# Empirical Industrial Organization (ECO 310) <br> Fall 2019. Victor Aguirregabiria 

Problem Set \#2
Due on Thursday, December 5th, 2019 [before 11:59pm]

INSTRUCTIONS. Please, follow the following instructions for the submission of your completed problem set.

1. Write your answers electronically in a word processor.
2. For the answers that involve coding in STATA, include in the document the code in STATA that you have used to obtain your empirical results.
3. Convert the document to PDF format.
4. Submit your problem set (in PDF) online via Quercus.
5. You should submit your completed problem set before 11:59pm of Monday, December 5th, 2019.
6. Problem sets should be written individually.

## The total number of marks is 150 .

QUESTION 1. [70 points]. Consider an industry with a differentiated product. There are two firms in this industry, firms $A$ and $B$. Each firm produces and sells two brands of the differentiated product: brands $A 1$ and $A 2$ are produced by firm $A$, and brands $B 1$ and $B 2$ by firm $B$. The demand system is a logit demand model, where consumers choose between five different alternatives: $j=0$, represents the consumer decision of no purchasing any product; and $j=A 1, j=A 2, j=B 1$, and $j=B 2$ represent the consumer purchase of product $A 1, A 2, B 1$, and $B 2$, respectively. The utility of no purchase $(j=0)$ is zero. The utility of purchasing product $j \in\{A 1, A 2, B 1, B 2\}$ is $\beta x_{j}-\alpha p_{j}+\varepsilon_{j}$, where the variables and parameters have the interpretation that we have seen in class. Variable $x_{j}$ is a measure of the quality of product $j$, e.g., the number of stars of the product according to consumer ratings. Therefore, we have that $\beta>0$. The random variables $\varepsilon_{1}$ and $\varepsilon_{2}$ are independently and identically distributed over consumers with a type I extreme value distribution, i.e., Logit model of demand. Let $H$ be the number of consumers in the market. Let $s_{0}, s_{A 1}, s_{A 2}$, $s_{B 1}$, and $s_{B 2}$ be the market shares of the five choice alternatives, such that $s_{j}$ represents the proportion of consumers choosing alternative $j$ and $s_{0}+s_{A 1}+s_{A 2}+s_{B 1}+s_{B 2}=1$.

Q1.1. (5 points) Based on this model, write the equation for the market share $s_{A 1}$ as a function of the prices and the qualities $x$ 's of all the products.
Q1.2. (5 points) Obtain the expression for the derivatives: (a) $\frac{\partial s_{j}}{\partial p_{j}}$; and (b) $\frac{\partial s_{j}}{\partial p_{k}}$ for $j \neq k$. Write the expression for these derivatives in terms only of the market shares $s_{j}$ and $s_{k}$ and the parameters of the model.

The profit function of firm $A$ is $\pi_{A}=p_{A 1} q_{A 1}+p_{A 2} q_{A 2}-c_{A 1} q_{A 1}-c_{A 2} q_{A 2}-F C\left(x_{A 1}\right)-$ $F C\left(x_{A 2}\right)$, where: $q_{j}$ is the quantity sold by firm $j$ (i.e., $q_{j}=H s_{j}$ ); $c_{j}$ is the marginal cost of producing good $j$, that is assumed constant, i.e., linear cost function; and $F C\left(x_{j}\right)$ is the fixed cost of producing a good with quality $x_{j}$.

Q1.3. (20 points) Suppose that firms take the qualities $x$ of their products as given and compete in prices ala Bertrand.
(a) Obtain the two equations that describe the marginal condition of profit maximization of firm $A$ in this Bertrand game. Write this equation taking into account the specific form of $\frac{\partial s_{j}}{\partial p_{j}}$ and $\frac{\partial s_{j}}{\partial p_{k}}$ in the Logit model.
(b) Given this equation, write two equations for the equilibrium price-cost margins $P C M_{A 1} \equiv$ $p_{A 1}-c_{A 1}$ and $P C M_{A 2} \equiv p_{A 2}-c_{A 2}$, respectively. In these equations, the price cost margins are functions of $s_{A 1}, s_{A 2}$, and the demand parameter $\alpha$. [Hint: Solve a system of two linear equations and two unknowns where the unknowns are the price cost margins and $P C M_{A 1}$ and $P C M_{A 2}$ ].

Q1.4. (10 points) Suppose that products $A 1$ and $A 2$ were produced by two different firms that operate separately and maximize their respective profits. Answer the same Questions as in Q1.3(a) and Q1.3(B) but for these independent firms.

Q1.5. (10 points) Compare the expressions for the equilibrium price cost margins in Questions Q1.3(b) and Q1.4(b).
(a) Does the multi-product firm change higher or lower price-cost margins than the single product firm?
(b) Based on this result, explain in words the implications of multiproduct firms on prices, firms' profits, and consumer surplus.
(c) Suppose that there are economies of scope in the production of the two goods. Explain in words how the presence of economies of scope affect your answer to question (b).

Q1.6. (20 points) Suppose that firms A and B merge into a single firm that maximizes the total profits from the four products.
(a) Obtain the expression for the price cost margin of each of the four products.
(b) Under the assumption of no economies of scope, explain in words the implications of this merge on prices, profits, and consumer surplus.
(c) Suppose that there are economies of scope. Explain in words the implications on the welfare effects of a merger.

QUESTION 2. [80 points]. To answer the questions in this part of the problem set you need to use the dataset verboven_cars.dta Use this dataset to implement the estimations describe below. Please, provide the STATA code that you use to obtain the results. For all the models that you estimate below, impose the following conditions:

- For market size (number of consumers), use Population/4, i.e., pop/4
- Use prices measured in euros (eurpr).
- For the product characteristics in the demand system, include the characteristics: hp, li, wi, cy, le, and he.
- Include also as explanatory variables the market characteristics: $\log (\mathrm{pop})$ and $\log (\mathrm{gdp})$.
- In all the OLS estimations include fixed effects for market (ma), year (ye), and brand (brd).
- Include the price in logarithms, i.e., $\log$ (eurpr).
- Allow the coefficient for log-price to be different for different markets (countries). That is, include as explanatory variables the $\log$ price, but also the log price interacting (multiplying) each of the market (country) dummies except one country dummy (say the dummy for Germany) that you use as a benchmark.


## Q2.1. (10 points)

(a) Obtain the OLS-Fixed effects estimator of the Standard logit model. Interpret the results.
(b) Test the null hypothesis that all countries have the same price coefficient.
(c) Based on the estimated model, obtain the average price elasticity of demand for each country evaluated at the mean values of prices and market shares for that country.
(d) Interpret the results

Q2.2. (10 points) Consider the equilibrium condition (first order conditions of profit maximization) under the assumption that each product is produced by only one firm.
(a) Write the equation for this equilibrium condition. Write this equilibrium condition as an equation for the Lerner Index, $\frac{p_{j}-M C_{j}}{p_{j}}$.
(b) Using the previous equation in Q2.2(a) and the estimated demand in Q2.1, calculate the Lerner index for every car-market-year observation in the data.
(c) Report the mean values of the Lerner Index for each of the counties/markets. Comment the results.

## Q2.3. (20 points)

(a) Using the equilibrium condition and the estimated demand, obtain an estimate of the marginal cost for every car-market-year observation in the data.
(b) Run an OLS-Fixed effects regression where the dependent variable is the estimated value of the marginal cost, and the explanatory variables (regressors) are the product characteristics hp, li, wi, cy, le, and he. Interpret the results.

Q2.4. (40 points) Consider the equilibrium condition (first order conditions of profit maximization) taking into account that each firm is a multi-product firm.
(a) Consider one firm, say 1 , that sells products $1,2, \ldots, J_{1}$. Given our demand system here - that is different to the one in Question 1 - obtain the first order conditions of profit maximization for this firm. Use these conditions to obtain the following expression for the equilibrium value of the Lerner Index of a product $j$ :

$$
L_{j} \equiv \frac{p_{j}-M C_{j}}{p_{j}}=\frac{1}{\alpha}\left[1+\frac{1}{p_{j}} \frac{\sum_{k=1}^{J_{1}} s_{j} p_{j}}{1-\sum_{k=1}^{J_{1}} s_{j}}\right]
$$

[Hint: First, use f.o.c. for product j to obtain an equation that relates $L_{j}$ with the variable $\Sigma \equiv \sum_{k=1}^{J_{1}} L_{k} s_{k} p_{k}$. Second, aggregate this equation over products and solve for $\Sigma$ to obtain its equilibrium values. Finally, plug in this value of $\Sigma$ in the f.o.c. for product j]
(b) Using this equilibrium condition for multi-product firms, the estimated demand, data on prices and market shares, and the actual ownership structure of products by the different firms, calculate the Lerner-indexes, and the marginal costs for every car-market-year observation in the data.
(c) Compare the estimated Lerner indexes under the assumption of single-product and under the assumption of multi-product. Present an xy scatterplot of these two variables. Comment the results.
(d) Compare the estimated Marginal Costs under the assumption of single-product and under the assumption of multi-product. Present an xy scatterplot of these two variables. Comment the results.

