Empirical Industrial Organization: Models, Methods, and Applications

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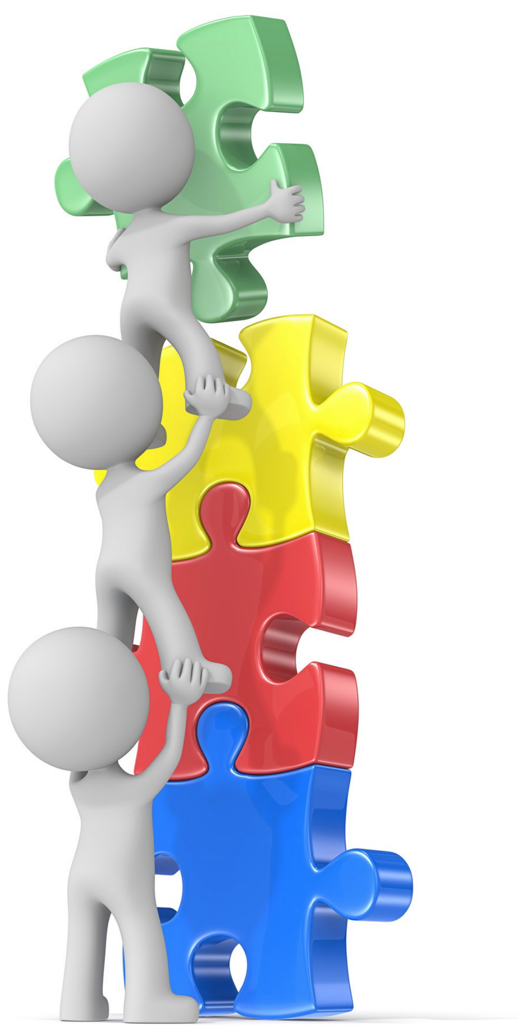


Contents

CHAPTER 4

Static Models of Competition in Prices and Quantities

1. Introduction

In most industries, the decisions of how much to produce and the price to charge are fundamental determinants of the firm's profit. These decisions are also main sources of strategic interactions between firms. In the market for an homogeneous good, the market price of the product depends on the total quantity produced by all the firms in the industry. In a differentiated product industry, demand for a firm's product depends on the prices of products sold by other firms in the industry. This type of strategic interactions have first order importance to understand competition and outcomes in most industries. For this reason, models of competition where firms choose prices or quantities are at the core of Industrial Organization.

The answer to many economy questions in IO require not only the estimation of demand and cost functions but also the explicit specification of an equilibrium model of competition. For instance, evaluating the effects of a merger, or a sales tax, or the entry in the market of a new product, require the explicit specification of a model of competition.

From an empirical point view, there are several purposes in the estimation of models of competition in prices or quantities. In many empirical applications, the researcher has information on firms' prices and quantities sold, but information on firms' costs is not always available. The researcher may not observe even the amounts of firms' inputs, such that it is not possible to obtain costs by estimating the production function as described in chapter 3. In this context, empirical models of competition in prices or quantities may provide an approach to obtain estimates of firms' marginal costs and the structure of the marginal cost function. Given an assumption about competition (for instance, perfect competition, Cournot, Bertrand, Stackelberg, collusion), the model predicts that a firm's marginal cost should be equal to a particular (model specific) marginal revenue. This is the key condition that is used to estimate firms' marginal costs in this class of models. Typically, the first step in the econometric analysis of these models consists in the estimation of the demand function or demand system. Given the estimated demand, we can construct an estimate of the realized marginal revenue for every observation in the sample. This measure of marginal revenue provides, directly, an estimate of the realized marginal cost at each sample observation.

Finally, we use this sample of realized marginal costs to estimate the marginal cost function, and in particular how the marginal cost depends on firm's output of different products (that is, economies of scale and scope), and possibly on other firm's characteristics such as historical cumulative output (that is, learning by doing), installed capacity, or geographic distance between the firm's production plants (that is, economies of density).

The value of a firm's marginal revenue depends on the form of competition in the industry, or *the nature of competition*. Given the same demand function, the marginal revenue is different under perfect competition, Cournot, Bertrand, or collusion. The researcher's selection of a model of competition typically implies answers the following choices: (a) is the product homogeneous or differentiated; (b) do firms compete in prices or in quantities?; (c) is there collusion between some or all the firms in the industry?; and (d) what does a firm believe about the behavior of other firms in the market? For instance, if the researcher assumes that the product is homogenous, firms compete in quantities, there is no collusion in the industry, and firms choose their levels of output under the belief that the other firms will not change their respective output levels (that is, Nash assumption), then the form of competition is the one in the Cournot model. In principle, some of these assumptions may be supported by the researcher's knowledge of the industry. However, in general, some of these assumptions are difficult to justify. Ideally. we would like to learn from our data about the nature of competition. Suppose that the researcher has data on firms' marginal costs (or estimates of these costs based on a production function) and an estimation of the demand system. Then, given an assumption about the form of competition in this industry (for instance, perfect competition, Cournot, collusion), the researcher can use the demand to obtain firms' marginal revenues and check whether they are equal to the observed marginal costs. That is, the researcher can test if a particular form of competition is consistent with the data. In this way, it is possible to find the form of competition that is consistent with the data, for instance, identify if there is evidence of collusive behavior. We will see in this chapter that, even if the researcher does not have data on firms' costs, it is still possible to combine the demand system and the equilibrium conditions to jointly identify marginal costs and the *nature of competition* in the industry. This is the main purpose of the so called *conjectural variation approach*.

In this chapter, we describe the specification and estimation of empirical models of Cournot competition in an homogenous product industry, Bertrand competition in a differentiated product industry, and the conjectural variation approach both in homogenous and differentiated product industries.

2. Homogenous product industry

2.1. Estimating marginal costs from equilibrium conditions. First, we consider the situation where the researcher does not have direct measures of marginal costs and uses the equilibrium conditions to estimate these costs.

2.1.1. *Perfect competition.* We first illustrate this approach in the context of a perfectly competitive industry for an homogeneous product. Suppose that the researcher knows, or is willing to assume, that the industry under study is perfectly competitive, and she has data on the market price and on firms' output for T periods of time (or T geographic markets) that we index by t. The dataset consists of $\{p_t, \, q_{it}\}$ for $i = 1, 2, ..., N_t$ and $t = 1, 2, ..., T$, where N_t is the number of firms active at period t. The variable profit of firm i is $p_t \; q_{it} - C_i(q_{it})$. Under perfect competition, the marginal revenue of any firm i is the market price, p_t. The marginal condition of profit maximization for firm i is $p_t = MC_i(q_{it})$ where $MC_i(q_{it})$ is the marginal cost, $MC_i(q_{it}) \equiv C_i'(q_{it})$. Under perfect competition, all the firms should have the same marginal costs. This is a clear testable restriction of the assumption of perfect competition with homogeneous product.

Consider a particular specification of the cost function. With a Cobb-Douglas production function, we have that:

$$MC_i(q_{it}) = q_{it}^{\theta} \; W_{1it}^{\alpha_1} \; ...W_{Jit}^{\alpha_J} \; \exp\{\varepsilon_{it}^{MC}\} \tag{2.1}$$

W_{jit} is the price of variable input j for firm i, and α's are technological parameters in the Cobb-Douglas production function. ε_{it}^{MC} is an unobservable to the researcher that captures the cost (in)efficiency of a firm and that depends on the firm's total factor productivity, unobserved input prices and unobserved fixed inputs. The technological parameter θ is equal to $\frac{1}{\alpha_V} - 1$, and α_V is the sum of the Cobb-Douglas coefficients of all the variable inputs. Therefore, the equilibrium condition $p_t = MC_i(q_{it})$ implies the following regression model in logarithms:

$$\ln(p_t) = \theta \; \ln(q_{it}) + \alpha_1 \; \ln(W_{1it}) + ... + \alpha_J \; \ln(W_{Jit}) + \varepsilon_{it}^{MC} \tag{2.2}$$

We can distinguish three cases for parameter θ. Constant Returns to Scale (CRS): $\alpha_V = 1$ such that $\theta = 0$ and this implies that the marginal cost function is a constant function. Decreasing (Increasing) Returns to Scale: $\alpha_V < 1$ ($\alpha_V > 1$) such that $\theta > 0$ ($\theta < 0$) and the log-marginal cost function is an increasing (decreasing) linear function of log-output. The

Using data on prices and quantities, we can estimate the slope parameter θ in this regression equation. Given estimates of the parameters θ and α's we can estimate ε_{it}^{MC} as a residual from this regression. Therefore, we can estimate the marginal cost function function of each firm. Since the dependent variable of the regression, $\ln(p_t)$, is constant over firms, then, by construction, firms that produce more are more cost-efficient according to the term $\alpha_1 \ln(W_{1it}) + ... + \alpha_J \ln(W_{Jit}) + \varepsilon_{it}^{MC}$.

Estimation of equation (2.2) by OLS suffers of an endogeneity problem. The equilibrium condition implies that firms with a large value of ε_{it}^{MC} are less cost-efficient and, and all else equal, should have a lower level of output. Therefore, the regressor $\ln(q_{it})$ is negatively correlated with the error term ε_{it}^{MC}. This negative correlation between the regressor and the error term implies that the OLS estimator provides a downward biased estimate of the true θ. For instance, the OLS estimate could show increasing returns to scale, $\theta < 0$, when the true technology has decreasing returns to scale, $\theta > 0$. This endogeneity problem does not disappear if we consider the model in market means.

We can deal with this endogeneity problem by using instrumental variables. Suppose that X_t^D is an observable variable (or vector of variables) that affects the demand of the product but not the marginal costs of the firms. The equilibrium of the model implies that these demand variables should be correlated with firms' outputs, $\ln(q_{it})$. Of course, this condition is testable. Under the assumption that these observable demand variables X_t^D are not correlated with the unobserved term in the marginal cost, $\mathbb{E}(X_t^D \ \varepsilon_{it}^{MC}) = 0$, we can use these variables as instruments for log-output in the regression equation (**??**) for the consistent estimation of θ.

2.1.2. *Cournot competition.* Now, suppose that the researcher assumes that the market is not perfectly competitive and that firms compete a la Nash-Cournot. The demand can be represented using the inverse demand function $p_t = P\left(Q_t, X_t^D\right)$, where $Q_t \equiv \sum_{i=1}^N q_{it}$ is the market total output, and X_t^D is a vector of exogenous market characteristic that affect demand. Each firm chooses its own output q_{it} to maximize profit. Profit maximization implies the condition of marginal revenue equal to marginal cost, and the marginal revenue function is:

$$MR_{it} = p_t + P_Q'\left(Q_t, X_t^D\right) \left[1 + \frac{dQ_{(-i)t}}{dq_{it}}\right] q_{it} \tag{2.3}$$

$P_Q'\left(Q_t, X_t^D\right)$ is the derivative of the inverse demand function with respect to total output. Variable $Q_{(-i)t}$ is the aggregate output of firms other than i, and the derivative $\dfrac{dQ_{(-i)t}}{dq_{it}}$ represents the *belief* or *conjecture* that firm i has about how other firms will respond by changing their output when this firm changes marginally its own output. Under the assumption of Nash-Cournot competition, this *belief* or *conjecture is zero:*

$$Nash - Cournot \Leftrightarrow \frac{dQ_{(-i)t}}{dq_{it}} = 0 \tag{2.4}$$

Firm i takes as fixed the quantity produced by the rest of the firms, $Q_{(-i)t}$, and chooses her own output q_{it} to maximize her profit. Therefore, the first order condition of optimality under Nash-Cournot competition is:

$$MR_{it} = p_t + P'_Q\left(Q_t, X_t^D\right)\ q_{it} = MC_i(q_{it}) \tag{2.5}$$

We assume that the profit function is globally concave in q_{it} for any positive value of $Q_{(-i)t}$ such that there is a unique value of q_{it} that maximizes the firm's profit, and it is fully characterized by the marginal condition of optimality that establishes that marginal revenue equals marginal cost.

Consider the same specification of the cost function as before. Suppose that the demand function has been estimated in a first step such that there is a consistent estimate of the demand function. Therefore, the researcher can construct consistent estimates of marginal revenues $p_t + P'_Q\left(Q_t, X_t^D\right)\ q_{it}$ for every firm i. Then, the econometric model can be described in terms of the following linear regression model in logarithms:[1]

$$\ln\left(MR_{it}\right) = \theta\ \ln(q_{it}) + \alpha_1\ \ln(W_{1it}) + ... + \alpha_J\ \ln(W_{Jit}) + \varepsilon_{it}^{MC} \tag{2.6}$$

We are interested in the estimation of the parameter θ and α's and of the firms' relative efficiency, ε_{it}^{MC}.

OLS estimation of this regression function suffers of the same endogeneity problem as in the perfect competition case described above. The model implies that $E(\ln(q_{it})\ \varepsilon_{it}^{MC}) \neq 0$, and more specifically there is a negative correlation between a firm's output and its unobserved inefficiency. To deal with this endogeneity problem, we can use instrumental variables. As in the case of perfect competition, we can use observable variables that affect demand but not costs, X_t^D, as instruments. With Cournot competition, we may have additional types of instruments.

Suppose that the researcher observes some exogenous input prices $W_{it} = (W_{1it}, ..., W_{Jit})$ and that at least one of these prices has cross-sectional variation over firms. For instance, suppose that there is information at the firm level on the firm's wage rate, or its capital stock, or its installed capacity. Note that, in equilibrium the input price of the competitors have an effect on the level of output of a firm. That is, given its own input prices W_{it}, log-output $\ln(q_{it})$ still depends on the input prices of other firms competing in the market, W_{jt} for $j \neq i$. A firm's output increases if, all else equal, the wage rates of a competitor increases. Note that the partial correlation between W_{jt} and $\ln(q_{it})$ is a testable condition. Under the assumption that the vector W_{jt} is exogenous, that is, $E(W_{jt}\ \ \varepsilon_{it}^{MC}) = 0$, a natural approach to estimate this model is using IV or GMM based on moment conditions that use the characteristics of other firms as an instrument for output. For instance, the moment

[1] For notational simplicity, here I omit the estimation error from the estimation of the demand function in the first step. Note that, in this case, this estimation error only implies measurement error in the dependent variable and it does not affect the consistency of the instrumental variables estimator described below or the estimation of robust standard errors.

conditions can be:

$$E\left(\left[\begin{array}{c}\ln(W_{it})\\ \sum_{j\neq i}\ln(W_{jt})\end{array}\right]\ \left[\ln\left(MR_{it}\right)-\theta\ \ln(q_{it})-W_{it}\ \alpha\right]\right)=\mathbf{0} \tag{2.7}$$

2.2. Identification of the nature of competition: Conjectural variation model.

2.2.1. *Model.* Consider an industry where, at period t, the inverse demand curve is $p_t = P\left(Q_t, X_t^D\right)$, and firms, indexed by i, have cost functions $C_i(q_{it})$. Every firm i, chooses its amount of output, q_{it}, to maximize its profit, $p_t\ q_{it} - C_i(q_{it})$. Without further assumptions, the marginal condition for the profit maximization of a firm is marginal revenue equal to marginal cost, where the marginal revenue of firm i is:

$$MR_{it} = p_t + P_Q'\left(Q_t\ X_t^D\right)\ \left[1+\frac{\partial Q_{(-i)t}}{\partial q_{it}}\right]\ q_{it} \tag{2.8}$$

As mentioned above, the term $\dfrac{\partial Q_{(-i)t}}{\partial q_{it}}$ represents the **belief** that firm i has about how the other firms in the market will respond if she changes its own amount of output marginally. We denote this **conjecture** or **belief** as the *conjectural variation* of firm i at period t, and denote it as $CV_{it} \equiv \dfrac{\partial Q_{(-i)t}}{\partial q_{it}}$.

As researchers, we can consider different assumptions about firms' beliefs or conjectural variations. Different assumptions on CVs imply different models of competition with their corresponding equilibrium outcomes. John Nash (1951) proposed the following conjecture: when a player constructs her best response, she believes that the other players will not response to a change in her decision. In the Cournot model, Nash conjecture implies that $CV_{it} = 0$. For every firm i, the "perceived" marginal revenue is $MR_{it} = p_t + P_Q'\left(Q_t\ X_t^D\right)$ q_{it}, and the condition $p_t + P_Q'\left(Q_t\ X_t^D\right)\ q_{it} = MC_i(q_{it})$ implies the Cournot equilibrium.

There are CVs that generate the perfect competition equilibrium and the collusive or cartel equilibrium.

Perfect competition. For every firm i, $CV_{it} = -1$. Note that this conjecture implies that: $MR_{it} = p_t + P_Q'\left(Q_t\ X_t^D\right)\ [1-1]\ q_{it} = p_t$, and the conditions $p_t = MC_i(q_{it})$ imply the perfect competition equilibrium.

Collusion (Cartel). For every firm i, $CV_{it} = N_t - 1$. This conjecture implies, $MR_{it} = p_t + P_Q'\left(Q_t\ X_t^D\right)\ N_t\ q_{it}$, that generates that equilibrium conditions $p_t + P_Q'\left(Q_t\ X_t^D\right)\ N_t$ $q_{it} = MC_i(q_{it})$. When firms have constant and homogeneous MCs, this condition implies $p_t + P_Q'\left(Q_t\ X_t^D\right)\ Q_t = MC_t$, which is the equilibrium condition for the Monopoly (collusive or cartel) outcome.

The value of the beliefs / CV parameters are related to the nature of competition.

$$\begin{aligned}
\text{Perfect competition:} &\qquad CV_{it} = -1; \quad MR_{it} = p_t \\
\text{Nash-Cournot:} &\qquad CV_{it} = 0; \quad MR_{it} = p_t + P_Q'(Q_t)\ q_{it} \\
\text{Cartel all firms:} &\quad CV_{it} = N_t - 1; \quad MR_{it} = p_t + P_Q'(Q_t)\ Q_t
\end{aligned} \tag{2.9}$$

Given this result, one can argue that CV is closely related to the nature of competition, and therefore with equilibrium price and quantities. If CV is negative, the degree of competition is stronger than Cournot. The closer to -1, the more competitive. If CV is positive, the degree of competition is weaker than Cournot. The closer to $N_t - 1$, the less competitive.

Interpreting CV_{it} as an exogenous parameter is not correct. Conjectural variations represent firms' beliefs, and as such they are **endogenous outcomes** from the model.

2.2.2. *Estimation with information on marginal costs.* Consider an homogeneous product industry and a researcher with data on firms' quantities and marginal costs, and market prices over T periods of time: $\{p_t, MC_{it}, q_{it}\}$ for $i = 1, 2, ..., N_t$ and $t = 1, 2, ..., T$. Under the assumption that every firm chooses the amount of output that maximizes its profit given its belief CV_{it}, we have that the following condition holds:

$$p_t + P_Q'\left(Q_t\ X_t^D\right)\ \ [1 + CV_{it}]\ q_{it} = MC_{it} \tag{2.10}$$

And solving for the conjectural variation, we have:

$$CV_{it} = \frac{p_t - MC_{it}}{-P_Q'\left(Q_t\ X_t^D\right)\ q_{it}} - 1 = \left[\frac{p_t - MC_{it}}{p_t}\right]\ \left[\frac{1}{q_{it}/Q_t}\right]\ |\eta_t| - 1 \tag{2.11}$$

where η_t is the demand elasticity. Note that $\frac{p_t - MC_{it}}{p_t}$ is the Lerner index and q_{it}/Q_t is the market share of firm i. This equation shows that, given data on quantities, prices, demand and marginal costs, we can identify the firms' beliefs that are consistent with these data and with profit maximization. Let us denote $\left[\frac{p_t - MC_{it}}{p_t}\right]\ \left[\frac{1}{q_{it}/Q_t}\right]$ as the Lerner-index-to-market-share ratio of a firm. If the Lerner-index-to-market-share ratios are close zero, then the estimated values of CV will be close to -1 unless the absolute demand elasticity is large. In contrast, if the Lerner-index-to-market-share ratios are large (that is, larger than the inverse demand elasticity), then estimated CV values will be greater than zero, and can reject the hypothesis of Cournot competition in favor of collusion.

Under the restriction that all the firms have the same marginal costs and conjectural variations, equation (2.11) that relates the Lerner index with the conjectural variation becomes:

$$\frac{p_t - MC_t}{p_t} = \left[\frac{1 + CV_t}{N_t}\right]\ \frac{1}{|\eta_t|} \tag{2.12}$$

where N_t is the number of firms in the market. This is the equation that we use in the empirical application that we describe at the end of this section. According to this expression, market power, as measured by the Lerner Index, depends on the elasticity of demand (negatively), the number of firms in the market (negatively), and the conjectural variation (positively).

2.2.3. *Estimation without information on marginal costs.* So far, we have considered the estimation of CV parameters when the researcher knows both demand and firms' marginal costs. We now consider the case where the researcher knows the demand, but it does not know firms' marginal costs. Identification of CVs requires also de identification of marginal costs. Under some conditions, we can jointly identify CVs and MCs using the marginal conditions of optimality and the demand.

The researcher observes data $\left\{p_t,\ q_{it},\ X_t^D,\ X_t^{MC} :\ i = 1, ...N_t; t = 1, ..., T\right\}$, where X_t^D are variables affecting consumer demand, for instance, average income, population, and W_t are variables affecting marginal costs, for instance, some input prices. Consider the linear (inverse) demand equation:

$$p_t = \alpha_0 + \alpha_1\ X_t^D - \alpha_2\ Q_t + \varepsilon_t^D \tag{2.13}$$

with $\alpha_2 \geq 0$, and ε_t^D is unobservable to the researcher. Consider the marginal cost function:

$$MC_{it} = \beta_0 + \beta_1\ W_t + \beta_2\ q_{it} + \varepsilon_{it}^{MC} \tag{2.14}$$

with $\beta_2 \geq 0$, and ε_{it}^{MC} is unobservable to the researcher. Profit maximization implies $p_t + \frac{dP_t}{dQ_t}\left[1 + CV_{it}\right] q_{it} = MC_{it}$. Since the demand function is linear and $\dfrac{dP_t}{dQ_t} = -\alpha_2$, we have:

$$p_t = \beta_0 + \beta_1\ W_t + \left[\beta_2 + \alpha_2(1 + CV_{it})\right]\ q_{it} + \varepsilon_{it}^{MC} \tag{2.15}$$

This equation describes the marginal condition for profit maximization. We assume now that $CV_{it} = CV$ for every observation i, t in the data. The structural equations of the model are the demand equation in (2.13) and the equilibrium condition in (2.15).

Using this model and data, can we identify (that is, estimate consistently) the CV parameter? For the structural model described by equations (2.13) and (2.15), the answer to this question is negative. However, we will see that a simple modification of this model implies separate identification of CV and MC parameters. We first describe the identification problem.

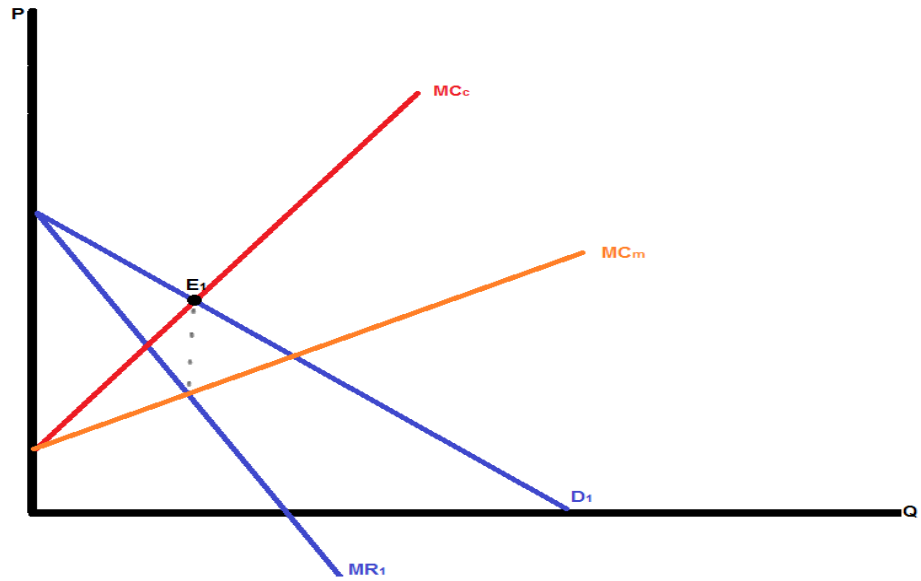
Identification of demand parameters. The estimation of the regression equation for the demand function needs to deal with the well-known simultaneity problem. In equilibrium, output Q_t is correlated with the error term ε_t^D. The model implies a valid instrument to estimate demand. In equilibrium, Q_t depends on the exogenous cost variable W_t. This variable does not enter in the demand equation. If W_t is not correlated with ε_t^D, then this variable satisfies all the conditions for being a valid instrument. Parameters α_0, α_1, and α_2 are identified using this IV estimator.

Identification of CV and MCs. In the regression equation (2.15) we also need to deal with a simultaneity problem. In equilibrium, output q_{it} is correlated with the error term ε_{it}^{MC}. The model implies a valid instrument to estimate this equation. In equilibrium, q_{it} depends on the exogenous demand shifter X_t^D. Note that X_t^D does not enter in the marginal cost and in the right hand side of the regression equation (2.15). If X_t^D is not correlated with ε_{it}^{MC}, then this variable satisfies all the conditions for being a valid instrument such that the parameters β_0, β_1, and $\gamma \equiv \beta_2 + \alpha_2(1 + CV)$ are identified using this IV estimator.

Note that we can identify the parameter $\gamma \equiv \beta_2 + \alpha_2(1 + CV)$ and the slope of inverse demand function, α_2. However, knowledge of γ and α_2 is not sufficient to identify separately CV and the slope of the MC, β_2. Given estimated values for γ and α_2, equation $\gamma = \beta_2 + \alpha_2(1 + CV)$ implies a linear relationship between CV and β_2 and there are infinite values of these parameters that satisfy this restriction. Even we restrict CV to belong to the values with a clear economic interpretation, such that $CV \in \{-1, 0, N-1\}$ and β_2 to greater or equal than zero, we do not have identification of these parameters. For instance, suppose that $N = 2$, $\gamma = 2$, and $\alpha_2 = 1$ such that we have the constraint $2 = \beta_2 + (1 + CV)$ or equivalently, $\beta_2 + CV = 1$. This equation is satisfied by any of the following forms of competition and values of $\beta_2 \geq 0$. Perfect competition: $CV = -1$ and $\beta_2 = 2$. Cournot competition: $CV = 0$ and $\beta_2 = 1$. And perfect collusion: $CV = N - 1 = 1$ and $\beta_2 = 0$.

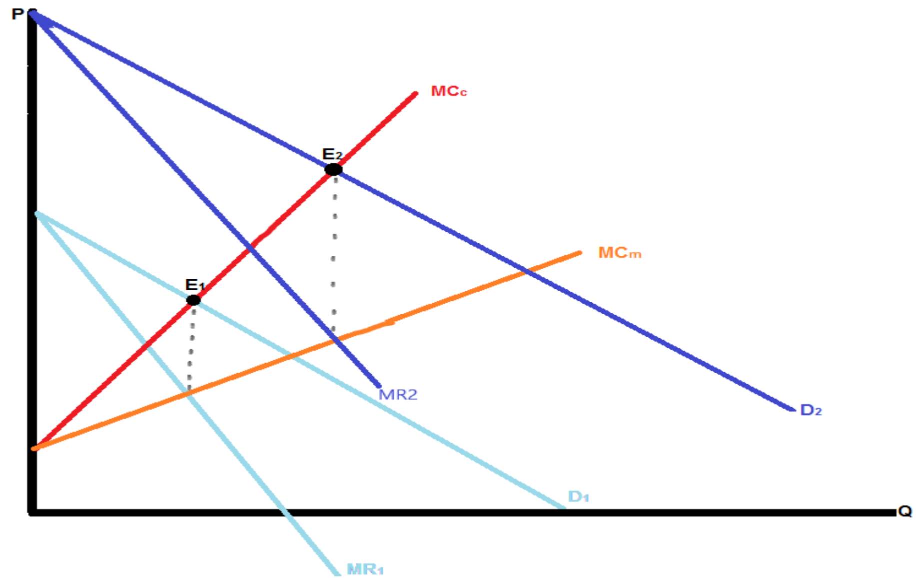
Following Bresnahan (1981), we can provide a graphical representation of this identification problem. Suppose that we have followed the approach described above to estimate consistently demand parameters, marginal cost parameters β_0 and β_1, and the parameter γ. We can define two hypothetical marginal cost functions: the marginal cost under the hypothesis of perfect competition ($CV = -1$ such that $\beta_2 = \gamma$), $MC_c \equiv \beta_0 + \beta_1\ W + \gamma\ q$; and the marginal cost under the hypothesis of monopoly or perfect collusion ($CV = N - 1$ such that $\beta_2 = \gamma - \alpha_2 N$), $MC_m \equiv \beta_0 + \beta_1\ W + (\gamma - \alpha_2 N)\ q$. That is, MC_c and MC_m are the marginal cost functions that rationalize the observed values (p_t, q_{it}) under the hypotheses of perfect competition and monopoly, respectively. Figure 5.1 shows that the observed price and quantity in market 1, say (p_1, q_1), can be rationalized either as the point where the demand function D_1 crosses the marginal cost MC_c, or as the monopoly outcome defined by the marginal revenue MR_1 and the marginal cost MC_m.

Figure 4.1: One data point: No identification of PC vs. collusion



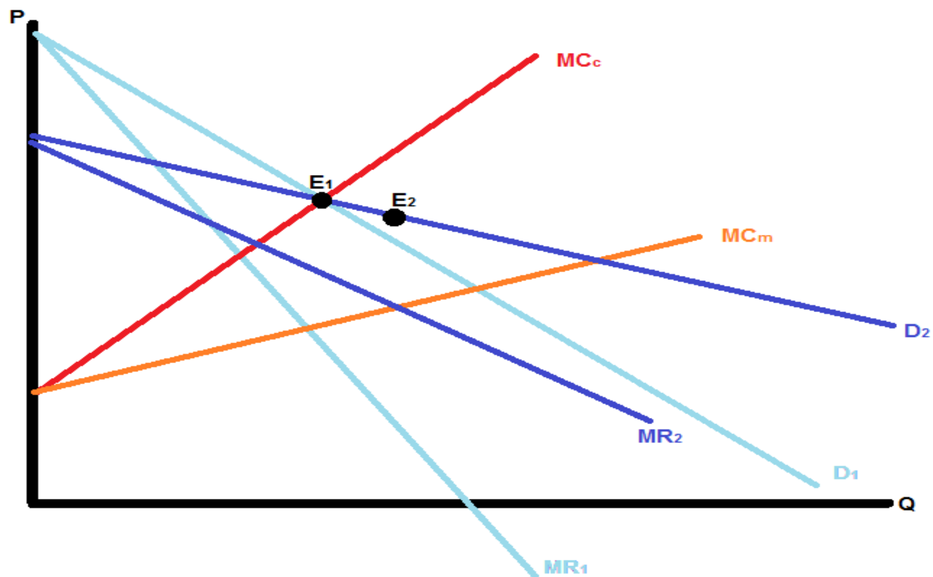
If the model is described by equations (2.13) and (2.15), the observation of prices and quantities (p_t, q_t) from multiple markets does not help to solve this identification problem. Suppose that we keep W_t constant such that the observations (p_t, q_t) for t = 1, 2, ..., T are generated by different values of the demand shifters X_t^D and ε_t^D. This implies parallel vertical shifts in the demand curve and in the corresponding marginal revenue curve, as represented in Figure 5.2. As explained above for observation (p_1, q_1), all the observations $\{p_t, q_t : t = 1, 2, ..., T\}$ can be rationalized either as perfect competive equilibria that come from the intersection of demand curves $\{D_t : t = 1, 2, ..., T\}$ and marginal cost MC_c, or as monopoly outcomes that are determined by the intersection of the marginal revenue curves $\{MR_t : t = 1, 2, ..., T\}$ and the marginal cost MC_m.

Figure 4.2: Multiple data points: No identification of PC vs. collusion



This graphical analysis provides also an intuitive interpretation of a solution to this identification problem. This solution involves generalizing demand function so that changes in exogenous variables do more than just a parallel shift in the demand curve and the marginal revenue. We introduce additional exogenous variables that are capable of **rotating** the demand curve. Consider Figure 5.3. We have two data points as represented by points E_1 and E_2. Now, point E_2 is associated with a change in the demand curve that consists in a rotation around point E_1. Under perfect competition, this rotation in the demand curve should not have any effect in equilibrium prices and quantities. Therefore, under perfect competition the value of (P, q) in market 2 should be the same as in market 1. Since point E_2 is different to E_1, we can reject the hypothesis of perfect competition. Changes in the slope of the demand have and effect on prices and quantities only if firms market power.

Figure 4.3: Multiple data points: Identification of PC vs. collusion



We now present more formally the identification of the model illustrated in Fugure 5.3. Consider now the following demand equation:

$$p_t = \alpha_0 + \alpha_1\ X_t^D - \alpha_2\ Q_t - \alpha_3\,[R_t\ Q_t] + \varepsilon_t^D \tag{2.16}$$

R_t is an observable variable that affects the slope of the demand. Some possible candidates for these variables are the price of a substitute or complement product, seasonal variables, or the consumer demographics. The key condition is that the parameter α_3 is different to zero. That is, when R_t varies, there is a rotation in the demand curve, that is, a change in the slope of the demand curve. Note that this condition is testable. Given this demand model, we have that $\frac{dP_t}{dQ_t} = -\alpha_2 - \alpha_3\ R_t$, and the marginal condition for profit maximization implies the following regression model:

$$p_t = \beta_0 + \beta_1\ W_t + \gamma_1\ q_{it} + \gamma_2\ \ (R_t\ q_{it}) + \varepsilon_{it}^{MC} \tag{2.17}$$

with $\gamma_1 \equiv \beta_2 + \alpha_2\ [1 + CV]$ and $\gamma_2 \equiv \alpha_3\ [1 + CV]$.

Equations (2.16) and (2.17) describe the structural model. Using this model and data, we can identify separately CV and MC parameters. Demand parameters can be identified similarly as before, using W_t as an instrument for output. Parameters α_0, α_1, α_2, and α_3 are identified using this IV estimator. The model also implies a valid instrument to estimate the parameters in the equilibrium equation in (2.17). We can instrument q_{it} using X_t^D. Parameters β_0, β_1, γ_1, and γ_2 are identified. Note that $\gamma_1 = \beta_2 + \alpha_2\,[1 + CV]$ and $\gamma_2 = \alpha_3\,[1 + CV]$ such that that given γ_2 and α_3 we identify CV, and given γ_1, α_2, and CV we identify β_2. The identification of CV is very intuitive: $1 + CV = \gamma_2/\alpha_3$. It measures the ratio between the sensitivity of price with respect to $(R_t\ q_{it})$ in the equilibrium equation relative to the sensitivity of price with respect to $(R_t\ Q_t)$ in the demand equation.

The sample variation in the slope of the inverse demand plays a key role in the identification of the CV parameter. An increase in the slope means that the demand becomes less price sensitive, more inelastic. For a monopolist, when the demand becomes more inelastic, the optimal price should increase. In general, for a firm with high level of market power (high CV), we should observe an important increase in prices associated with an increase in the slope. On the contrary, if the industry is characterized by very low market power (low CV) the increases in prices should be practically zero. Therefore, the response of prices to an exogenous change in the slope of the demand contains key information for the estimation of θ.

2.2.4. *An application: The sugar industry.* Genesove and Mullin (GM) study competition in the US sugar industry during the period 1890-1914. Why this period? The reason is that for this period they can collect high quality information on the value of marginal costs. Two aspects play are important in the collection of information on marginal costs. First, the production technology of refined sugar during this period was very simple and the marginal cost function can be characterized in terms of a simple linear function of the cost of raw sugar, the main intermediate input in the production of refined sugar. Most importantly, during this period there was an important investigation of the industry by the US anti-trust authority. As a result of that investigation, there are multiple reports from expert witnesses that provide estimates about the structure and magnitude of production costs in this industry.

As we describe below, GM use this information on marginal costs to test the validity of the standard conjectural variation approach for estimation of price cost margins and marginal costs. Here I describe briefly the main idea for this approach.

Let $p_t = P(Q_t)$ be the inverse demand function in the industry. Under the conjectural variation approach, and under the assumption that all the firms are identical in their marginal costs and in their conjectural variations, the marginal revenue at period t is:

$$MR_t = p_t - [1 + CV_t]\ \frac{Q_t}{N_t}\ \frac{dP(Q_t)}{dQ_t} \tag{2.18}$$

where $dP(Q_t)/dQ_t$ is the derivative of the inverse demand function. The condition for profit maximization (marginal revenue equals marginal cost) is $p_t - [1 + CV_t]\ \frac{Q_t}{N_t}\ \frac{dP(Q_t)}{dQ_t} = MC_t$, and it implies the following condition for the Lerner Index:

$$\frac{p_t - MC_t}{p_t} = \left[\frac{1 + CV_t}{N_t}\right]\ \frac{1}{|\eta_t|} \tag{2.19}$$

Therefore, if we observe prices and can estimate the demand elasticity and the marginal cost, then we have a simple and direct estimate of the conjectural variation. Without information on MCs, the estimation of the CV should be based: (a) on our estimation of demand, and in particular, on exclusion restrictions that permit the identification of demand parameters; and (b) on our estimation of the MC function, on exclusion restrictions that permit the identification of this function. If assumptions (a) or (b) are not correct, our estimation of the CV and therefore of the Lerner Index, will be biased. GM evaluate these assumptions by comparing the estimates of CV using information on MCs and not using that information.

The rest of this section describes the following aspects of this empirical application: (a) The industry; (b) The data; (c) Estimates of demand parameters; and (d) Estimation of CV.

The industry. Homogeneous product industry. Highly concentrated during the sample period, 1890-1914. The industry leader, American Sugar Refining Company (ASRC), had more than 65% of the market share during most of these years.

Production technology. Refined sugar companies buy "raw sugar" from suppliers in national or international markets, transformed it into refined sugar, and sell it to grocers. They sent sugar to grocers in barrels, without any product differentiation. Raw sugar is 96% sucrose and 4% water. Refined sugar is 100% sucrose. The process of transforming raw sugar into refined sugar is called "melting", and it consists of eliminating the 4% of water in raw sugar. Industry experts reported that the industry is a "fixed coefficient" production technology:[2]

$$Q^{refined} = \lambda \ Q^{raw}$$

where $Q^{refined}$ is refined sugar output, Q^{raw} is the input of raw sugar, and $\lambda \in (0,1)$ is a technological parameter. That is, 1 ton of raw sugar generates λ tons units of refined sugar.

Marginal cost function. Given this production technology, the marginal cost function is:

$$MC = c_0 + \frac{1}{\lambda} \ p^{raw}$$

where p^{raw} is the price of the input raw sugar (in dollars per pound), and c_0 is a component of the marginal cost that depends on labor and energy. Industry experts unanimously report that the value of the parameter λ was close to 0.93, and c_0 was around \$0.26 per pound. Therefore, the marginal cost at period (quarter) t, in dollars per pound of sugar, was:

$$MC_t = 0.26 + 1.075 \ p_t^{raw}$$

[2]Actually, the fixed coefficient Leontieff production function is $Q^{refined} = \min\{\lambda \ Q^{raw} \ ; \ f(L,K)\}$ where $f(L,K)$ is a function of labor and capital inputs. However, cost minimization will generally imply that $Q^{refined} = \lambda \ Q^{raw} = f(L,K)$.

The data. Quarterly US data for the period 1890-1914. The dataset contains 97 quarterly observations on industry output, price, price of raw sugar, imports of raw sugar, and a seasonal dummy.

$$\text{Data} = \{\ Q_t,\ p_t,\ p_t^{raw},\ IMP_t,\ S_t : t = 1, 2, ..., 97\}$$

IMP_t represents the imports of raw sugar from Cuba. And S_t is a dummy variable for the Summer season: $S_t = 1$ is observation t is a Summer quarter, and $S_t = 0$ otherwise. The summer was a high demand season for sugar because most the production of canned fruits was concentrated during that season, and the canned fruit industry accounted for an important fraction of the demand of sugar.

Based on this data, we can also obtain a measure of marginal cost as $MC_t = 0.26 + 1.075$ p_t^{raw}.

Estimates of demand parameters. GM estimate four different models of demand. The main results are consistent for the four models. Here I concentrate on the linear demand, $Q_t = \beta_t\ (\alpha_t - p_t)$, and the inverse demand equation is:

$$p_t = \alpha_t - \frac{1}{\beta_t}\ Q_t$$

We can refer to β_t to the price sensitivity of demand, which is the inverse of the slope of the demand curve, that is, higher price sensitivity implies a smaller slope of the demand curve. GM consider the following specification for α_t and β_t:

$$\begin{aligned} \alpha_t &= \alpha_L\ (1 - S_t) + \alpha_H\ S_t + e_t^D \\ \beta_t &= \beta_L\ (1 - S_t) + \beta_H\ S_t \end{aligned}$$

α_L, α_H, β_L, and β_H are parameters. α_L and β_L are the intercept and the slope of the demand during the "Low Season" (when $S_t = 0$). And α_H and β_H are the intercept and the slope of the demand during the "High Season" (when $S_t = 1$). e_t^D is an error term that represents all the other variables that affect demand and that we do not observe. Therefore, we can write the following inverse demand equation:

$$p_t = \alpha_L + (\alpha_H - \alpha_L)S_t + \frac{1}{\beta_L}(-Q_t) + \left(\frac{1}{\beta_H} - \frac{1}{\beta_L}\right)(-S_t Q_t) + e_t^D$$

This is a regression equation where the explanatory variables are a constant term, S_t, $(-Q_t)$, and $(-S_t Q_t)$, and the parameters are α_L, $(\alpha_H - \alpha_L)$, $\frac{1}{\beta_L}$, and $\left(\frac{1}{\beta_H} - \frac{1}{\beta_L}\right)$. From the estimation of these parameters, we can recover α_L, α_H, β_L, and β_H.

As we have discussed before, Q_t is an endogenous regressor in this regression equation. We need to use IV to deal with this endogeneity problem. In principle, it seems that we could we p_t^{raw} as an instrument. However, GM have a reasonable concern about the validity of this instrument. The demand of raw sugar from the US accounts for a significant fraction of the world demand of raw sugar. Therefore, exogenous shocks in the demand of refined sugar (e_t^D) might generate an increase if the world demand of raw sugar and in p_t^{raw} such that $Cov(e_t^D, p_t^{raw}) \neq 0$. Instead they use imports of raw sugar from Cuba as an instrument: almost 100% of the production of raw sugar in Cuba was exported to US, and the authors claim that variations in Cuban production of raw sugar was driven by supply/weather conditions and not by the demand from US.

These are the parameter estimates.

Table 4.1: Genesove & Mullin: Demand estimates

Demand Estimates		
Parameter	Estimate	Standard Error
α_L	5.81	(1.90)
α_H	7.90	(1.57)
β_L	2.30	(0.48)
β_H	1.36	(0.36)

In the high season the demand shifts upwards and becomes less elastic. The estimated price elasticities of demand in the low and the high season are $|\eta_L| = 2.24$ and $|\eta_H| = 1.04$, respectively. According to this, any model of oligopoly competition where firms have some market power predicts that the price cost margin should increase during the price season due to the lower price sensitivity of demand.

Before we discuss the estimates of the conjectural variation parameter, it is interesting to illustrate the errors that researchers can make if in the absence of information about marginal costs they estimate price cost margins by making an adhoc assumption about the value of CV in the industry. As mentioned above, the industry was highly concentrated during this period. Though there were approximately 6 firms active during most of the sample period, one of the firms accounted for more than two-thirds of total output. Suppose three different researchers of this industry, that we label as researchers M, C, and S. Researcher M considers that the industry was basically a Monopoly/Cartel during this period (in fact, there was anti-trust investigation, so there may be some suspicions of collusive behavior). Therefore, she assumes that $[1+CV]/N = 1$. Researcher C considers that the industry can be characterized by Cournot competition between the 6 firms, such that $[1+CV]/N = 1/6$. Finally, researcher S thinks that this industry can be better described by a Stackelberg model with 1 leader and 5 Cournot followers, and therefore $[1+CV]/N = 1/(2*6-1) = 1/11$. What are the respective predictions of these researchers about market power as measured by the Lerner index? The following table presents the researchers' predictions and also the actual value of the Lerner index based on our information on marginal costs (that we assume is not available for these 3 researchers). Remember that $Lerner = \frac{p-MC}{p} = \left[\frac{1+CV}{N}\right] \frac{1}{|\eta|}$.

Table 4.2: Genesove & Mullin: Markups under different conduct parameters

Predicted Market Power Based on Different Assumptions on $\frac{1+CV}{N}$				
Assumed $\frac{1+CV}{N}$	Predicted Lerner Low season: $\left[\frac{1+CV}{N\ \lvert\eta_L\rvert}\right]$	Actual Lerner Low season: $\frac{p_L-MC}{p_L}$	Predicted Lerner High season: $\left[\frac{1+CV}{N\ \lvert\eta_H\rvert}\right]$	Actual Lerner High season: $\frac{p_H-MC}{p_H}$
Monopoly: $\frac{1+CV}{N}=1$	$\frac{1}{2.24}=44.6\%$	3.8%	$\frac{1}{1.04}=96.1\%$	6.5%
Cournot: $\frac{1+CV}{N}=\frac{1}{6}$	$\frac{1/6}{2.24}=7.4\%$	3.8%	$\frac{1/6}{1.04}=16.0\%$	6.5%
Stackelberg: $\frac{1+CV}{N}=\frac{1}{11}$	$\frac{1/11}{2.24}=4.0\%$	3.8%	$\frac{1/11}{1.04}=8.7\%$	6.5%

This table shows that the researcher M will make a very seriously biased prediction of market power in the industry. Since the elasticity of demand is quite low in this industry, especially during the high season, the assumption of Cartel implies a very high Lerner index, much higher than the actual one. Researcher C also over-estimates the actual Lerner index. The estimates of researcher S are only slightly upward biased.

Consider the judge of an anti-trust case where there is very little reliable information on the actual value of MCs. The picture of industry competition that this judge gets from the three researchers is very different. This judge would be interested in measures of market power in this industry that do not depend on an adhoc assumption about the value of CV.

Estimation of conjectural variation. Suppose that we do not observe the MC and we use the approach described Section 2 to estimate the CV and then the lerner index. The condition marginal revenue equal to marginal cost implies the following equation:

$$p_t = c_0 + c_1\ p_t^{raw} + \theta\ \frac{Q_t}{\beta_t} + e_t^{MC}$$

with $\theta \equiv (1+CV)/N$. We treat c_0 and c_1 (the parameters in the marginal cost function) as parameter to estimate because we do not know that $c_0 = 0.26$ and $c_0 = 1.075$. We interpret e_t^{MC} as an error term in the marginal cost. After the estimation of the demand equation, we have $\hat{\beta}_t = 2.30(1-S_t) + 1.36\ S_t$. Therefore, we estimate the equation:

$$p_t = c_0 + c_1\ p_t^{raw} + \theta\ \frac{Q_t}{\hat{\beta}_t} + e_t^{MC}$$

Since Q_t is endogeneously determined, it should be correlated with e_t^{MC}. To deal with this endogeneity problem, GM use instrumental variables. Again, the use imports from Cuba as an instrument for Q_t. In principle, they might have considered the seasonal dummy S_t as an instrument, but they were probably concerned that there may be also seasonality in the marginal cost such that e_t^{MC} and S_t might be correlated (for instance, wages of seasonal workers). The following table presents these IV estimates of c_0, c_1 and θ, their standard errors (in parentheses) and the "true" values of these parameters based on the information on marginal costs.

Table 4.3: Genesove & Mullin: Estimates of conduct and marginal cost parameters

Estimates of Marginal Costs and θ			
Parameter	Estimate	(s.e.)	"True" value$^{(Note)}$
$\frac{1+CV}{N}$	0.038	(0.024)	0.10
c_0	0.466	(0.285)	0.26
c_1	1.052	(0.085)	1.075

The "true" value of $\frac{1+CV}{N}$ using information of MC is obtained using the relationship $\frac{1+CV}{N} = (\frac{p-MC}{p})\ |\eta|$. The estimates of $\frac{1+CV}{N}$, c_0, and c_1, are not too far from their "true" values. This seems a validation of the CV approach for this particular industry. Based on this estimate of $\frac{1+CV}{N}$, the predicted values for the Lerner index in the low season is $\left[\frac{1+CV}{N}\right] \frac{1}{|\eta_L|} = \dfrac{0.038}{2.24} = 1.7\%$, and the predicted Lerner Index in high season $\left[\frac{1+CV}{N}\right] \frac{1}{|\eta_H|} = \dfrac{0.038}{1.04} = 3.6\%$. Remember that the true values of the Lerner index using information on marginal costs were 3.8% in the low season and 6.5% in the high season. Therefore, the estimates using the CV approach under-estimate the actual market power in the industry, but by a relatively small magnitude.

3. Differentiated product industry

3.1. Model. Consider an industry with J differentiated products (for instance, automobiles) indexed by $j \in \mathcal{J} = \{1, 2, ..., J\}$. Consumer demand for each of these products can be represented using the demand system:

$$q_j = D_j\left(\mathbf{p}, \mathbf{x}\right) \qquad \text{for } j \in \mathcal{J} \tag{3.1}$$

where $\mathbf{p} = (p_1, p_2, ..., p_J)$ is the vector of product prices, and $\mathbf{x} = (x_1, x_2, ..., x_J)$ is a vector of other product attributes. There are F firms in the industry, indexed by $f \in \{1, 2, ..., F\}$. Each firm f owns a subset $\mathcal{J}_f \subset \mathcal{J}$ of the brands. The profit of firm f is:

$$\Pi_f = \sum_{j \in \mathcal{J}_f} p_j\ q_j - C_j(q_j) \tag{3.2}$$

where $C_j(q_j)$ is the cost of producing a quantity q_j of product j. Firms compete in prices.

Bertrand. For the moment, we assume Nash-Bertrand competition: each firm chooses its own prices to maximize profits and takes the prices of other firms as given. The first order conditions of optimality for profit maximization of firm f are: for any $j \in \mathcal{J}_f$

$$q_j + \sum_{k \in \mathcal{J}_f} [p_k - MC_k] \ \frac{\partial D_k}{\partial p_j} = 0 \tag{3.3}$$

where MC_j is the marginal cost $C_j'(q_j)$. We can write this system in vector form. For firm f:

$$\mathbf{q}^f + \Delta\mathbf{D}^f \ \left[\mathbf{p}^f - \mathbf{MC}^f\right] = 0 \tag{3.4}$$

where $\mathbf{q}^f$, $\mathbf{p}^f$, and $\mathbf{MC}^f$ are column vectors with the quantities, prices, and marginal costs, respectively, for every product $j \in \mathcal{J}_f$, and $\Delta\mathbf{D}^f$ is the square matrix with the demand-price derivatives $\dfrac{\partial D_k}{\partial p_j}$ for every $j, k \in \mathcal{J}_f$. Solving for price-cost margins in this system:

$$\mathbf{p}^f - \mathbf{MC}^f = -\left[\Delta\mathbf{D}^f\right]^{-1} \mathbf{q}^f \tag{3.5}$$

The RHS of this equation depends only on demand parameters, not costs. Given an estimated demand system, the vector of Price-Cost Margins under Nash-Bertrand competition (and a particular ownership structure of brands),is known to the researcher.

EXAMPLE. Single product firms & Logit model. For single product firms, the marginal condition of optimality is:

$$p_j - MC_j = -\left[\frac{\partial D_j}{\partial p_j}\right]^{-1} q_j \tag{3.6}$$

In the logit demand system, we have that:

$$D_j\left(\mathbf{p}, \mathbf{x}\right) = H \ \frac{\exp\left\{x_j'\beta - \alpha \ p_j\right\}}{1 + \sum_{k=1}^{J} \exp\left\{x_k'\beta - \alpha \ p_k\right\}} \tag{3.7}$$

where H represents market size, and β and α are parameters. This demand system implies that $\dfrac{\partial D_j}{\partial p_j} = -\alpha \ H \ s_j(1 - s_j)$ where s_j is the market share $s_j \equiv q_j/H$. Therefore, in this model:

$$PCM_j \equiv p_j - MC_j = \frac{1}{\alpha(1 - s_j)} \tag{3.8}$$

We see that in this model the price-cost margin of a firm declines with the price sensitivity of demand, α, and increases with the own market share, s_j. ■

EXAMPLE. Logit model with Multi-product firms. With multiproduct firms we have that, the F.O.C. is $q_j + \sum_{k \in \mathcal{J}_f} PCM_k \ \dfrac{\partial D_k}{\partial p_j} = 0$. In Logit demand system: $\dfrac{\partial D_j}{\partial p_j} = -\alpha \ H \ s_j(1 - s_j)$ and for $k \neq j$, $\dfrac{\partial D_j}{\partial p_k} = \alpha \ H \ s_j \ s_k$. And this implies:

$$PCM_j \ = \ \frac{1}{\alpha} + \sum_{k \in \mathcal{J}_f} PCM_k \ s_k \tag{3.9}$$

The RHS is firm specific but it does not vary across products within the same firm. This condition implies that all the products owned by a firm have the same price-cost margin. This condition implies that the price-cost margin is:

$$PCM_j = \overline{PCM}_f = \frac{1}{\alpha\left(1 - \sum_{k\in\mathcal{J}_f} s_k\right)} \tag{3.10}$$

For the Logit demand model, a multi-product firm charges the same price-cost margin to all its products. This prediction does not extend to more general/flexible demand systems. Note also that a multi-product firm charges higher prices than a single-product firm:

$$\frac{1}{\alpha\left(1 - \sum_{k\in\mathcal{J}_f} s_k\right)} > \frac{1}{\alpha\left(1 - s_j\right)} \tag{3.11}$$

This prediction is robust and it extends to Bertrand competition when products are substitutes. ■

Multiproduct as source of market power. We can write F.O.C. for firm f product j as:

$$\begin{array}{rcl} PCM_j & = & \left[\dfrac{-\partial D_j}{\partial p_j}\right]^{-1} q_j \\ & + & \left[\dfrac{-\partial D_j}{\partial p_j}\right]^{-1} \left[\sum\limits_{k\in J_f;\ k\neq j} PCM_k \ \dfrac{\partial D_k}{\partial p_j}\right] \end{array} \tag{3.12}$$

With substitutes, $\dfrac{\partial D_k}{\partial p_j} > 0$ for $k \neq j$, and the second term is positive. Selling multiple products contribute to increase the price-cost margin of each of the products.

Collusion and other ownership structures. Suppose that there is collusion between some or all the firms. We can represent a collusive setting as a partition of the set of firms, into a number R of groups or "rinks". Let $\mathcal{F} = \{1, 2, ..., F\}$ be the set of all the firms, and let $\mathcal{R}_1$, $\mathcal{R}_2$, ..., $\mathcal{R}_R$ be a partition of the set $\mathcal{F}$ such that $\mathcal{R}_1 \cup\ \mathcal{R}_2 \cup\ ... \cup\ \mathcal{R}_R = \mathcal{F}$ and $\mathcal{R}_r \cap \mathcal{R}_{r'} = \varnothing$ for $r \neq r'$. According to this partition, a firm belongs to one and only one rink. We also use $\mathcal{R}(f)$ to denote the rink the rink to which firm f belongs.

A collusion rink, together with the ownership structure of the products, implies a set of products from all the firms in the rink. Then, we can define:

$$\mathcal{J}_{\mathcal{R}(f)} \equiv \{j : j \in \mathcal{J}_{f'} \text{ for some } f' \in \mathcal{R}(f)\} \equiv \bigcup\nolimits_{f'\in\mathcal{R}(f)} \ \mathcal{J}_{f'}$$

We also define the dummy variables $\Theta_j^{\mathcal{R}(f)} \equiv 1\{j \in \mathcal{J}_{\mathcal{R}(f)}\}$. No extreme example are no collusion and collusion of all the firms. With no collusion we have that the number of rinks R is equal to the number of firms F, and $\mathcal{J}_{\mathcal{R}(f)} = \mathcal{J}_f$, and $\Theta_j^{\mathcal{R}(f)} = 1\{j \in \mathcal{J}_f\}$. With collusion of all firms, we have that there is only one rink, $R = 1$, $\mathcal{R}_1 = \mathcal{F}$, $\mathcal{J}_{\mathcal{R}(1)} = \mathcal{J}$, and all the indicators $\Theta_j^{\mathcal{R}}$ are equal to one.

Firm f maximizes its collusion rink profit:

$$\sum_{j=1}^{J} \Theta_j^{\mathcal{R}(f)} \ [p_j \ q_j - C_j(q_j)]$$

The F.O.C.s for firm f: for $j \in \mathcal{J}_f$

$$q_j + \sum_{k=1}^{J} [p_k - MC_k] \ \Theta_k^{R(f)} \ \frac{\partial D_k}{\partial p_j} = 0$$

In vector form, using all the products that belong to the collusion rink $R(f)$

$$\mathbf{q}^{R(f)} + \left[\Delta \mathbf{D}^{R(f)}\right] \ \left[\mathbf{PCM}^{R(f)}\right] = 0$$

$\Delta \mathbf{D}^{R(f)}$ = matrix of demand-price derivatives $\dfrac{\partial D_k}{\partial p_j}$ for every j, k in the collusion rink of firm f. Such that:

$$\mathbf{PCM}^{R(f)} = -\left[\Delta \mathbf{D}^{R(f)}\right]^{-1} \mathbf{q}^{R(f)}$$

3.2. Estimating MCs based on assumption on form of competition. The researcher has data from J products over T markets, and knows the ownership structure:

$$\text{Data} = \{p_{jt}, \ q_{jt}, \ x_{jt} : \ j = 1, ..., J; \ t = 1, 2, ..., T\}$$

Suppose that the demand function has been estimated in a fist step, such that there is a consistent estimator of the demand system $D_j \left(\mathbf{p}_t, \mathbf{x}_t\right)$. For every firm f, the research has an estimate of vector $-\left[\Delta \mathbf{D}_t^f\right]^{-1} \mathbf{q}_t^f$ for every firm f. Therefore, under the assumption of Bertrand competition she has consistent estimates of the vectors of MCs:

$$\mathbf{MC}_t^f = \mathbf{p}_t^f + \left[\Delta \mathbf{D}_t^f\right]^{-1} \mathbf{q}_t^f$$

Similarly, given an hypothetical collusion rink $R(f)$ represented by the indicators $\Theta_j^{R(f)}$, the researcher can construct $\left[\Delta \mathbf{D}^{R(f)}\right]^{-1} \mathbf{q}^{R(f)}$ and obtain the estimate of marginal costs:

$$\mathbf{MC}_t^{R(f)} = \mathbf{p}_t^{R(f)} + \left[\Delta \mathbf{D}_t^{R(f)}\right]^{-1} \mathbf{q}_t^{R(f)}$$

Different hypothesis about collusion, or ownership structures of products (for instance, mergers), imply different Price-Cost margins and different estimates of marginal costs. After estimating the realized values of MCs, we can estimate the marginal cost function.

Consider the following cost function:

$$C(q_{jt}) = \frac{1}{\gamma + 1} q_{jt}^{\gamma+1} \ \exp\{x_{jt}'\alpha + \omega_{jt}\}$$

Such that:

$$MC_{jt} = q_{jt}^{\gamma} \exp\{x_{jt}'\alpha + \omega_{jt}\}$$

where ω_{jt} is unobservable to the researcher. The econometric model is:

$$\ln(MC_{jt}) = \gamma \ \ln(q_{jt}) + x'_{jt}\alpha + \omega_{jt}$$

We are interested in the estimation of the parameters α and γ.

Endogeneity: The equilibrium model implies that $E(\ln(q_{tj}) \ \omega_{jt}) \neq 0$. Firms/products with larger ω_{jt} are less efficient in terms of costs (or products are more costly to produce), and this, all else equal, implies a smaller amount of output.

Instrumental variables. Suppose that $E(x_{kt} \ \omega_{jt}) = 0$ for any (k, j). We can use as instruments for $\ln(q_{jt})$ the characteristics of other firms/products.

$$E\left(\begin{bmatrix} x_{jt} \\ \sum_{k\neq j} x_{kt} \end{bmatrix} \ \left[\ln(MC_{jt}) - \gamma \ \ln(q_{jt}) - x'_{jt}\alpha\right]\right) = \mathbf{0}$$

3.3. Testing the nature of competition. Suppose that the researcher observes the true MC_{jt}. Or more realistically, observes a measure of costs, S^{MC}_{obs}, for instance, the mean value of the MCs of all products and firms in the industry; the mean value of the MC of one particular firm. Given an estimated demand system and an hypothesis about collusion, represented by a matrix of collusion rink dummies $\mathbf{\Theta}^R = \{\Theta^{R(f)}_j\}$, we can obtain the MCs under this hypothesis: $MC_j(\mathbf{\Theta}^R)$.

Let $S^{MC}(\mathbf{\Theta}^R)$ the value of the statistic (for instance, mean value of all MCs) under the hypothesis $\mathbf{\Theta}^R$. We can use $S^{MC}(\mathbf{\Theta}^R)$ and S^{MC}_{obs} to construct a test of the null hypothesis $\mathbf{\Theta}^R$. For instance, if S^{MC} is a vector of sample means, we could use a Chi-square test. This is the approach in Nevo (2001). It is possible to consider $\Theta^{R(f)}_j$ as parameters to estimate, similarly as the conjectural variation parameters in the homogeneous product case. Using the estimated demand, our specification of the MC function, and the F.O.C.s of profit maximization, it is possible to jointly identify $\Theta^{R(f)}_j$ and parameters in MCs. We need similar rotation demand variables as in the homogeneous demand case (Nevo, 1998).

Testing form of competition: Without info on MCs. Instead of estimating $\Theta^{R(f)}_j$ some papers have used non-nested hypothesis tests to test null hypothesis of Collusion against the alternative of Bertrand (or viceversa). The most commonly used non-nested tests procedures are: Cox-Test and Vuong-Test. Davidson & McKinnon provide an intuitive interpretation of these tests: Obtain residuals from the model under H_0; Run regression of the residuals on variable in the model under H_1; Under null, #obs $\times$ R-squared of this regression is Chi-square.

3.3.1. *Competition and Collusion in the American Automobile Industry.*

Table 4.4: Bresnahan (1987) : Descriptive Statistics

TABLE I

Year	(1) Auto Production[a]	(2) Real Auto Price-CPI[b]	(3) % Change Auto Price-Cagan[c]	(4) Auto Sales[d]	(5) Auto Quantity Index[e]
1953	6.13	1.01	NA	14.5	86.8
1954	5.51	0.99	NA	13.9	84.9
1955	7.94	0.95	−2.5	18.4	117.2
1956	5.80	0.97	6.3	15.7	97.9
1957	6.12	0.98	6.1	16.2	100.0

Notes: [a] Millions of units over the model year. [Source: *Automotive News.*]
[b] (CPI New automobile component)/CPI. [Source: *Handbook of Labor Statistics.*]
[c] Adjusted for quality change. [See Cagan (1971), especially pp. 232–3.]
[d] Auto output in constant dollars, *QIV* of previous year through *QIII* of named year, in billions of 1957 dollars. [Source: *National Income and Product Accounts.*]
[e] (4)/(2), normalized so 1957 = 100.

Table 4.5: Bresnahan (1987) : Tests of conduct parameters

TABLE III
COX TEST STATISTICS

Hypotheses	*C*	*N–C*	'*p*'	*H*
a—1954				
Collusion	—	0.8951	0.9464	−1.934
Nash-Competition	−2.325	—	−0.8878	−2.819
"Products"	−3.978	3.029	—	−1.604
Hedonic	−12.37	−10.94	−13.02	—
b—1955				
Collusion	—	−10.36	−9.884	−13.36
Nash-Competition	−1.594	—	1.260	0.6341
"Products"	−0.7598	−4.379	—	−1.527
Hedonic	−3.353	−8.221	−5.950	—
c—1956				
Collusion	—	1.227	0.8263	1.629
Nash-Competition	−2.426	—	−4.586	0.8314
"Products"	−3.153	0.9951	—	4.731
Hedonic	−5.437	−9.671	−11.58	—

Table 4.6: Bresnahan (1987) Estimates demand & MCs (collusion 1954 & 1956)

TABLE IV
PARAMETER ESTIMATES 1954–56, MAINTAINED SPECIFICATION

Parameters	*1954*[a]	*1955*[b]	*1956*[a]
Physical Characteristics			
Quality Proxies			
Constant	47.91	48.28	50.87
	(32.8)	(43.2)	(29.4)
Weight #/1000	0.3805	0.5946	0.5694
	(0.332)	(0.145)	(0.374)
Length "/1000	0.1819	0.1461	0.1507
	(0.128)	(0.059)	(0.146)
Horsepower/100	2.665	3.350	3.248
	(0.692)	(0.535)	(0.620)
Cylinders	−0.7387	−0.9375	−0.9639
	(0.205)	(0.115)	(0.186)
Hardtop Dummy	0.9445	0.4531	0.4311
	(0.379)	(0.312)	(0.401)
Demand/Supply			
μ—*Marginal Cost*	0.1753	0.1747	0.1880
	(0.024)	(0.020)	(0.035)
γ—*Lower Endpoint*	4.593	3.911	4.441
	(1.49)	(2.08)	(1.46)
V_{max}—*Upper Endpoint*	$1.92E+7$	$2.41E+7$	$2.83E+7$
	$(8.44E+6)$	$(9.21E+6)$	$(7.98E+6)$
δ—*Taste Density*	0.4108	0.4024	0.4075
	(0.138)	(0.184)	(0.159)

Estimates Demand & MCs: Bertrand 1954, 1955, 1956. The estimated structural model under the maintained assumption of collusion in years 1954 & 1956 and Bertrand competition in1955 implies very stable coefficient estimates over the three years. That is, the observed changes in quantity and prices in 1955 can be fully explained by the change in conduct, and not by a change in demand or costs parameters. Instead, the models that impose Collusion over the three years, or Bertrand over the three years imply estimates of structural parameters with strong and implausible changes in demand and costs in year 1955.

3.3.2. *Conduct in the Ready-to-eat cereal industry.*

Nevo (2001). Ready-to-Eat (RTE) cereal market: highly concentrated; many apparently similar products, and yet price-cost margins (PCM) are high. What are the sources of market power? Product differentiation? Multi-product firms? Collusion? Nevo: (1) estimates a demand system of differentiated products for this industry; (2) recovers PCMs and compare them to rough/aggregate estimates of PCM at the industry level; (3) based on this comparison, tests Bertrand vs (full) Collusion [and rejects collusion]; (4) Under Bertrand, compares estimated PCMs with the counterfactual with single-product firms.

Data. A market is a city-quarter. IRI data on market shares and prices. 65 cities x 20 quarters [Q188-Q492] x 25 brands [total share is 43-62%]. Most of the price variation is cross-brand (88.4%), the remainder is mostly cross-city, and a small amount is cross-quarter. Relatively poor brand characteristics so model includes brand fixed effects.

Table 4.7: Nevo (2001) : Market shares

TABLE I

VOLUME MARKET SHARES

	88Q1	88Q4	89Q4	90Q4	91Q4	92Q4
Kellogg	41.39	39.91	38.49	37.86	37.48	33.70
General Mills	22.04	22.30	23.60	23.82	25.33	26.83
Post	11.80	10.30	9.45	10.96	11.37	11.31
Quaker Oats	9.93	9.00	8.29	7.66	7.00	7.40
Ralston	4.86	6.37	7.65	6.60	5.45	5.18
Nabisco	5.32	6.01	4.46	3.75	2.95	3.11
C3	75.23	72.51	71.54	72.64	74.18	71.84
C6	95.34	93.89	91.94	90.65	89.58	87.53
Private Label	3.33	3.75	4.63	6.29	7.13	7.60

Source: IRI Infoscan Data Base, University of Connecticut, Food Marketing Center.

Table 4.8: Nevo (2001): Demand estimates

TABLE VI

RESULTS FROM THE FULL MODEL[a]

Variable	Means (β's)	Standard Deviations (σ's)	Interactions with Demographic Variables: Income	Income Sq	Age	Child
Price	−27.198	2.453	315.894	−18.200	—	7.634
	(5.248)	(2.978)	(110.385)	(5.914)		(2.238)
Advertising	0.020	—	—	—	—	—
	(0.005)					
Constant	−3.592[b]	0.330	5.482	—	0.204	—
	(0.138)	(0.609)	(1.504)		(0.341)	
Cal from Fat	1.146[b]	1.624	—	—	—	—
	(0.128)	(2.809)				
Sugar	5.742[b]	1.661	−24.931	—	5.105	—
	(0.581)	(5.866)	(9.167)		(3.418)	
Mushy	−0.565[b]	0.244	1.265	—	0.809	—
	(0.052)	(0.623)	(0.737)		(0.385)	
Fiber	1.627[b]	0.195	—	—	—	−0.110
	(0.263)	(3.541)				(0.0513)
All-family	0.781[b]	0.1330	—	—	—	
	(0.075)	(1.365)				
Kids	1.021[b]	2.031	—	—	—	
	(0.168)	(0.448)				
Adults	1.972[b]	0.247	—	—	—	
	(0.186)	(1.636)				
GMM Objective (degrees of freedom)			5.05 (8)			
MD χ^2			3472.3			
% of Price Coefficients > 0			0.7			

[a] Based on 27,862 observations. Except where noted, parameters are GMM estimates. All regressions include brand and time dummy variables. Asymptotically robust standard errors are given in parentheses.
[b] Estimates from a minimum-distance procedure.

Direct measure of mean value of the price-cost margin in the industry: 31%

Table 4.9: Nevo (2001): Average markups under different conduct hypotheses

TABLE VIII

MEDIAN MARGINS[a]

	Logit (Table V column ix)	Full Model (Table VI)
Single Product Firms	33.6% (31.8%–35.6%)	35.8% (24.4%–46.4%)
Current Ownership of 25 Brands	35.8% (33.9%–38.0%)	42.2% (29.1%–55.8%)
Joint Ownership of 25 Brands	41.9% (39.7%–44.4%)	72.6% (62.2%–97.2%)
Current Ownership of All Brands	37.2% (35.2%–39.4%)	—
Monopoly/Perfect Price Collusion	54.0% (51.1%–57.3%)	—

[a] Margins are defined as $(p - mc)/p$. Presented are medians of the distribution of 27,862 (brand-city-quarter) observations. 95% confidence intervals for these medians are reported in parentheses based on the asymptotic distribution of the estimated demand coefficients. For the Logit model the computation is analytical, while for the full model the computation is based on 1,500 draws from this distribution.

Michel & Weiergraeber (2018). This paper studies competition in the US cereal industry during 1991-96. Two important events during this period: (1) Merger of Post & Nabisco in 1993; (2) massive wholesale price reduction in 1996. The paper emphasizes importance of allowing **conduct "parameters" to vary over time and across firms** in the same industry. The authors are also particularly concern with finding **powerful instruments** to separately identify conduct and marginal costs. They propose novel instruments that exploit information on **firms' promotional activities**.

Data. Consumer level scanner data from the Dominick's Finer Food (DFF). 58 **supermarket stores** located in the Chicago metropolitan area. Sample Period: February 1991 to October 1996. Data aggregated at the monthly level (69 months). Focus on **26 brands** of cereals from the **6 nationwide manufacturers**: Kellogg's, General Mills, Post, Nabisco, Quaker Oats, and Ralston Purina. Brands classified in 3 groups: adult, family, and kids. Importantly: dataset contains information on **wholesale prices** (not only retail prices), and **in-store promotional activities**.

Market shares. Very concentrated industry: $CR1 \simeq 45\%$; $CR2 \simeq 75\%$. Firms market shares are more or less stable over the sample period, though with some changes after 1993 merger.

Post & Nabisco merger is 1993. Main concern of antitrust authority was the strong substitutability in the adult cereal segment between Post's and Nabisco's products (price increase after merger). The merger did not lead to any product entry or exit or any changes in existing products. Following the merger, Post+Nabisco increased significantly its prices, and this price increased was followed by the other firms. In principle, this could be explained under Bertrand-Nash competition, without any change in conduct.

On April 1996, Post decreased its wholesale prices by 20%. This was followed, a few weeks later, by significant price cuts by the other firms. Average decrease in the wholesale price between April and October 1996 of **9.66**% (and 7.5% in retail price). Reduced form regressions for wholesale prices: $\log(p_{jst}^{w}) = PROMO_{jst}\ \beta + \delta_1\ AFTMER_t + \delta_2\ Y96_t + \alpha_j^{(1)} + \alpha_s^{(2)} + \varepsilon_{jst}$. 96,512 observations. Rich controls for promotions variables, and store-level aggregate demand. Estimated effects: after-merger (δ_1) 0.0609 (s.e. = 0.0023); year-1996 (δ_2) -0.0983 (s.e. = 0.0015).

Structural model.

Demand: Random coefficients nested logit model. Random coefficients logit model discrete choice demand model: $u_{ijt} = x_j\ \beta_i + \beta_i^{PRO}\ PRO_{jt} + \alpha_i\ p_{jt} + \xi_{jt} + \varepsilon_{ijt}$. PRO_{jt} total (aggregated over stores and type of promotion) in-store promotions of product j during month t. Similar to Nevo (2001) but, very importantly, including in-store promotional variables. BLP-Instruments: Characteristics of other products. The authors exploit the substantial amount of sample variation in Promotion variables. **Results:** Price coefficient is highly negative; High-income consumers are less price-sensitive; Promotions have a significant positive effect.

Supply side: Flexible conduct parameter framework that specifies the **degree of cooperation** by a matrix of parameters that capture the degree to which firms internalize their rivals' profits. **Manufacturers' marginal costs:** Constant, $MC_{jt} = W_{jt}\ \gamma + \omega_{jt}$, and ω_{jt} follows AR(1) process. **Conduct parameters:** Parameter $\lambda_{ff't} \in [0,1]$ represents the degree to which firm f internalizes the profits of firm f' when setting its whole sale price in month t. Identification of MCs and Conduct. Consider the case of two single-product firms. Their pricing equations are:

$$\begin{aligned} p_1 - \left(\frac{\partial s_1}{\partial p_1}\right)^{-1} s_1 &= MC_1 + \lambda_{12}\ (p_2 - MC_2)\ \left(\frac{\partial s_1}{\partial p_1}\right)^{-1} \frac{\partial s_2}{\partial p_1} \\ p_2 - \left(\frac{\partial s_2}{\partial p_2}\right)^{-1} s_2 &= MC_2 + \lambda_{21}\ (p_1 - MC_1)\ \left(\frac{\partial s_2}{\partial p_2}\right)^{-1} \frac{\partial s_1}{\partial p_2} \end{aligned}$$

We need instruments. The number of instruments needed increases with the number of firms, because we have more λ parameters. "BLP instruments" = characteristics of other products. Two issues with this type of instruments: (1) often are weak instruments; (2) if product characteristics do not vary across markets or time, these instruments are collinear with brand fixed effects. Instead the authors use promotional variables of other products as instruments. Demand elasticities are significantly affected by these variables. They have substantial variation across products, over time, and markets.

Still, there is the concern that promotional variables are endogenous, that is, correlated with the unobservable component of the marginal cost. Promotions are chosen by firms: it is more profitable to make promotions when marginal costs are low. To deal with this endogeneity, the authors make the following assumptions on the **error structure** and the **timing of promotion decisions**. **Error structure:** Fixed effects (product, store and seasonal) and AR(1) shock: $\omega_{jt} = \rho\ \omega_{jt-1} + v_{jt}$. . The model is estimated using quasi-differences: $(y_{jt} - \rho\ y_{jt-1}) = (x_{jt} - \rho\ x_{jt-1})\,\beta + v_{jt}$. **Timing assumption:** Promotions are negotiated between manufacturers and retailers at least one month in advanced. Therefore, $PROMO_{jt}$ is not correlated with the i.i.d. shock v_{jt}.

Empirical results. Strong evidence for coordination between 1991-1992. On average the conduct parameter is 0.277: that is, a firm values \$1 of its rivals' profits as much as \$0.277 of its own profits. Because this coordination, pre-merger price-cost margins are **25.6% higher** than under multiproduct Bertrand-Nash pricing. After the Post + Nabisco merger in 1993, the degree of coordination increased significantly, on average to 0.454. Towards year 1996, the degree of coordination becomes close to 0, consistent with multiproduct Bertrand-Nash pricing.

Counterfactuals: if firms had competed ala Bertrand-Nash before 1996: Consumer welfare would have increased by between \$1.6 − \$2.0 million per year; Median wholesale prices would have been 9.5% and 16.3% lower during the pre-merger and post-merger periods.

3.4. Conjectural variation model with differentiated products. Consider an industry with a differentiated product. There are two firms in this industry, firm 1 and firm 2. Each firm produces and sells only one brand of the differentiated product: brand 1 is produced by firm 1, and brand 2 is produced by firm 2. The demand system has the structure of a logit demand model, where consumers choose between three different alternatives: $j = 0$, represents the consumer decision of not purchasing any of the two products; and $j = 1$ and $j = 2$ represent the consumer purchase of product 1 and 2, respectively. The utility of no purchase ($j = 0$) is zero. The utility of purchasing product $j \in \{1, 2\}$ is $\beta\, x_j - \alpha\, p_j + \varepsilon_j$, where the variables and parameters have the interpretation that we have seen in class. Variable x_j is a measure of the quality of product j, for instance, the number of stars of the product according to consumer ratings. Therefore, we have that $\beta > 0$. The random variables ε_1 and ε_2 are independently and identically distributed over consumers with a type I extreme value distribution, that is, Logit model of demand. Let H be the number of consumers in the market. We define the market shares s_0, s_1, and s_2 such that $s_0 + s_1 + s_2 = 1$ and s_j represents the proportion of consumers choosing alternative j.

The logit model implies that the market share of product 1, s_1, is:

$$s_1 = \frac{\exp\{\beta\ x_1 - \alpha\ p_1\}}{1 + \exp\{\beta\ x_1 - \alpha\ p_1\} + \exp\{\beta\ x_2 - \alpha\ p_2\}}$$

The profit function of firm $j \in \{0, 1\}$ is $\pi_j = p_j\ q_j - c_j\ q_j$, where: q_j is the quantity sold by firm j (that is, $q_j = H\ s_j$); and c_j is firm $j's$ marginal cost, that is assumed constant, that is, linear cost function.

Suppose that firms take their qualities x_1 and x_2 as given and compete in prices ala Bertrand. The F.O.C. implies:

$$p_1 - c_1 = \frac{1}{\alpha\ (1 - s_1)}$$

Now, suppose that the researcher is not willing to impose the assumption of Bertrand competition and considers a conjectural variations model. Define the conjecture parameter CV_1 as the belief or conjecture that firm 1 has about how firm 2 will change its price when firm 1 changes marginally its price. That is, CV_1 represents the belief or conjecture of firm 1 about $\frac{\partial p_2}{\partial p_1}$. Similarly, CV_2 represents the belief or conjecture of firm 2 about $\frac{\partial p_2}{\partial p_1}$. Then, the f.o.c. for profit maximization is:

$$q_1 + (p_1 - c_1)\left[\frac{\partial q_1}{\partial p_1} + \frac{\partial q_1}{\partial p_2} CV_1\right] = 0$$

Solving for the price-cost margin, we have that:

$$p_1 - c_1 = \frac{1}{\alpha\ (1 - s_1 - s_2 CV_1)}$$

Suppose that the researcher does not know the magnitude of the marginal costs c_1 and c_2, but she knows that the two firms use the same production technology, the same type of variable inputs, and purchase these inputs in the same markets where they are price takers. Therefore, the researcher knows that $c_1 = c_2 = c$, though she does not know the magnitude of c. The marginal conditions for profit maximization for the two firms, together with the condition $c_1 = c_2 = c$, imply that price difference between these two firms, $p_1 - p_2$, is a particular function of their markets shares and the conjectural variations. The marginal conditions for firms 1 and 2 are $p_1 - c = \frac{1}{\alpha\ (1 - s_1 - s_2 CV_1)}$ and $p_2 - c = \frac{1}{\alpha\ (1 - s_2 - s_1 CV_1)}$, respectively. The difference between these two equations implies:

$$p_1 - p_2 = \frac{1}{\alpha\ (1 - s_1 - s_2 CV_1)} - \frac{1}{\alpha\ (1 - s_2 - s_1 CV_2)}$$

The researcher observes prices $p_1 = \$200$ and $p_2 = \$195$ and market shares $s_1 = 0.5$ and $s_2 = 0.2$. Firm 1 has both a larger price and a larger market share because its product has better quality, that is, $x_1 > x_2$. Though not really relevant to answer this question, note that in this industry the higher quality product does not imply a larger marginal cost but only a larger fixed cost. The researcher has estimated the demand system and knows that $\alpha = 0.01$. Solving the data into the previous equation, we have:

$$\$200 - \$195 = \frac{100}{1 - 0.5 - 0.2\ CV_1} - \frac{100}{1 - 0.2 - 0.5\ CV_2}$$

This is a condition that the parameters CV_1 and CV_2 should satisfy. Using this equation we can show that the hypothesis of Nash-Bertrand competition (that requires $CV_1 = CV_2 = 0$) implies a prediction about the price difference $p_1 - p_2$ that is substantially larger than the price difference that we observe in the data. The hypothesis of Nash-Bertrand competition, $CV_1 = CV_2 = 0$, implies that the right hand side of the equation in Q15(b) is:

$$\frac{100}{0.5} - \frac{100}{0.8} = 200 - 125 = \$75$$

That is, Nash-Bertrand implies a price difference of \$75 but the price difference in the data is only \$5. The hypothesis of Collusion, $CV_1 = CV_2 = 1$, implies that the right hand side of the equation in Q15(b) is:

$$\frac{100}{0.5 - 0.2} - \frac{100}{0.8 - 0.5} = \$0$$

That is, Collusion implies a price difference of \$0, which is close to the price difference of \$5 that we observe in the data.

4. Quantity and price competition with incomplete information

In this chapter, we have considered different factors that can affect price and quantity competition and market power in an industry. Economies of scale and scope, firms' heterogeneity in marginal costs, product differentiation, multi-product firms, or conduct/form of competition are among the most important features that we have considered so far. All the models that we have considered assume that firms have perfect knowledge about demand, their own costs, and the costs of their competitors. In game theory, this type of models are denoted as *games of complete information*. This assumption can quite unrealistic in some industries. Firms have uncertainty about current and future realizations of demand, costs, market regulations, or the behavior of competitors. This uncertainty can have substantial implications for their decisions and profits, and for the efficiency of the market. For example, firms may gather better information and use it in their pricing or production strategies to improve their profits and the probability of survival in the market.

The assumption of firms' complete information has been the status quo in empirical models of Cournot or Bertrand competition. In reality, firms often face significant uncertainty about demand and about their rivals costs and strategies. Firms are different in their ability and their costs for collecting and processing information, for similar reasons as they are heterogeneous in their costs of production or investment. In this section, we study models and empirical applications of price and quantity competition that allow for firms' incomplete and asymmetric information. Our main purpose is to study how limited information affects competition and market outcomes.

4.1. Cournot competition with private information: Theory. Vives (2002) studies theoretically the importance of firms' private information as a determinant of prices, market power, and consumer welfare. He considers a market in which firms compete ala Cournot and have private information. Then, he studies the relative weights of private information and market power in accounting for the welfare losses at the market outcome. He shows that in large enough markets, abstracting from market power provides a much better approximation than abstracting from private information. If M represents market size, then the effect of market power is of the order of $1/M$ for prices and $1/M^2$ on per-capita deadweight loss, while the effect of private information is of the order of $1/\sqrt{M}$ for prices and $1/M$ for per-capita deadweight loss. Numerical simulations of the model show that there is a critical value for market size M^* (that depends on the values of structural parameters) such that the effect of private information dominates the effect of market power if and only if market size is greater than this threshold value.

Consider the market of an homogeneous product where firms compete a la Cournot and there is free market entry. A firm's marginal cost is subject to idiosyncratic shocks that are private information of the firm. The demand function and the marginal cost functions are linear such that the model is linear-quadratic. This feature facilitates substantially the characterization of a Bayesian Nash equilibrium in this model with incomplete information. There are M consumers in the market and each consumer has an indirect utility function $U(x) = \alpha\ x - \beta\ x^2/2 - p\ x$, where x is the consumption of the good, p is the market price, and $\alpha > 0$ and $\beta > 0$ are parameters. This utility function implies the market level inverse demand function, $p = P(Q) = \alpha - (\beta/M)\ Q$. Firms are indexed by i. If firm i is actively producing in the market its cost function is $C(q_i, \theta_i) = \theta_i\ q_i + (\gamma/2)\ q_i^2$ such that its marginal cost is $MC_i = \theta_i + \gamma\ q_i$. Variable θ_i represents a random shock that is private information of firm i. These random shocks are i.i.d. over firms with a distribution with mean μ_θ and variance σ_θ^2 that are common knowledge for all the firms. Every active firm producing in the market should pay a finxed cost $F > 0$.

The model is a two-stage game. At the first stage, firms decide whether to enter the market or not. If a firm decides to enter, it pays a fixed cost $F > 0$. When a firm makes its entry decision it does not know yet the realization of it idiosyncratic θ_i. Therefore, the entry decision is based on the maximization of expected profits. At the second stage, each active firm i that has decided to enter observes its own θ_i but not the θ's of the other active firms, and compete according to a Bayesian Nash-Cournot equilibrium.

We now solve the equilibrium of the model starting at the second stage. Suppose that there are n firms active in the market. The expected profit of firm i is:

$$\begin{aligned}\pi_i(\theta_i) &= \mathbb{E}\left[P(Q) \mid \theta_i\right]\ q_i - \theta_i\ q_i - \frac{\gamma}{2}q_i^2 \\ &= \left(\alpha - \beta_M\ \left(q_i + \mathbb{E}\left[\sum_{j\neq i} q_j\right]\right)\right)\ q_i - \theta_i\ q_i - \frac{\gamma}{2}q_i^2\end{aligned} \tag{4.1}$$

where $\beta_M \equiv \beta/M$, and the expectation $\mathbb{E}\left[.\right]$ is over the distribution of the variables θ_j for $j \neq i$, which are not known to firm i. A *Bayesian Nash Equilibrium (BNE)* is an n-tuple of of strategy functions, $[\sigma_1(\theta_1),\ \sigma_2(\theta_1),\ ...,\sigma_n(\theta_n)]$, such that for every firm i:

$$\sigma_i(\theta_i) = \arg\max_{q_i}\ \mathbb{E}\left[P(Q) \mid \theta_i,\, \sigma_j \text{ for } j\neq i\right]\ q_i - \theta_i\ q_i - \frac{\gamma}{2}q_i^2 \tag{4.2}$$

We first order condition of optimality for the best response of firm i implies:

$$q_i = \sigma_i(\theta_i) = \left[\gamma + 2\beta_M\ \right]^{-1}\left[\alpha - \theta_i - \beta_M\ \sum_{j\neq i}\mathbb{E}\left(\sigma_j(\theta_j)\right)\right] \tag{4.3}$$

Since firms are identical up to the private information θ_i, it seems reasonable to focus on symmetric BNE such that $\sigma_i(\theta_i) = \sigma(\theta_i)$ for every firm i. Imposing this restriction in the best response condition (4.3), taking expectations over the distribution of θ_i, and solving for $\sigma^e \equiv \mathbb{E}\left(\sigma(\theta_i)\right)$, we obtain that:

$$\sigma^e \equiv \mathbb{E}\left(\sigma(\theta_i)\right) = \left[\gamma + \beta_M\ (n+1)\right]^{-1}\left[\alpha - \mu_\theta\right] \tag{4.4}$$

And solving this expression in (4.3), we obtain the following closed-form expression for the equilibrium strategy function under BNE:

$$q_i = \sigma(\theta_i) = \frac{\alpha - \mu_\theta}{\gamma + \beta_M\ (n+1)} - \frac{\theta_i - \mu_\theta}{\gamma + 2\beta_M} \tag{4.5}$$

Under this equilibrium, the expected profit of an active firm, before knowing the realization of θ_i is:

$$\mathbb{E}\left[\pi(\theta_i)\right] = \left[\beta_M + \gamma/2\right]\ \mathbb{E}\left[\sigma(\theta_i)^2\right] = \frac{\left[\alpha - \mu_\theta^2\right]}{\left[\gamma + \beta_M\ (n+1)\right]^2} + \frac{\sigma_\theta^2}{\left[\gamma + 2\beta_M\right]^2} \tag{4.6}$$

Given this expected profit, we can obtain the the equilibrium number of entrants in the first stage of the game. Given a market of size M, the free-entry number of firms $n^*(M)$ is approximated by the solution to $\mathbb{E}\left[\pi(\theta_i)\right] - F = 0$. Given the expression for the equilibrium profit, it is simple to verify that $n^*(M)$ is of the same order as market size M. That is, the ratio $n^*(M)/M$ of the firms per consumer is bounded away from zero and infinity. It is interesting to compare this equilibrium to the Cournot equilibrium with complete information (CI). In this full information model, we have that:

$$q_i^{CI} = \frac{\alpha - \widetilde{\theta}_n}{\gamma + \beta_M\ (n+1)} - \frac{\theta_i - \widetilde{\theta}_n}{\gamma + 2\beta_M} \tag{4.7}$$

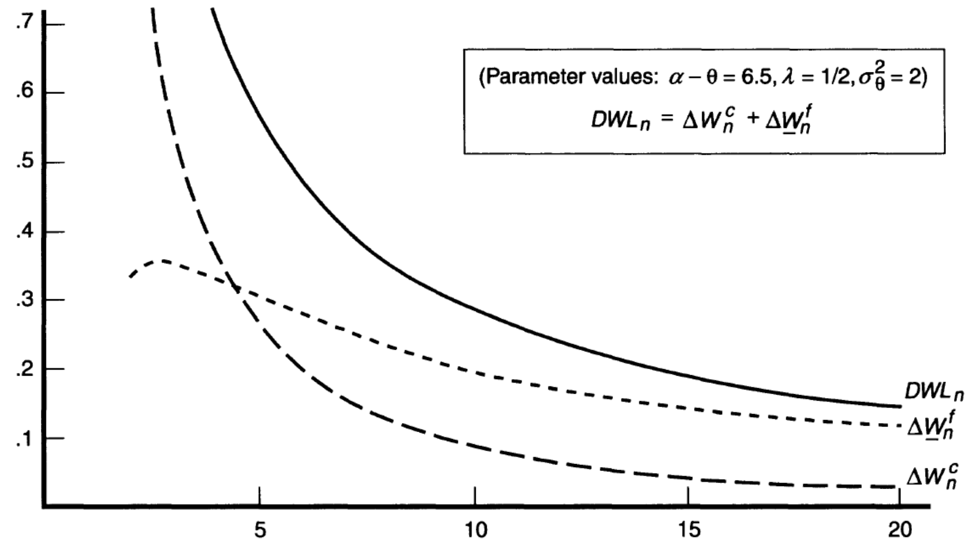
where $\widetilde{\theta}_n \equiv (n-1)^{-1} \sum_{j \neq i} \theta_j$.

From the point of view of a social planner, the optimal allocation in this industry can be achieved if firms share all their information and behave as price takers. Let us label this equilibrium as $CI - PT$ form *complete information* with *price taking* behavior. If p and W are the price and the total welfare, respectively, under the "true" model (with both Cournot conduct and private information), then the differences $p - p_{CI-PT}$ and $W - W_{CI-PT}$ represent the combined effect of incomplete information and Cournot behavior on prices and on welfare. To measure the specific effects of incomplete information and Cournot behavior, it is convenient to define two models. A model that considers Cournot competition in the market but ignores the existence of asymmetric information, that we label as CI for complete information. And a model that considers incomplete information but assumes that firms are price takers, that we label as PT for complete information. Consider the decomposition:

$$\begin{aligned} p - p_{CI-PT} &= [p - p_{PT}] + [p_{PT} - p_{CI-PT}] \\ W_{CI-PT} - W &= [W_{CI-PT} - W_{PT}] + [W_{PT} - W] \end{aligned} \tag{4.8}$$

The term $p - p_{PT}$ captures the effect of Cournot behavior (market power) on prices, and the term $p_{PT} - p_{CI-PT}$ captures the effect of incomplete information. Similarly, $W_{CI-PT} - W$ is the total deadweight loss, $[W_{CI-PT} - W_{PT}]$ is the contribution of incomplete information, and $[W_{PT} - W]$ is the constribution of Cournot competition. Note that this is one of different ways we can decompose these effects. For instance, we could also consider the decompositions, $p - p_{CI-PT} = [p - p_{CI}] + [p_{CI} - p_{CI-PT}]$ and $W_{CI-PT} - W = [W_{CI-PT} - W_{CI}] + [W_{CI} - W]$. The main results are the same using one or the other decomposition.

In this model, as market size M (and therefore n) goes to inifinity, these differences go to zero: $[p - p_{CI-PT}] \longrightarrow 0$ and $\dfrac{W_{CI-PT} - W}{M} \longrightarrow 0$. As market size increases, market price and welfare per capita converge to the optimal allocation. That is, private information and Cournot behavior have an effect only when the market is not too large. **Main result.** There is a critical value for market size, M^* (that depends on the values of structural parameters), such that the effect of private information, on prices and consumer welfare, dominates the effect of market power if and only if market size is greater than this threshold value.



Policy implications. Antitrust authorities look with suspicion the information exchanges between firms because they can help collusive agreements. The collusion concern is most important in the presence of a few players because collusion is easier to be sustained in this case.(repeated game). This paper shows that with few firms market power (Cournot) has the most important contribution to the DWL, so it seems reasonable to control these information exchanges. When market size and the number of firms increase, information asymmetry becomes a more important factor in the the DWL. In this case, it seems optimal to allow for some information sharing between firms.

4.2. Cournot competition with private information: Application. Armantier and Richard (2003) study empirically how firms' asymmetric information on marginal costs affects competition and outcomes in the US airline industry. They also investigate how marketing alliances that facilitate information sharing can affect competition.

They investigate American Airlines' (AA) and United Airlines' (UA) duopoly petition at Chicago O'Hare airport during the third quarter of 1993

5. Exercises

5.1. Exercise 1. Consider an industry with a differentiated product. There are two firms in this industry, firms 1 and 2. Each firm produces and sells only one brand of the differentiated product: brand 1 is produced by firm 1, and brand 2 by firm 2. The demand system is a logit demand model, where consumers choose between three different alternatives: $j = 0$, represents the consumer decision of no purchasing any product; and $j = 1$ and $j = 2$ represent the consumer purchase of product 1 and 2, respectively. The utility of no purchase $(j = 0)$ is zero. The utility of purchasing product $j \in \{1, 2\}$ is $\beta\ x_j - \alpha\ p_j + \varepsilon_j$, where the variables and parameters have the interpretation that we have seen in class. Variable x_j is a measure of the quality of product j, for instance, the number of stars of the product according to consumer ratings. Therefore, we have that $\beta > 0$. The random variables ε_1 and ε_2 are independently and identically distributed over consumers with a type I extreme value distribution, that is, Logit model of demand. Let H be the number of consumers in the market. Let s_0, s_1, and s_2 be the market shares of the three choice alternatives, such that s_j represents the proportion of consumers choosing alternative j and $s_0 + s_1 + s_2 = 1$.

Question 1.1. Based on this model, write the equation for the market share s_1 as a function of the prices and the qualities x's of all the products.

Question 1.2. Obtain the expression for the derivatives: (a) $\dfrac{\partial s_1}{\partial p_1}$; (b) $\dfrac{\partial s_1}{\partial p_2}$; (c) $\dfrac{\partial s_1}{\partial x_1}$; and (d) $\dfrac{\partial s_1}{\partial x_2}$. Write the expression for these derivatives in terms only of the market shares s_1 and s_2 and the parameters of the model.

The profit function of firm $j \in \{0, 1\}$ is $\pi_j = p_j\ q_j - c_j\ q_j - FC(x_j)$, where: q_j is the quantity sold by firm j (that is, $q_j = H\ s_j$); c_j is firm $j's$ marginal cost, that is assumed constant, that is, linear cost function; and $FC(x_j)$ is a fixed cost that depends on the level of quality of the firm.

Question 1.3. Suppose that firms take their qualities x_1 and x_2 as given and compete in prices ala Bertrand.
(a) Obtain the equation that describes the marginal condition of profit maximization of firm 1 in this Bertrand game. Write this equation taking into account the specific form of $\dfrac{\partial s_1}{\partial p_1}$ in the Logit model.
(b) Given this equation, write the expression for the equilibrium price-cost margin $p_1 - c_1$ as a function of s_1 and the demand parameter α.

Now, suppose that the researcher is not willing to impose the assumption of Bertrand competition and considers a conjectural variations model. Define the conjecture parameter CV_1

as the belief or conjecture that firm 1 has about how firm 2 will change its price when firm 1 changes marginally its price. That is, CV_1 represents the belief or conjecture of firm 1 about $\frac{\partial p_2}{\partial p_1}$. Similarly, CV_2 represents the belief or conjecture of firm 2 about $\frac{\partial p_2}{\partial p_1}$.

Question 1.4. Suppose that firm 1 has a conjectural variation CV_1.

(a) Obtain the equation that describes the marginal condition of profit maximization of firm 1 under this conjectural variation. Write this equation taking into account the specific form of $\frac{\partial s_1}{\partial p_1}$ in the Logit model. [Hint: Now, we have that: $\frac{dq_1}{dp_1} = \frac{\partial q_1}{\partial p_1} + \frac{\partial q_1}{\partial p_2}\frac{\partial p_2}{\partial p_1}$, where $\frac{\partial q_1}{\partial p_1}$ and $\frac{\partial q_1}{\partial p_2}$ are the expressions you have derived in Q1.2].

(b) Given this equation, write the expression for the equilibrium price-cost margin $p_1 - c_1$ as a function of the market shares s_1 and s_2, and the parameters α and CV_1.

Question 1.5. Suppose that the researcher does not know the magnitude of the marginal costs c_1 and c_2, but she knows that the two firms use the same production technology, they use the same type of variable inputs, and they purchase these inputs in the same markets where they are price takers. Under these conditions, the researcher knows that $c_1 = c_2 = c$, though she does not know the magnitude of the marginal cost c.

(a) The marginal conditions for profit maximization in Q1.4(b), for the two firms, together with the condition $c_1 = c_2 = c$, imply that price difference between these two firms, $p_1 - p_2$, is a particular function of their markets shares and their conjectural variations. Derive the equation that represents this condition.

(b) The researcher observes prices $p_1 = \$200$ and $p_2 = \$195$ and market shares $s_1 = 0.5$ and $s_2 = 0.2$. Firm 1 has both a larger price and a larger market share because its product has better quality, that is, $x_1 > x_2$. The researcher has estimated the demand system and knows that $\alpha = 0.01$. Plug in these data into the equation in Q1.5(a) to obtain a condition that the parameters CV_1 and CV_2 should satisfy in this market.

(c) Using the equation in Q1.5(b), show that the hypothesis of Nash-Bertrand competition (that requires $CV_1 = CV_2 = 0$) implies a prediction about the price difference $p_1 - p_2$ that is substantially larger than the price difference that we observe in the data.

(d) Using the equation in Q1.5(b), show that the hypothesis of Collusion (that requires $CV_1 = CV_2 = 1$) implies a prediction about the price difference $p_1 - p_2$ that is much closer to the price difference that we observe in the data.

5.2. Exercise 2. To answer the questions in this part of the problem set you need to use the dataset `verboven_cars.dta` Use this dataset to implement the estimations describe below. Please, provide the STATA code that you use to obtain the results. For all the models that you estimate below, impose the following conditions:

- For market size (number of consumers), use Population/4, that is, `pop/4`
- Use prices measured in euros (`eurpr`).
- For the product characteristics in the demand system, include the characteristics: `hp, li, wi, cy, le,` and `he.`
- Include also as explanatory variables the market characteristics: ln(pop) and log(gdp).
- In all the OLS estimations include fixed effects for market (`ma`), year (`ye`), and brand (`brd`).
- Include the price in logarithms, that is, ln(`eurpr`).
- Allow the coefficient for log-price to be different for different markets (countries). That is, include as explanatory variables the log price, but also the log price interacting (multiplying) each of the market (country) dummies except one country dummy (say the dummy for Germany) that you use as a benchmark.

Question 2.1.
(a) Obtain the OLS-Fixed effects estimator of the Standard logit model. Interpret the results.
(b) Test the null hypothesis that all countries have the same price coefficient.
(c) Based on the estimated model, obtain the average price elasticity of demand for each country evaluated at the mean values of prices and market shares for that country.

Question 2.2. Consider the equilibrium condition (first order conditions of profit maximization) under the assumption that each product is produced by only one firm.
(a) Write the equation for this equilibrium condition. Write this equilibrium condition as an equation for the Lerner Index, $\dfrac{p_j - MC_j}{p_j}$.
(b) Using the previous equation in Q2.2(a) and the estimated demand in Q2.1, calculate the Lerner index for every car-market-year observation in the data.
(c) Report the mean values of the Lerner Index for each of the counties/markets. Comment the results.
(d) Report the mean values of the Lerner Index for each of the top five car manufacturers (that is, the five car manufacturers with largest total aggregate sales over these markets and sample period). Comment the results.

Question 2.3.
(a) Using the equilibrium condition and the estimated demand, obtain an estimate of the marginal cost for every car-market-year observation in the data.
(b) Run an OLS-Fixed effects regression where the dependent variable is the estimated value of the marginal cost, and the explanatory variables (regressors) are the product characteristics `hp, li, wi, cy, le,` and `he.` Interpret the results.

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