#### ECO 310: Empirical Industrial Organization Lecture 8: Models of Competition in Prices or Quantities: Introduction

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#### Outline on today's lecture

- 1. Introduction
- 2. Estimating Marginal Costs given a form of competition
  - 2.1. Perfect competition
  - 2.2. Cournot competition
  - 2.3. Bertrand competition: differentiated prod.
- 3. Estimating the form of competition when MCs are observed
- 4. Estimating the form of competition & MCs
  - 4.1. Homogeneous product model
  - 4.2. Differentiated product model

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# 1. Introduction

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#### Introduction

- Firms' decisions of how much to produce (or sell) and the price to charge are fundamental determinants of firms' profits.
- These decisions are main sources of strategic interactions between firms.
- In the market for an homogeneous good, the price depends on the total quantity produced by all the firms in the industry.
- With differentiated products, demand for a firm's product depends on the prices of products sold by other firms in the industry.
- These **strategic interactions** have first order importance to understand competition and outcomes in most industries.
- For this reason, models of competition where firms choose prices or quantities are at the core of Industrial Organization.

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## Equilibrium model of competition

- The answer to many economy questions require not only the estimation of demand and cost functions but also the explicit specification of an equilibrium model of competition.
- For instance, suppose that we are interested in measuring the effects of:
  - a merger
  - a sales tax
  - firms' collusion
  - the entry of a new firm or product in the market

- ...

• Answering these questions requires the explicit specification of a model of competition.

#### Empirical models of Price or Quantity competition

- We can distinguish three general classes of applications of empirical models of competition in prices or quantities.
- [1] Estimation of firms' marginal costs.
- [2] Identification of the "form of competition".
- [3] Joint identification of marginal costs and "form of competition"

#### Estimation of firms' marginal costs

- In many empirical applications, the researcher has information on firms' prices and quantities sold, but **information on firms' costs is not always available**.
- In this context, empirical models of competition in prices or quantities may provide an approach to obtain estimates of firms' marginal costs, and of the structure of these costs.
- Given an assumption about competition (e.g., Cournot, Bertrand, Stackelberg, Collusion), the model predicts that for every firm *i*,  $MR_i = MC_i$ , where the concept of  $MR_i$  depends on the assumption of the model of competition.
- Based on a estimation of demand, we can construct estimates of firms' *MR*. Then, the equilibrium conditions of the model imply and estimate of *MCs*.

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#### Identification of the "Nature of competition"

- Suppose that the researcher has data to estimate separately the demand function and firms' marginal costs (e.g., from the production function and firms' input prices).
- Given an assumption about the form or nature of competition in this industry (e.g., Perfect competition, Cournot, Collusion), the researcher can use the demand to obtain firms' marginal revenues,  $MR_i$ , and check if they are equal to the observed marginal costs,  $MC_i$
- That is, the researcher can test if a particular form of competition is consistent with the data.
- In this way, the researcher can find the form of competition that is more consistent the data, e.g., identify if there is evidence of firms' collusion.

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#### Joint identification of MCs and Nature of competition

- Suppose that the researcher does not have data on firms' MCs (or estimates of these MCs from production function).
- We will see that, under some conditions, it is still possible to use the estimated demand and equilibrium conditions to jointly identify firms' marginal costs and the form of competition in the market.
- This is the purpose of the conjectural variation approach.

#### Main References

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# 2.1. Estimating marginal costs given a form of competition: Perfect competition

#### Estimating MCs: Perfect competition

- We first illustrate this approach in the context of a **perfectly** competitive industry for an homogeneous product.
- The research has data on the market price and on firms' output for T periods of time (or geographic markets):

Dataset = { $p_t$ ,  $q_{it}$ : for  $i = 1, 2, ..., N_t$  & t = 1, 2, ..., T }

where  $N_t$  is the number of firms active at period t.

• The variable profit of firm *i* is:

$$\Pi_{it} = p_t \ q_{it} - C_i(q_{it})$$

• Under perfect competition, the marginal revenue of any firm *i* is the market price,  $p_t$ . Profit maximization implies:

$$p_t = MC_i(q_{it})$$
 for every firm *i*

where  $MC_{it} \equiv C'_i(q_{it})$ .

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## Estimating MCs: Perfect competition [2]

Suppose that:

$$MC_i(q_{it}) = q_{it}^{\theta} \exp{\{\varepsilon_{it}^{MC}\}}$$

where  $\theta$  is a technological parameter and  $\varepsilon_{it}^{MC}$  is an unobservable that captures the cost efficiency of a firm.

• (i) Constant Returns to Scale (CRS), i.e., constant marginal cost or  $\theta = 0$ ;

(ii) Decreasing Returns to Scale (DRS), i.e., increasing marginal cost or  $\theta > 0$ ;

(iii) Increasing Returns to Scale (IRS), i.e., decreasing marginal cost or  $\theta < 0.$ 

• Using the equilibrium condition, we can estimate  $\theta$  and the cost efficiency  $\varepsilon_{it}^{MC}$  of every firm *i*.

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# Estimating MCs: Perfect competition [3]

• The equilibrium condition  $p_t = MC_i(q_{it})$  implies the following regression model in logarithms:

$$\ln(p_t) = \theta \ \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- Using data on prices and quantities, we can estimate the slope parameter  $\theta$  in this regression equation.
- Given an estimate of  $\theta$ , we can estimate  $\varepsilon_{it}^{MC}$  as a residual from this regression, i.e.,  $\varepsilon_{it}^{MC} = \ln(p_t) \theta \ln(q_{it})$ .
- Therefore, we can estimate the marginal cost function of each firm,  $MC_i(q_{it}) = q_{it}^{\theta} \exp{\{\varepsilon_{it}^{MC}\}}.$

[4]

## Estimating MCs: Perfect competition

$$\ln(p_t) = \theta \ \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- Estimation of this equation by OLS suffers of an **Endogeneity problem**.
- The equilibrium condition implies that the less efficient firms (with larger value of  $\varepsilon_{it}^{MC}$ ) have a lower level of output.
- Therefore, the regressor  $\ln(q_{it})$  is negatively correlated with the error term  $\varepsilon_{it}^{MC}$ .
- This negative correlation between the regressor and the error term implies that the OLS estimator provides a downward biased estimate of the true θ, e.g., the OLS estimate can show IRS (i.e., θ < 0) when the true technology has DRS (i.e., θ > 0).

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# Estimating MCs: Perfect competition

• This **Endogeneity problem** does not disappear if we consider the model in market means:

$$\ln\left(\boldsymbol{p}_{t}\right) = \theta \,\,\overline{\ln q}_{t} + \overline{\varepsilon}_{t}^{MC}$$

[5]

where  $\overline{\ln q}_t$  and  $\overline{\varepsilon}_t^{MC}$  represents the means values of the variables  $\ln(q_{it})$  and  $\varepsilon_{it}^{MC}$  over all the firms active at period t.

We still have that ln q<sub>t</sub> and ε<sub>t</sub><sup>MC</sup> are negatively correlated:

 in time periods with larger aggregate cost shocks ε<sub>t</sub><sup>MC</sup> there is lower average log-output ln q<sub>t</sub>.

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#### Estimating MCs: Perfect competition

$$\ln\left(\boldsymbol{p}_{t}\right) = \theta \,\,\overline{\ln q}_{t} + \overline{\varepsilon}_{t}^{MC}$$

- We can deal with this endogeneity problem by using **instrumental variables**.
- Suppose that  $X_t^D$  is a vector of observable variables that affect demand. These variables should be correlated with  $\overline{\ln q}_t$  because demand shocks affect firms' output decisions.
- Under the assumption that these observable demand variables  $X_t^D$  are not correlated with  $\overline{\varepsilon}_t^{MC}$ , we can use these variables as instruments for  $\overline{\ln q_t}$  for the consistent estimation of  $\theta$ .

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# 2.2. Estimating marginal costs given a form of competition: Cournot competition

#### Estimating MCs: Cournot competition

- We still have an homogeneous product industry and a researcher with data on quantities and prices over T periods of time:  $\{p_t, q_{it}\}$  for  $i = 1, 2, ..., N_t$  and t = 1, 2, ..., T.
- But now, the researcher assumes that the market is not perfectly competitive and that firms compete a la Nash-Cournot.
- The variable profit of firm *i* is  $\Pi_{it} = p_t q_{it} C_i(q_{it})$ .
- The demand can be represented using the inverse demand function,

$$p_t = P\left(Q_t, X_t^D\right)$$

where  $Q_t \equiv \sum_{i=1}^{N} q_{it}$  is the market total output, and  $X_t^D$  is a vector of exogenous market characteristic that affect demand.

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#### Estimating MCs: Cournot competition

- Each firm chooses its own output  $q_{it}$  to maximize profit.
- Since profit is equal to revenue minus cost, profit maximization implies the condition of marginal revenue equal to marginal cost.
- The marginal revenue function is:

$$\begin{aligned} MR_{it} &= \frac{d(p_t \ q_{it})}{dq_{it}} = p_t + \frac{dp_t}{dq_{it}}q_{it} \\ &= p_t + P_Q'\left(Q_t, X_t^D\right) \left[1 + \frac{dQ_{(-i)t}}{dq_{it}}\right]q_{it} \end{aligned}$$

where:

 $P'_Q(Q_t, X_t^D)$  is the derivative of the inverse demand function with respect to total output;

 $Q_{(-i)t}$  is the aggregate output of firms other than *i*.

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## Estimating MCs: Cournot competition [3]

$$MR_{it} = p_t + P'_Q \left(Q_t, X^D_t\right) \left[1 + \frac{dQ_{(-i)t}}{dq_{it}}\right] q_{it}$$

- $\frac{dQ_{(-i)t}}{dq_{it}}$  represents the **belief** or **conjecture** that firm *i* has about how other firms will respond by changing their output when this firm changes marginally its own output.
- Under the assumption of Nash-Cournot competition, this *belief* or *conjecture is zero:*

$$Nash - Cournot \Leftrightarrow rac{dQ_{(-i)t}}{dq_{it}} = 0$$

• Firm *i* takes as fixed the quantity produced by the rest of the firms,  $Q_{(-i)t}$ , and chooses his own output  $q_{it}$  to maximize his profit.

[4]

# Estimating MCs: Cournot competition

• Therefore, the first order condition of optimality under Nash-Cournot competition is:

$$MR_{it} = p_t + P_Q'\left(Q_t, X_t^D
ight) \ q_{it} = MC_i(q_{it})$$

- Since P'<sub>Q</sub> (Q<sub>t</sub>, X<sup>D</sup><sub>t</sub>) < 0 (downward sloping demand curve), it is clear that MR<sub>it</sub> < p<sub>t</sub>.
- Therefore, if the marginal cost  $MC_i(q_{it})$  is a non-decreasing function, we have that the optimal amount of output  $q_{it}$  under Cournot is smaller than under perfect competition.
- Oligopoly competition reduces output and consequently increases price.

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# Estimating MCs: Cournot competition

- Consider the same specification of the cost function as before, with  $MC_i(q_{it}) = q_{it}^{\theta} \exp{\{\varepsilon_{it}^{MC}\}}.$
- Suppose that the demand function has been estimated in a fist step, such that there is a consistent estimate of the demand function.
- The researcher can construct consistent estimates of marginal revenues  $MR_{it} = p_t + P'_O(Q_t, X^D_t) q_{it}$  for every firm *i*.

Then, the econometric model can be described in terms of the following linear regression model in logarithms:

$$\ln(MR_{it}) = \theta \ \ln(q_{it}) + \varepsilon_{it}^{MC}$$

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## Estimating MCs: Cournot competition

$$\ln(MR_{it}) = \theta \ \ln(q_{it}) + \varepsilon_{it}^{MC}$$

- OLS estimation of this regression function suffers of the same endogeneity problem as in the perfect competition case.
- To deal with this endogeneity problem, we can use instrumental variables.
- As in the case of perfect competition, we can use observable variables that affect demand but not costs,  $X_t^D$ , as instruments.
- In the case of Cournot competition we can have additional types of instruments.

# Estimating MCs: Cournot competition [7]

- Suppose that the researcher observes also some exogenous characteristics of firms that affect the marginal cost.
- For instance, suppose that there is information at the firm level on the firm's wage rate, or its capital stock, or its installed capacity.
- Let us represent these variables using the vector Z<sub>it</sub>.

Therefore, the marginal cost function is now  $MC_i(q_{it}) = q_{it}^{\theta} \exp\{Z_{it}\gamma + \varepsilon_{it}^{MC}\}$ , where  $\gamma$  is a vector of parameters. The marginal condition of optimality, in logarithms, becomes:

$$\ln (MR_{it}) = \theta \ \ln(q_{it}) + Z_{it} \ \gamma + \varepsilon_{it}^{MC}$$

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# Estimating MCs: Cournot competition

$$\ln \left( MR_{it} \right) = \theta \ \ln(q_{it}) + Z_{it} \ \gamma + \varepsilon_{it}^{MC}$$

- Note that the characteristics Z<sub>it</sub> of firms j other than i have an effect on the equilibrium amount of output of a firm *i*.
- The smaller Z<sub>it</sub> the more cost efficient firm j, the larger its output, the smaller price  $p_t$  and the marginal revenue  $MR_{it}$ , and the smaller  $q_{it}$  for any firm *i* other than *j*.
- Under the assumption that the vector of firm characteristics in Z are exogenous, i.e.,  $E(Z_{it} \ \varepsilon_{it}^{MC}) = 0$  for any (i, j), we can use the characteristics  $Z_{it}$  of other firms as instrumental variables.

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[9]

### Estimating MCs: Cournot competition

• For instance, we can use  $\sum_{j \neq i} Z_{jt}$  as an instrumental variables, and estimate  $\theta$  and  $\gamma$  using the moment conditions:

$$E\left(\left[\begin{array}{c}Z_{it}\\\sum_{j\neq i}Z_{jt}\end{array}\right] \left[\ln\left(MR_{it}\right)-\theta \ \ln(q_{it})-Z_{it} \ \gamma\right]\right)=\mathbf{0}$$

• Or equivalently, using a 2SLS estimator.

# 2.3. Estimating marginal costs given a form of competition: Bertrand with diff. product

#### Estimating MCs: Bertrand with diff. prod.

- Consider the industry of a differentiated product.
- The researcher has data on prices, quantities, and product characteristics for the J products in the industry, where J is large: {p<sub>i</sub>, q<sub>i</sub>, X<sub>i</sub>} for i = 1, 2, ..., J.
- For the moment, we consider that each product is produced by only one firm and each firm produces only one product.
- The profit of firm *i* is  $\Pi_i = p_i q_i C_i(q_i)$ .

[2]

#### Estimating MCs: Bertrand with diff. prod.

• The demand system comes from a discrete choice model of demand:

$$q_i = H s_i = H \sigma_i(\mathbf{p}, \mathbf{X})$$

- *H* is the number of consumers in the market, s<sub>i</sub> is the market share of product, i.e., s<sub>i</sub> ≡ q<sub>i</sub>/H.
- σ<sub>i</sub>(**p**, **X**) is the market share function in the demand model, and **p** and **X** are the vectors of prices and characteristics.
- For instance, under a logit demand system we have that,

$$\sigma_i(\mathbf{p}, \mathbf{X}) = \frac{\exp\left\{-\alpha \ p_i + X_i \ \beta\right\}}{1 + \sum_{j=1}^J \exp\left\{-\alpha \ p_j + X_j \ \beta\right\}}$$

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#### Estimating MCs: Bertrand with diff. prod.

- Under Bertrand competition, each firm chooses its price p<sub>i</sub> to maximize its profit.
- The marginal condition of optimality implies that  $\frac{d\Pi_i}{dp_i} = 0$ , or equivalently,  $\frac{d(p_iq_i)}{dp_i} = \frac{dC_i(q_i)}{dp_i}$ .
- Note that profit Π<sub>i</sub> depends on price p<sub>i</sub> both directly and indirectly through q<sub>i</sub>. Then we have that

$$rac{d(p_i q_i)}{dp_i} = q_i + p_i rac{dq_i}{dp_i}$$

And

$$\frac{dC_i(q_i)}{dp_i} = MC_i(q_i) \frac{dq_i}{dp_i}$$

# Estimating MCs: Bertrand with diff. prod. [4]

• Combining these equations,  $\frac{d(p_i q_i)}{dp_i} = \frac{dC_i(q_i)}{dp_i}$ , we have:

$$MR_i = p_i + \frac{q_i}{dq_i/dp_i} = MC_i(q_i)$$

• And taking into account that  $q_i = H \ s_i = H \ \sigma_i(\mathbf{p}, \mathbf{X})$ :

$$MR_i = p_i + \frac{s_i}{d\sigma_i/dp_i} = MC_i(q_i)$$

- The term  $\frac{s_i}{d\sigma_i/dp_i}$  is negative. Therefore,  $\frac{-s_i}{d\sigma_i/dp_i}$  is the price-cost margin  $p_i MC_i(q_i)$  in equilibrium.
- For instance, for the Logit demand system, we have that  $d\sigma_i/dp_i = -\alpha \ s_i(1-s_i)$ , such that:

$$p_i - rac{1}{lpha(1-s_i)} = MC_i(q_i)$$

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[5]

#### Estimating MCs: Bertrand with diff. prod.

• In general, we have that the marginal revenue,

$$MR_i = p_i + rac{s_i}{d\sigma_i/dp_i}$$

only depends on  $p_i$ ,  $s_i$ , and the demand function  $\sigma_i(\mathbf{p}, \mathbf{X})$ .

• After estimating the demand function, the researcher knows (or has estimates) of the marginal revenues *MR<sub>i</sub>* for every firm/product in the market.

#### Estimating MCs: Bertrand with diff. prod. [6]

- Suppose that the marginal cost function is  $MC_i(q_{it}) = q_{it}^{\theta} \exp\{X_{it} + \gamma + \varepsilon_{it}^{MC}\}$ .
- The marginal cost of producing a product depends on the characteristics of this product.
- Suppose that the demand function has been estimated in a fist step, such that there is a consistent estimate of the demand function.
- Then, the econometric model is:

$$\ln(MR_{it}) = \theta \ \ln(q_{it}) + X_{it} \ \gamma + \varepsilon_{it}^{MC}$$

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# Estimating MCs: Bertrand with diff. prod. [7]

$$\ln\left(MR_{it}\right) = \theta \ \ln(q_{it}) + X_{it} \ \gamma + \varepsilon_{it}^{MC}$$

- OLS estimation of this regression function suffers of the same endogeneity problem as in the PC or Cournot.
- To deal with this endogeneity problem, we can use instrumental variables.
- We can use the characteristics of products other than *i*, X<sub>jt</sub> j ≠ i, as instruments.

$$E\left(\left[\begin{array}{c}X_{it}\\\sum_{j\neq i}X_{jt}\end{array}\right]\left[\ln\left(MR_{it}\right)-\theta\ \ln(q_{it})-X_{it}\ \gamma\right]\right)=\mathbf{0}$$

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