability that a tail will eventually occur?

   (a) 0
   (b) 1
   (c) $\frac{p}{1-p}$
   (d) $\frac{1-p}{2-p}$
   (e) $\frac{1-p}{p}$

11. A medical trial into the effectiveness of a new medication for migraine pain was carried out. The investigators were interested in determining if there is a relationship between gender and the efficacy of the medication. 120 females and 90 males volunteered to take part in the trial. After two hours from taking the medication, the number of patients feeling relief from pain and the number of patients who felt no relief was recorded for each gender. The data are summarized in the table below:

	Female	Male	*Total*
Relief	50	30	80
No Relief	70	60	130
Total	120	90	210

If one of the patients selected at random did not feel relief, what is the probability that this patient is a male?

(a) $\frac{2}{7}$

(b) $\frac{7}{13}$

(c) $\frac{6}{13}$

(d) $\frac{9}{21}$

(e) $\frac{8}{21}$

12. Let A and B be events in a sample space, $\mathcal{S}$ such that:
$P(A) = 0.5$, $P((A \cup B)^c) = 0.1$, $P(B|A) = 0.4$. Then, $P(B|A^c)$ is

(a) 0.6

(b) 0.4

(c) 0

(d) 0.8

(e) 1

Write the letter coresponding to the correct answer for each question in the chart below:

Question	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Letter Answer	d	b	d	b	c	a	d	b	c	d

[5m each part; no part marks - final answers must be in the chart]